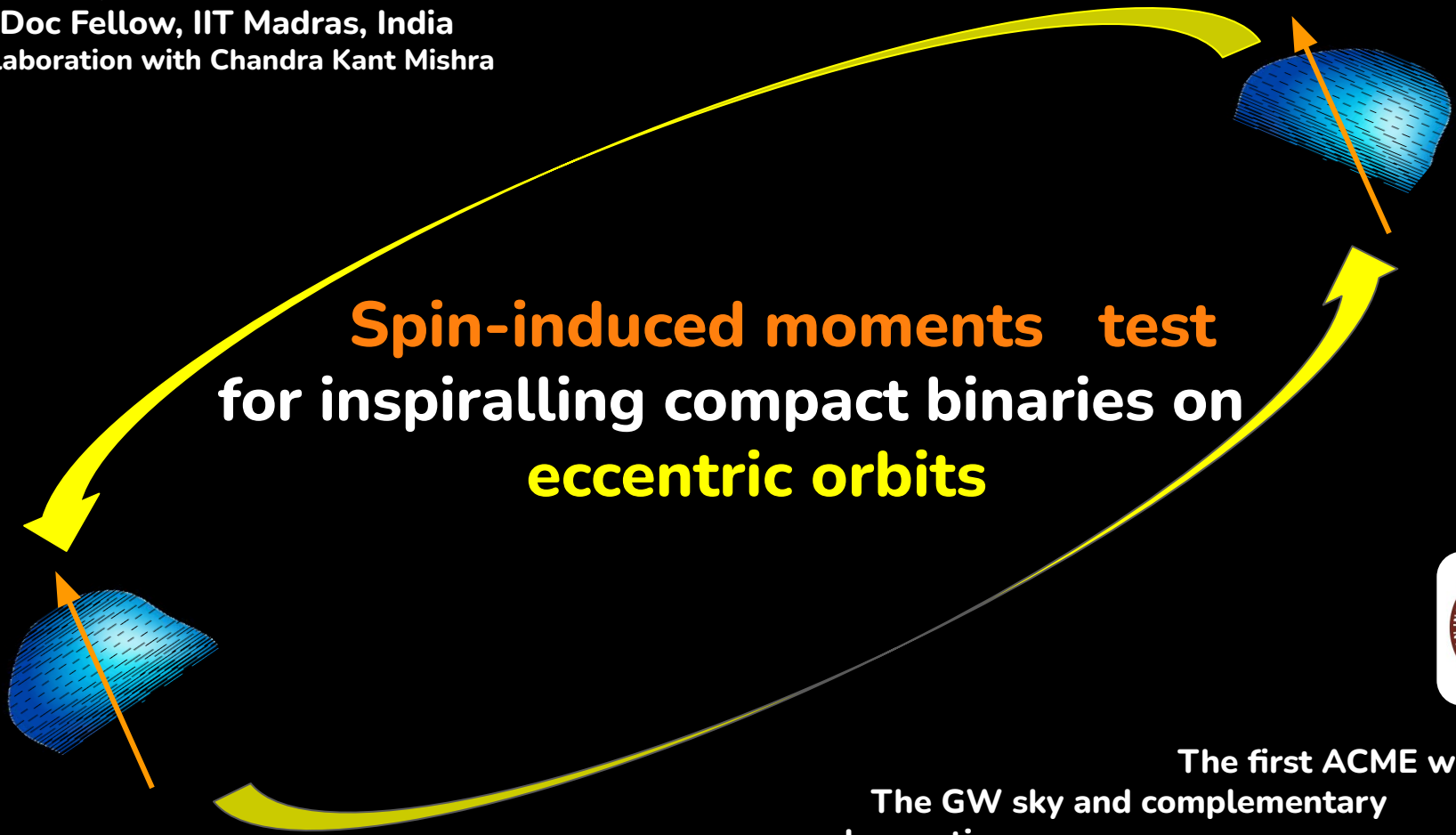
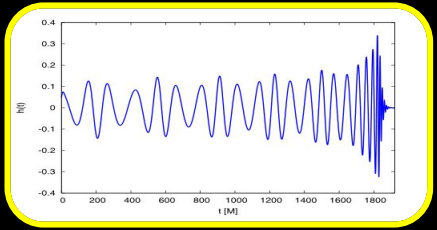
Syed Naqvi,
Post-Doc Fellow, IIT Madras, India
In Collaboration with Chandra Kant Mishra

Spin-induced moments test for inspiralling compact binaries on eccentric orbits



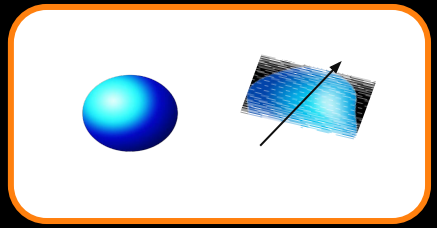
The first ACME workshop
The GW sky and complementary observations
Toulouse, France 7th-11th April 2025

Eccentricity



Test of Compact Binaries

SIQM



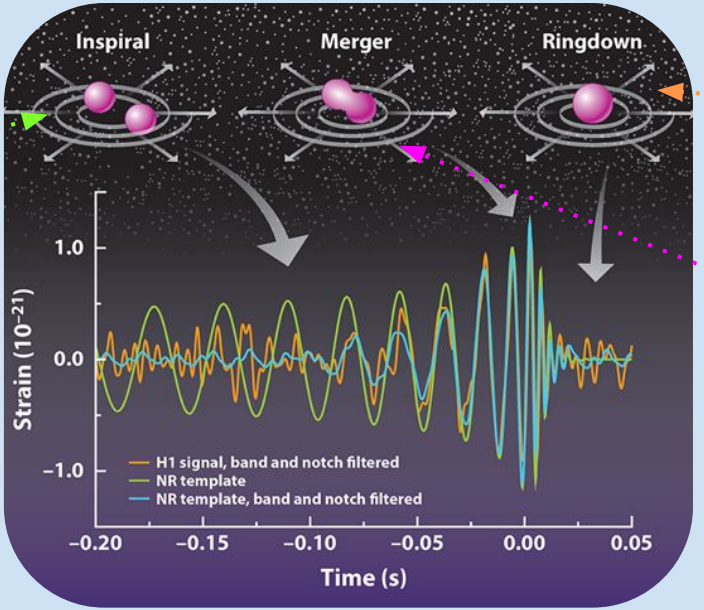
- Motivation : Eccentric GWs + Spin Induced Moments
- Post-Newtonian Waveform: SIQM (κ-parameter) +Ecc
- Fisher Matrix Analysis: Errors on κ

  i) 3.5PN Spinning Circular + 3PN Non-Spin Eccentricity

  ii)3.5PN Spinning Circular + 3PN (Eccentricity + κ)-dependent terms

GWs : Methods to compute templates

Post-Newtonian Theory



BH Perturbation Theory

Numerical Rel.

Eccentricity

$$h = Ae^{i\Psi}$$

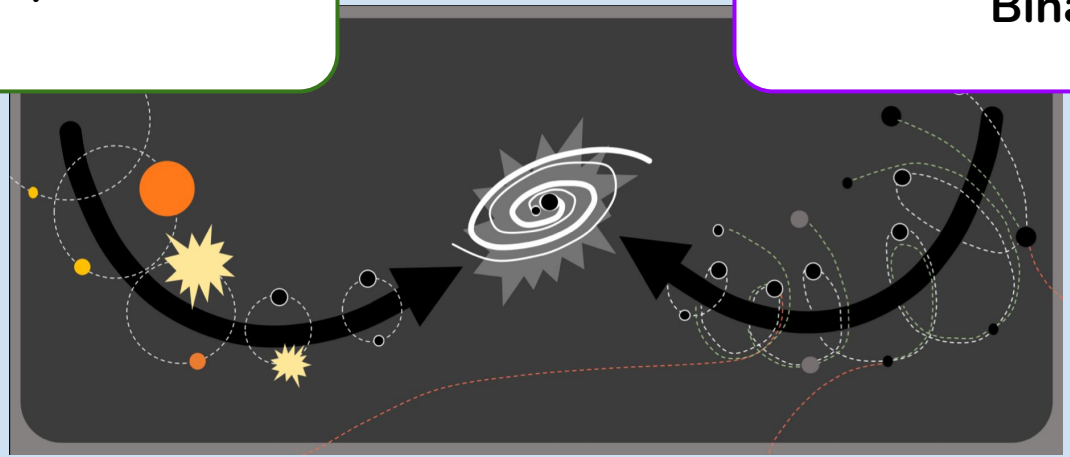
SIQM

Gravitational Waves and the Effort to Detect them By Peter Shawhan

GWs: The case for Eccentricity

Isolated Binary Evolution

- Shed their formation eccentricity
- Circularise when entering detector



Dynamically formed Binaries

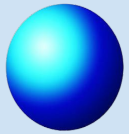
- Retain eccentricity
- In globular clusters, AGN...

*issue of model uncertainties + lack of GW statistics : BBH formation scenarios ???

Searching for eccentricity: Isobel M Romero-Shaw, 2019

Mapelli, Binary Black Hole Mergers: Formation and Populations

GWs: The case for SIQM



The spinning motion of companion A creates a distortion in its mass distribution => creates a distortion in the gravitational field outside the star, measured by Q_{ab}

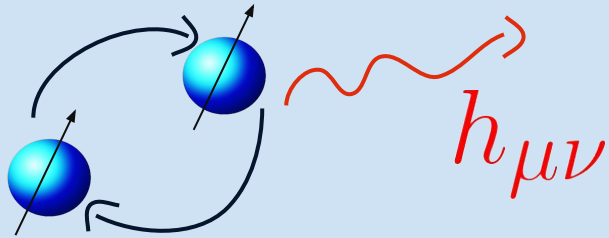
The quadrupole term, Q_{ab}, in the gravitational potential affects the orbital motion of the companions, and it affects also the emission of GWs.

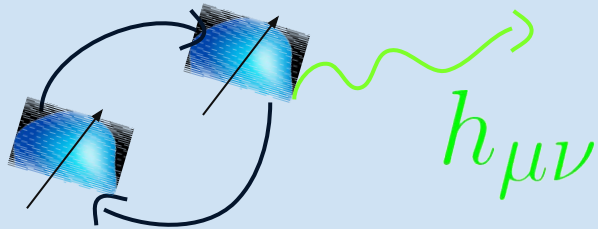
Poisson '98, Kidder '95

$$Q_A = -\kappa \chi_A^2 m_A^3$$

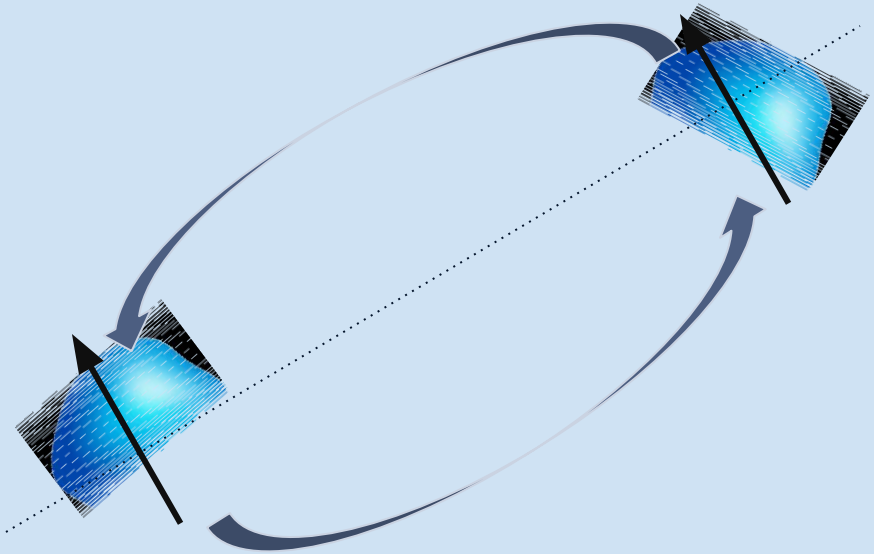
κ = 1, BH
κ ≠ 1, N.S etc

Kappa: Spin-induced moment constant





Testing the Compact binary nature: SIQM + Ecc



No-hair theorem test ➡ Is the object truly a Kerr BH?

Astrophysical identity test ➡ Neutron star vs boson star vs BH

Modified gravity/GR test ➡ Deviations from GR predictions

*References:

- *Post-Newtonian theory for gravitational waves, Living Reviews in Relativity, Luc Blanchet 2024*
- *Sources of Gravitational Waves: Theory and Observations, Alessandra Buonanno and B.S. Sathyaprakash*

PN Waveform: General overview + where SIQM Enters

GW Waveform (Freq. Domain)

$$\tilde{h}(f) = A\psi(f)$$

$$\Psi(f) = \frac{3}{128\eta v^5}\left\{1 + v^2\left[\text{1 PN}\right] + v^3\left[\text{1.5PN } (\beta)\right] + v^4\left[\text{2 PN } (\sigma)\right] + v^5(\ldots)\right.$$

$$v = (\pi M f)^{1/3} \qquad \eta = \mu/M$$

M = Total Mass
η = symm mass-ratio

Spin-Orbit: $\beta \sim \sum_A \hat{\mathbf{L}}.\chi_\mathbf{A}$

Spin-Spin: $\sigma = \sigma_{ss} + \sigma_{qm}$

$\sigma_{ss} \sim \chi_1 \cdot \chi_2$

κ appears at 2PN, 3PN, 4PN

$$Q_A = -\kappa \chi_A^2 m_A^3$$

SIQM - quasi circular binaries

Why is it imp?

- Probing mass gap bw massive NS and lightest BH
- Existence of BH mimickers (boson stars, κ ~ 10-100), gravastars
- Test of GR (Krishnendu 2022, arxiv:2201.05418)

Earlier Studies

- Krishnendu et al 2017, 2018: semianalytical P.E for κ (2PN Amplitude and 4PN phase)
- Divyajyoti et al 2024: Bayesian param. est. for spin-precession + higher order
- Lyu et al 2024: compare SIQM for precc + non-precess VS tidal heating based on IMRPhenomXPHM

Future Studies

SIQM for eccentric binaries ?

Inspiral Waveform: Adding Kappa-Ecc Terms*

- Inspiral Waveform : an update to the non-spinning 3PN eccentric phasing (Moore et al)
- Include aligned-spin effects to the small **eccentricity** expanded time and freq domain phases
- Valid for max e0=0.3.

κ_1, κ_2

$$\Psi = \frac{3}{128\nu y^5}\left\{\Psi_{\rm circ} + \frac{650}{731}e_0^2\left(\frac{y_0}{y}\right)^{19/3}\left[\Psi_{\rm SO,ecc} + \Psi_{\rm SS,ecc}\right] + e_0^4(\ldots)\right.$$

Aim

- Kappa errors for ET and Cosmic Explorer and compare with adv. LIGO
- Define symmetric and anti-symm combination: $\kappa_S = (\kappa_1 + \kappa_2)/2$
  (For BBH: κ_S=1, κ_A=0)

*Credit: Omkar, Soham Kaushik 2024

Fisher Analysis: Main Ingredients $\{\theta_i\}$

- Fisher Information Matrix (PSD:) $S_n(f)$ **(ET, CE)**

$$\Gamma_{ij} = 2\int_{f_{\rm lower}}^{f_{\rm upper}} df \frac{\tilde{h}_i(f)\tilde{h}_j^*(f) + \tilde{h}_j(f)\tilde{h}_i^*(f)}{S_n(f)},$$

- Error on each parameter is given by the square root of the diagonal entries of the covariance matrix.

$$\sigma_i = \sqrt{\Sigma_{ii}}.$$

- **Parameter Space** $\theta_i = \{t_c, \phi_c, \mathcal{M}_{\rm chirp}, \eta, \chi 1, \chi 2, \kappa_s, {\rm e0}\}$

Baseline Config:

3.5PN Spinning Circular

(q=1.2, Dist=400Mpc Analytical ET-D)

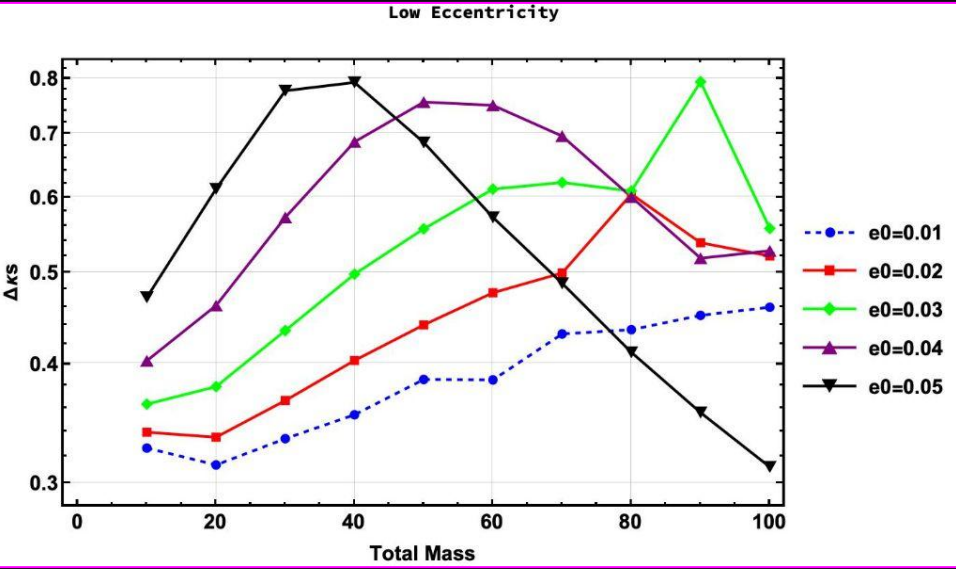
✚

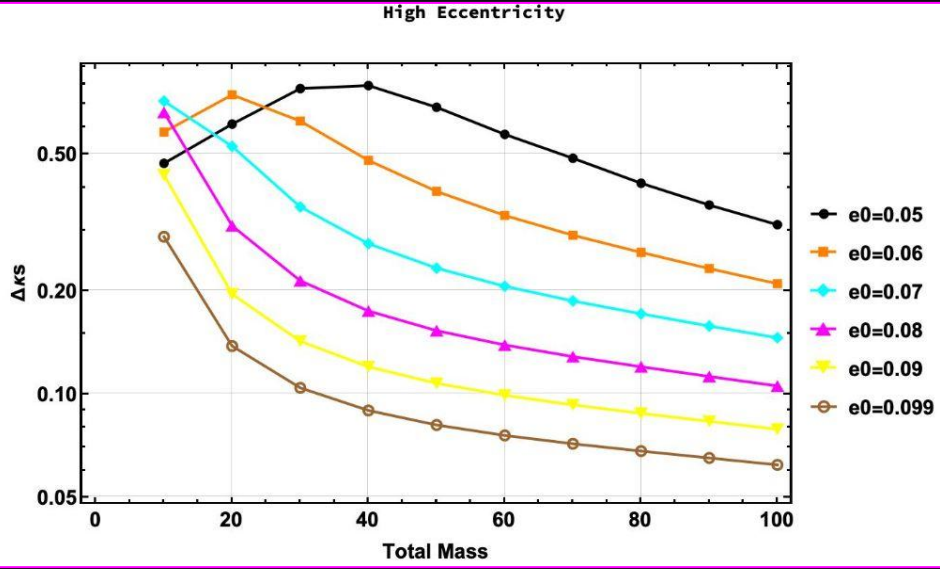
Case I) **Ecc** Terms in param space => Moore et al 3PN - Ecc terms

Case II) **Ecc-κ** in param space => Omkar et al 3PN - (Ecc+**κ**) terms

q=1.2

3.5 PN Spinning Circular + 3 PN Ecc Moore et al



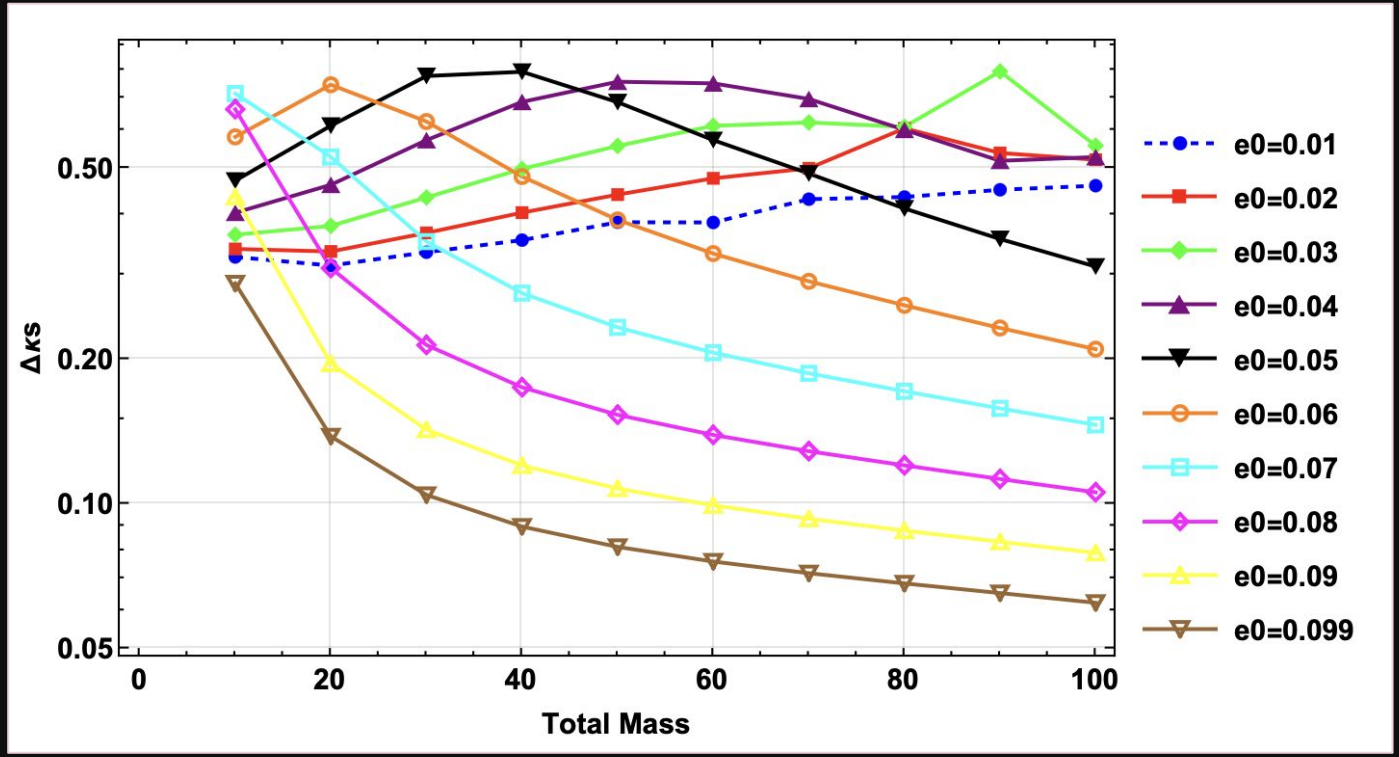


- κ_s Errors increase for low e0 [0.01,0.04] decrease for high e0 [0.05, 0.1]

Credit: arXiv:1605.00304

q=1.2

3.5 PN Spinning Circular + 3 PN Ecc Moore et al

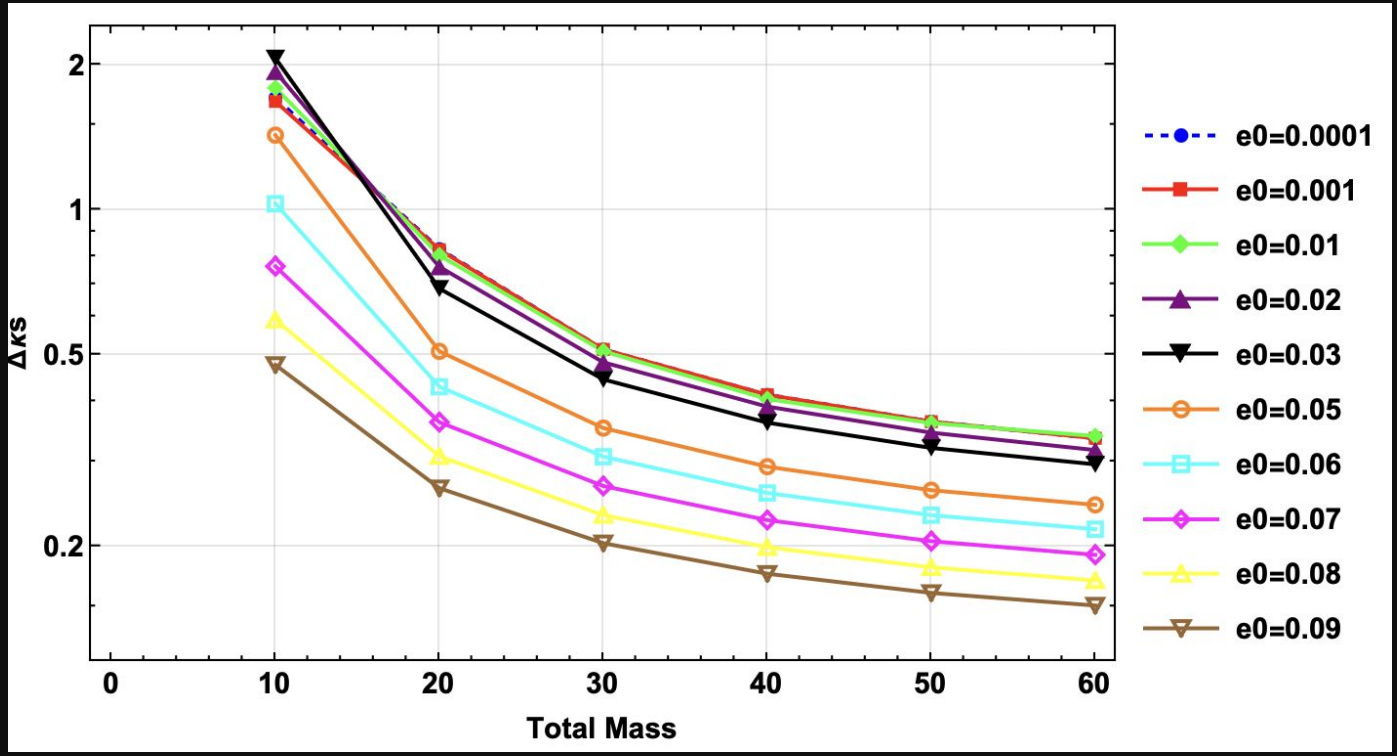


κ_s Errors improving for high e0 towards high total M => retain ecc

Credit: arXiv:1605.00304

q=1.2

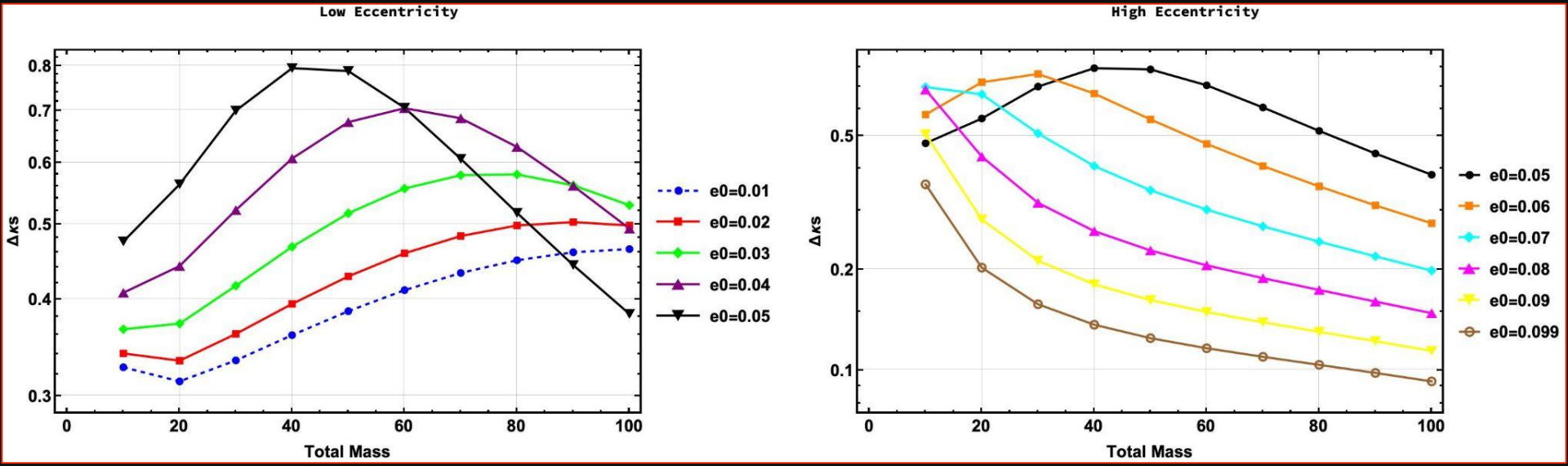
4 PN Spinning Circular + 3 PN Ecc Moore et al



Credit: arXiv:1605.00304

q=1.2

3.5 PN Spinning Circular + 3 PN Ecc-Kappa Omkar et al Including Ecc-Kappa Terms

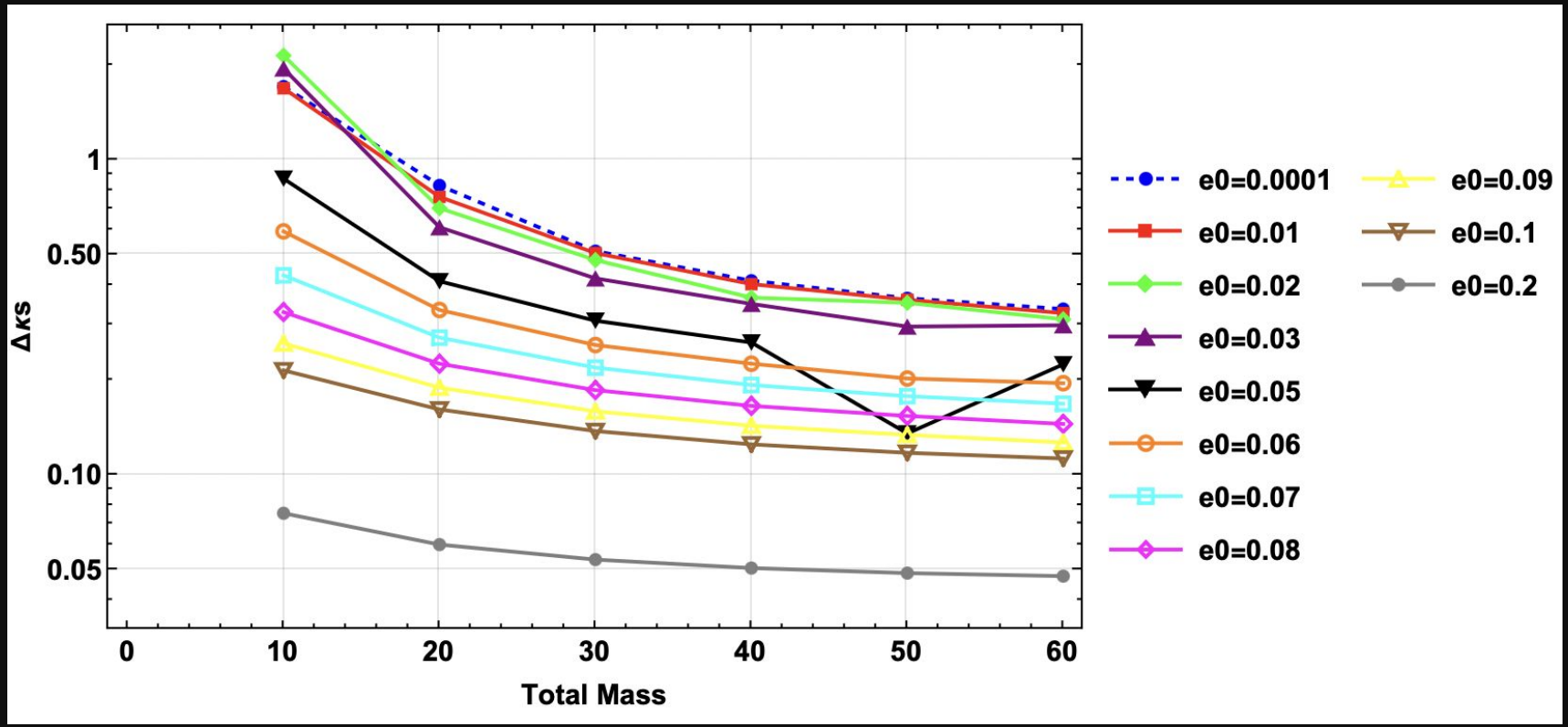


κ_s : Terms in 3.5PN spinning-circular, now also at 2PN Ecc and 3PN Ecc case!

Credit: arXiv:2412.10909

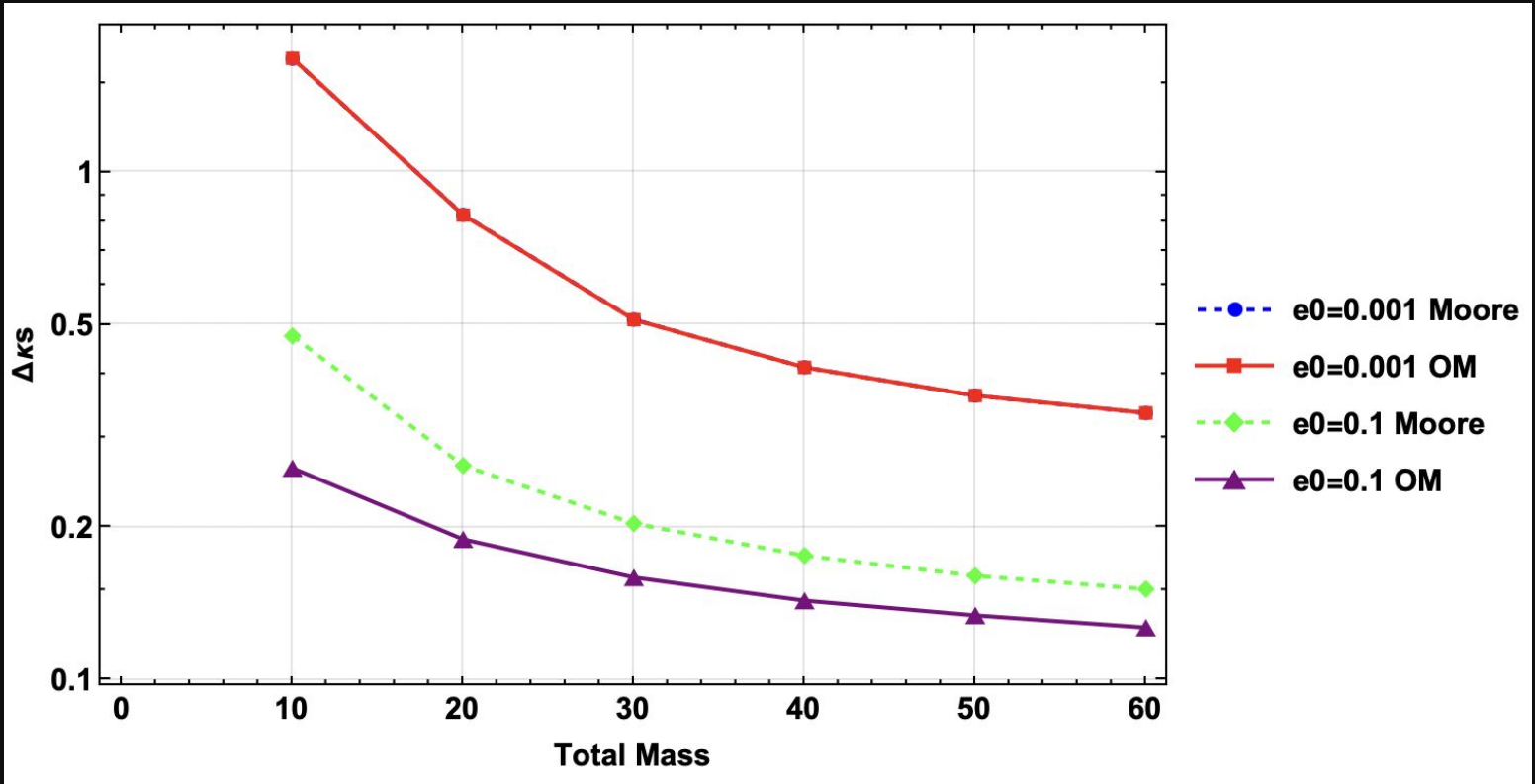
q=1.2

4 PN Spinning Circular + 3 PN Ecc Omkar et al:

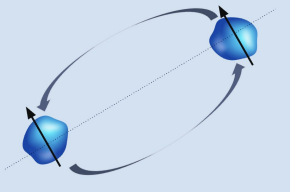


Credit: arXiv:2412.10909

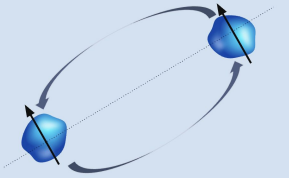
4 PN Spinning Circular: 3 PN Ecc Moore et al VS Omkar



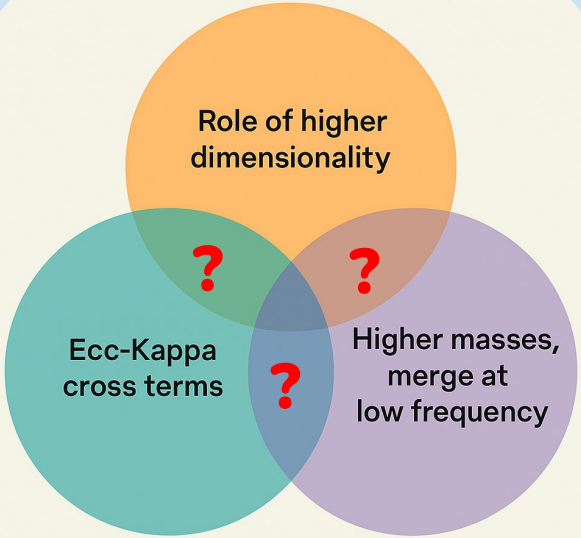
κ_s : Better for Omkar et al. => kappa dependent terms in the Ecc part



Conclusion



- Fisher analysis for SIQM + Ecc (only phasing corrections) =>
  To add amplitude corrections
- What we study: for mass-ratio q=1.2
  - i) 3.5PN spin-circ + 3PN only Ecc leading order (e0)^2
  - ii)3.5PN spin-circ + 3PN Ecc-kappa leading order (e0)^2
- κ_S errors improve with Eccentricity =>
  - i) need for careful treatment of parameter degeneracies !
  - ii) to understand correlation better + Numerical PSD
- Case for (e0)^4 . . .+ Full Bayesian Analysis needed



Thank You!

Many templates ~ O(millions)

Finding the correct template/needle!

Interplay of Eccentricity and Kappa in Fisher Analysis

Higher Dimensionality

- **Explanation**: Adding eccentricity introduces new parameters (like e0e_0e0), increasing dimensionality.
- **Implication**: This may **reduce degeneracies** between parameters like κ\kappaκ, spins, and chirp mass, improving measurability.
- **Caution**: This can also **introduce new degeneracies** (e.g. e0e_0e0 vs spin).

Ecc-Kappa Cross Terms

- **Explanation**: Eccentricity couples to κ\kappaκ in PN phase (e.g. at 3PN). These **mixed terms** modify how κ\kappaκ influences the waveform.
- **Implication**: You gain **new information about κ\kappaκ** via these cross terms, especially at low frequency.

"Higher Masses"

- **Explanation**: High-mass binaries merge earlier → waveform mostly in **low-frequency band**.
- **Implication**: Eccentric corrections (which are stronger at low frequencies) **dominate the waveform**, and thus the Fisher information.