Waveform systematics and sky localization for LISA signals of massive black hole binaries

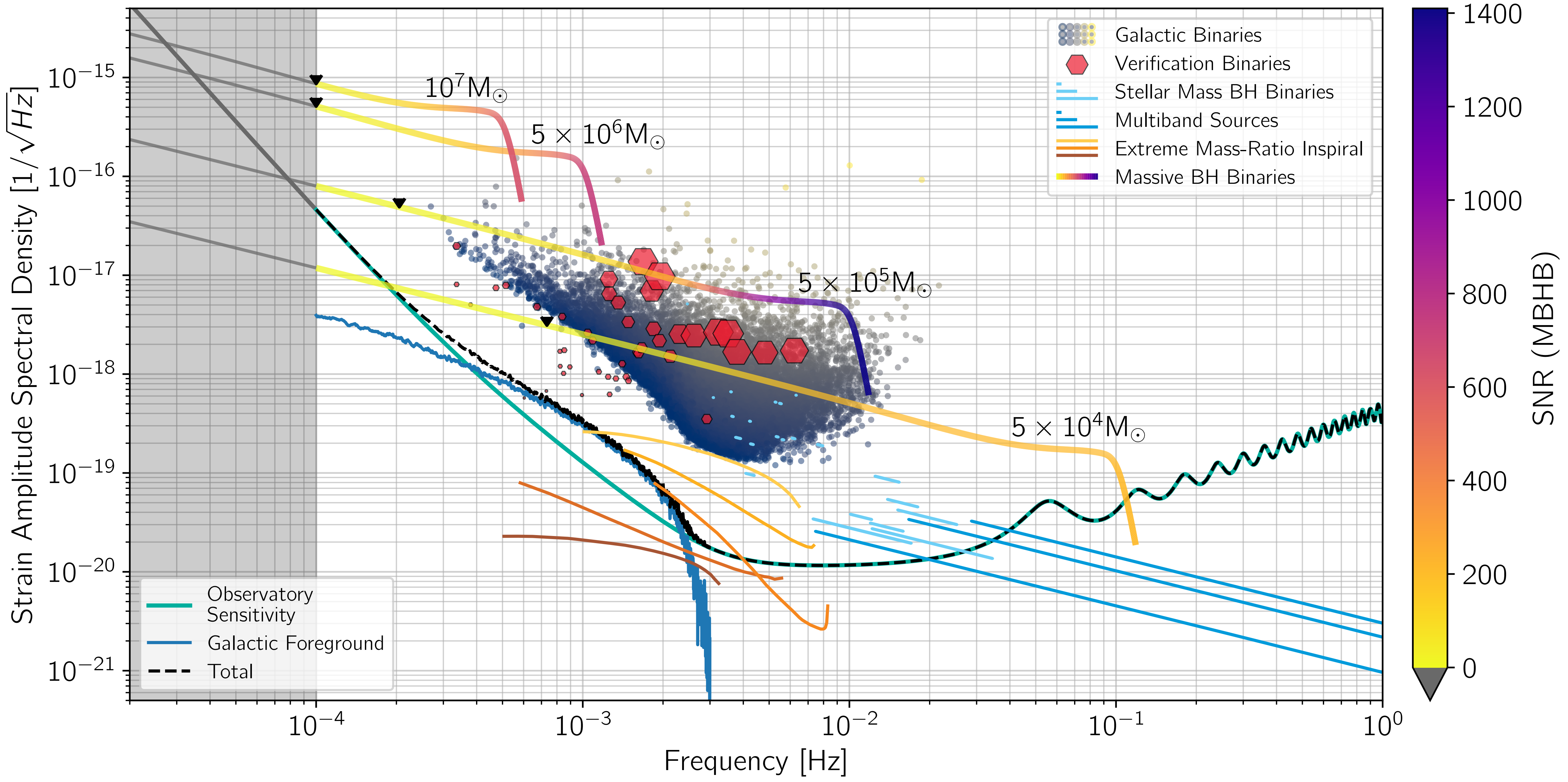
Sylvain Marsat (L2IT, Toulouse)

in collaboration with A. Mangiagli (AEI Potsdam), A. Toubiana (U. Milano Bicocca)

Outline

- **MBHBs and LISA response**
- Sky localization: main mode
- Sky localization: sky degeneracies
- Sky localization: galaxy counts
- Pre-merger sky localization
- Waveform systematics

Massive black hole binaries for LISA

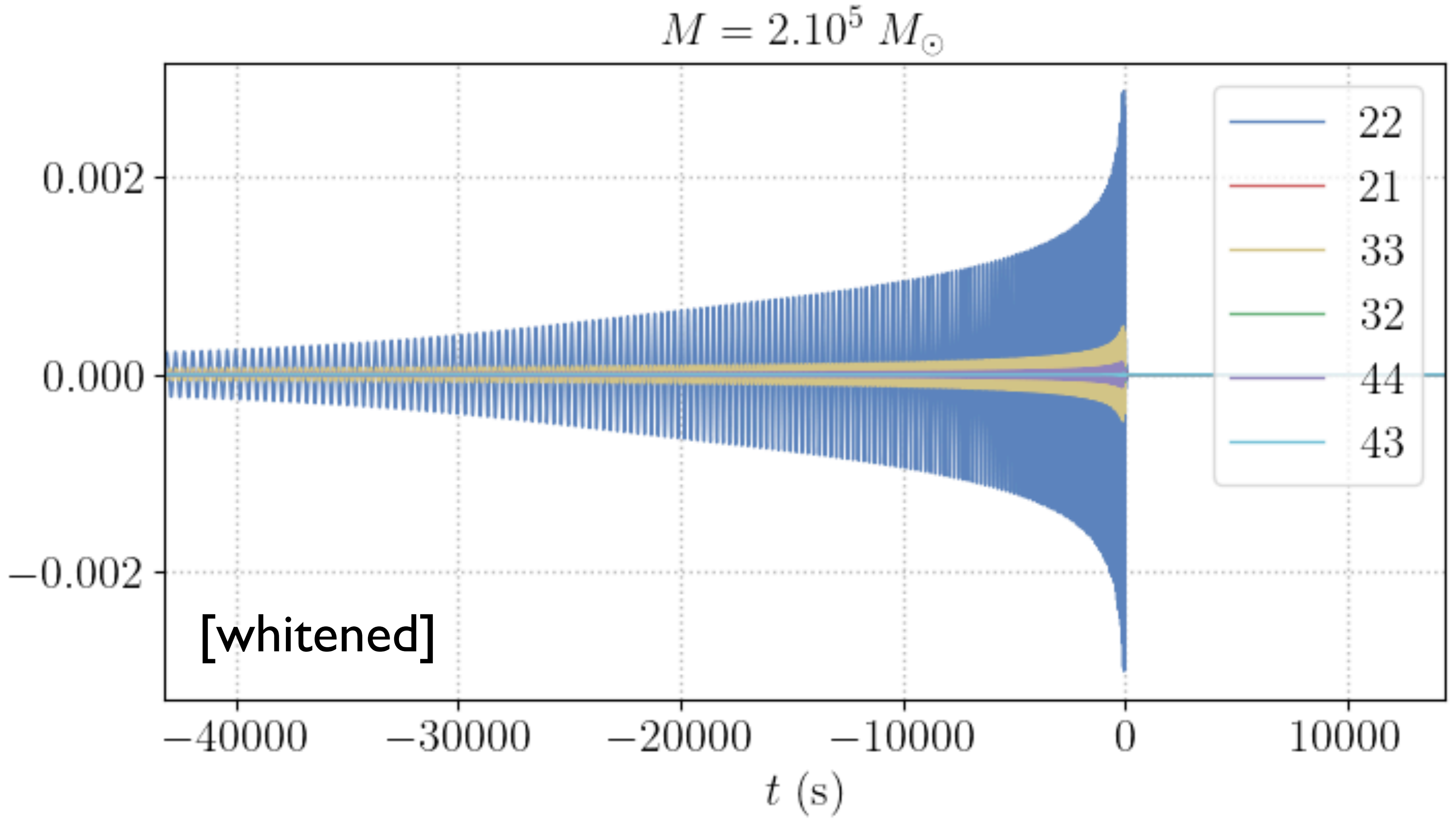


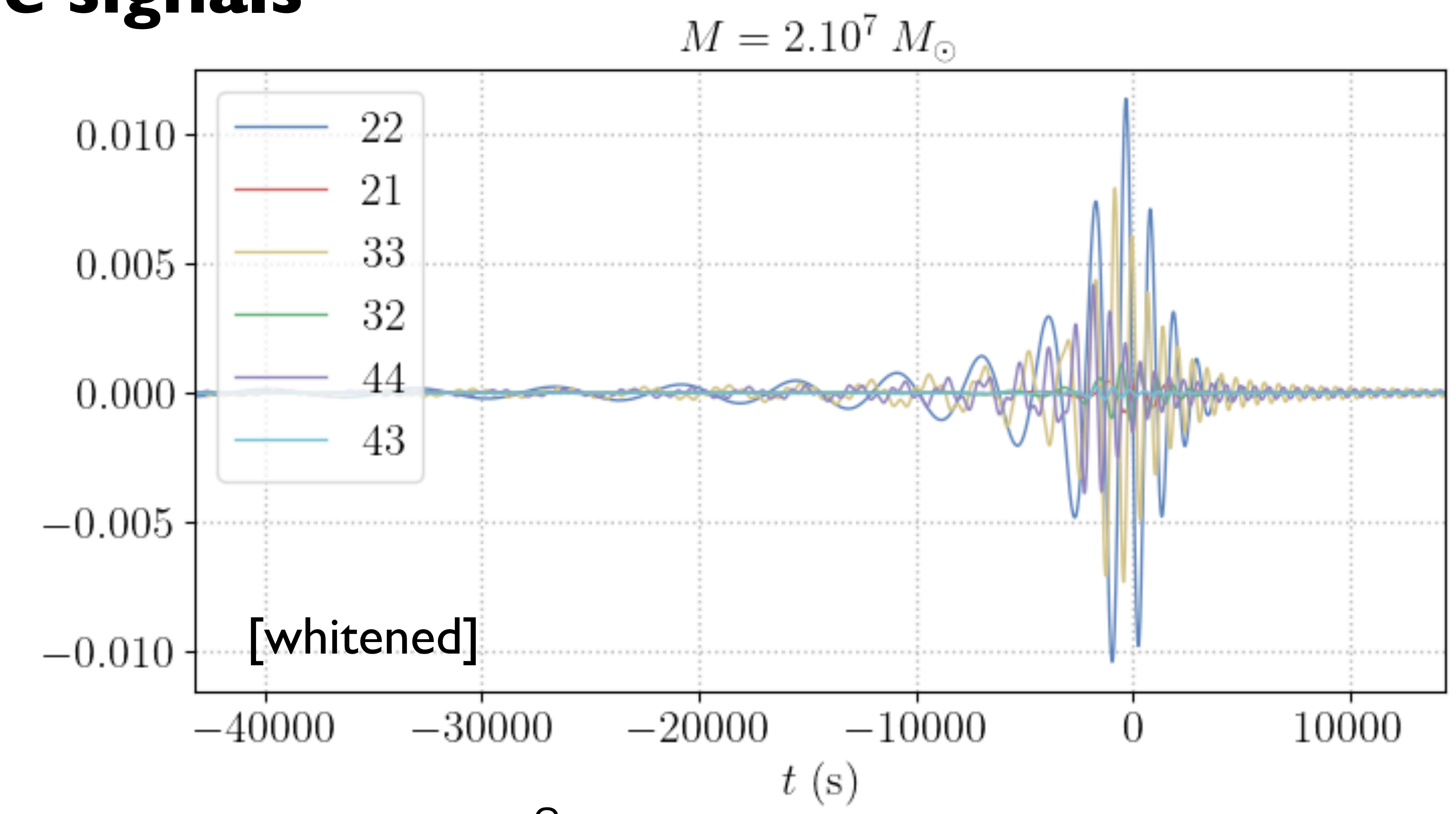
Science case

MBHBs cardinal sources for LISA.

- how well can LISA localize MBHBs ?
- localization of MBHBs in advance of the coalescence ?
- EM counterparts from ‘golden events’: could waveform systematics affect the sky localization ?

Example signals



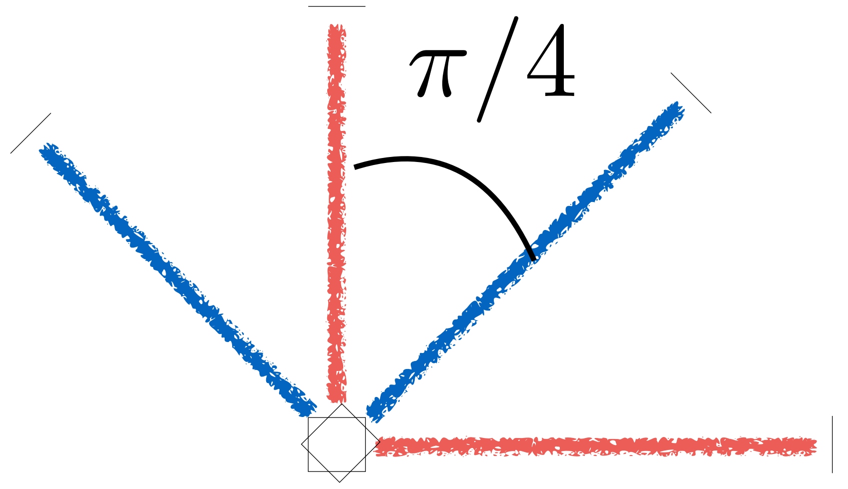


Higher harmonics strong at merger (break degeneracies)

Data analysis simulations still missing precession, eccentricity

The LISA instrumental response

The low-frequency response



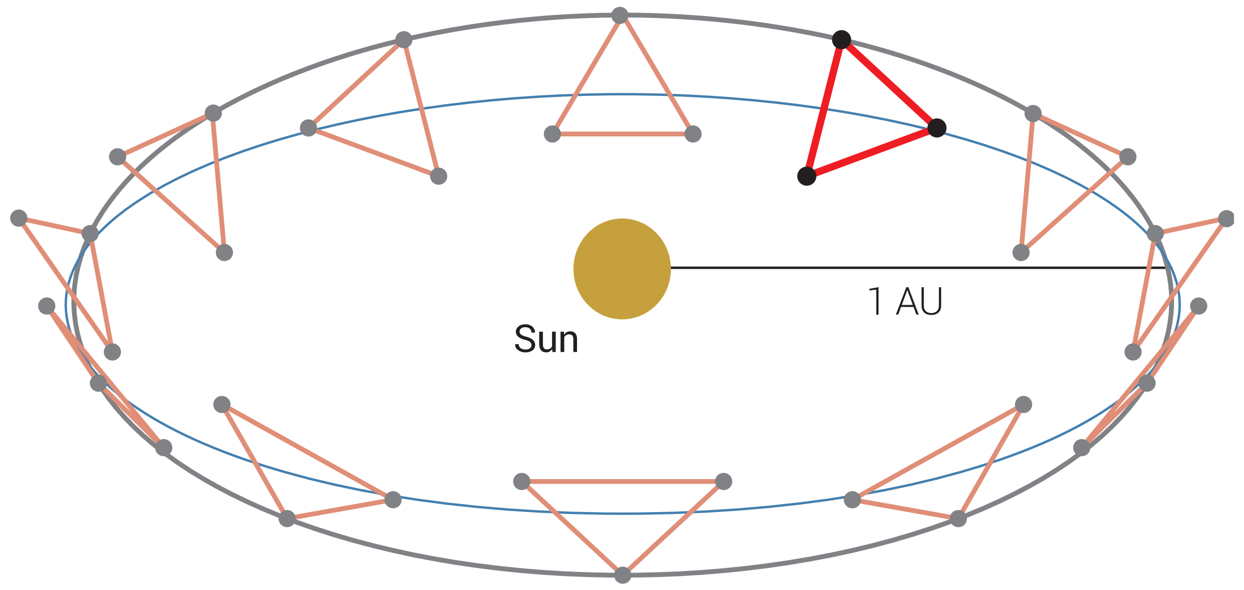
- At low frequencies TDIA, TDIE become equivalent to 2 LIGO-like channels
- For short transients the LISA motion has only a weak effect

Pattern function response with HM:

$$h = \sum_{a,e} \sum_{\ell m} \frac{1}{d} F_{a,e}^{\ell m} h_{\ell m}$$

constant prefactors dependent on geometry

The full response



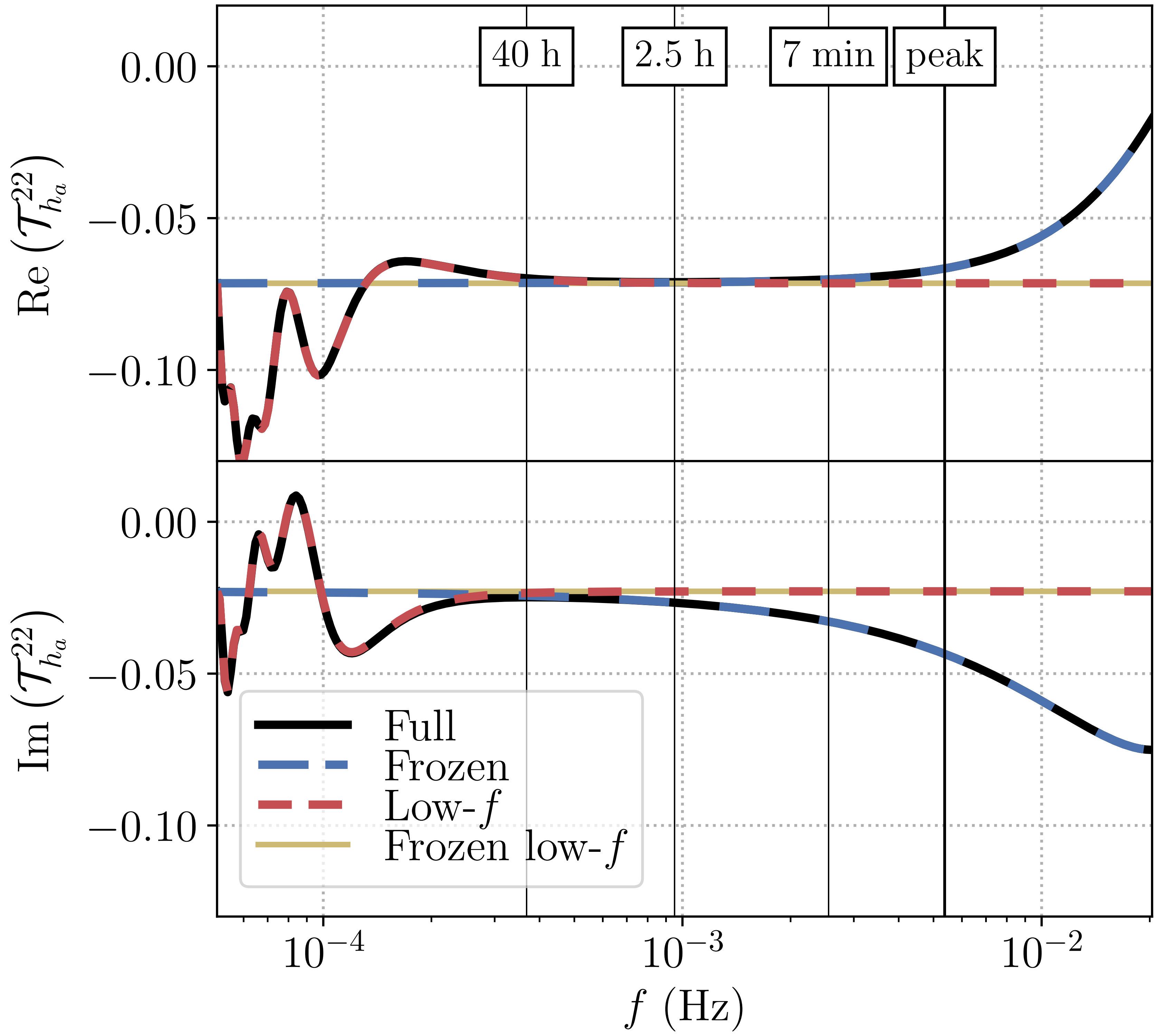
Single-link response:

$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \mathrm{sinc}\left[\pi f L \left(1 - k \cdot n_l\right)\right] \exp\left[i\pi f \left(L + k \cdot (p_r + p_s)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

+ Doppler phase: $\exp\left[2i\pi f k \cdot p_0(t_f)\right]$ + TDI combinations

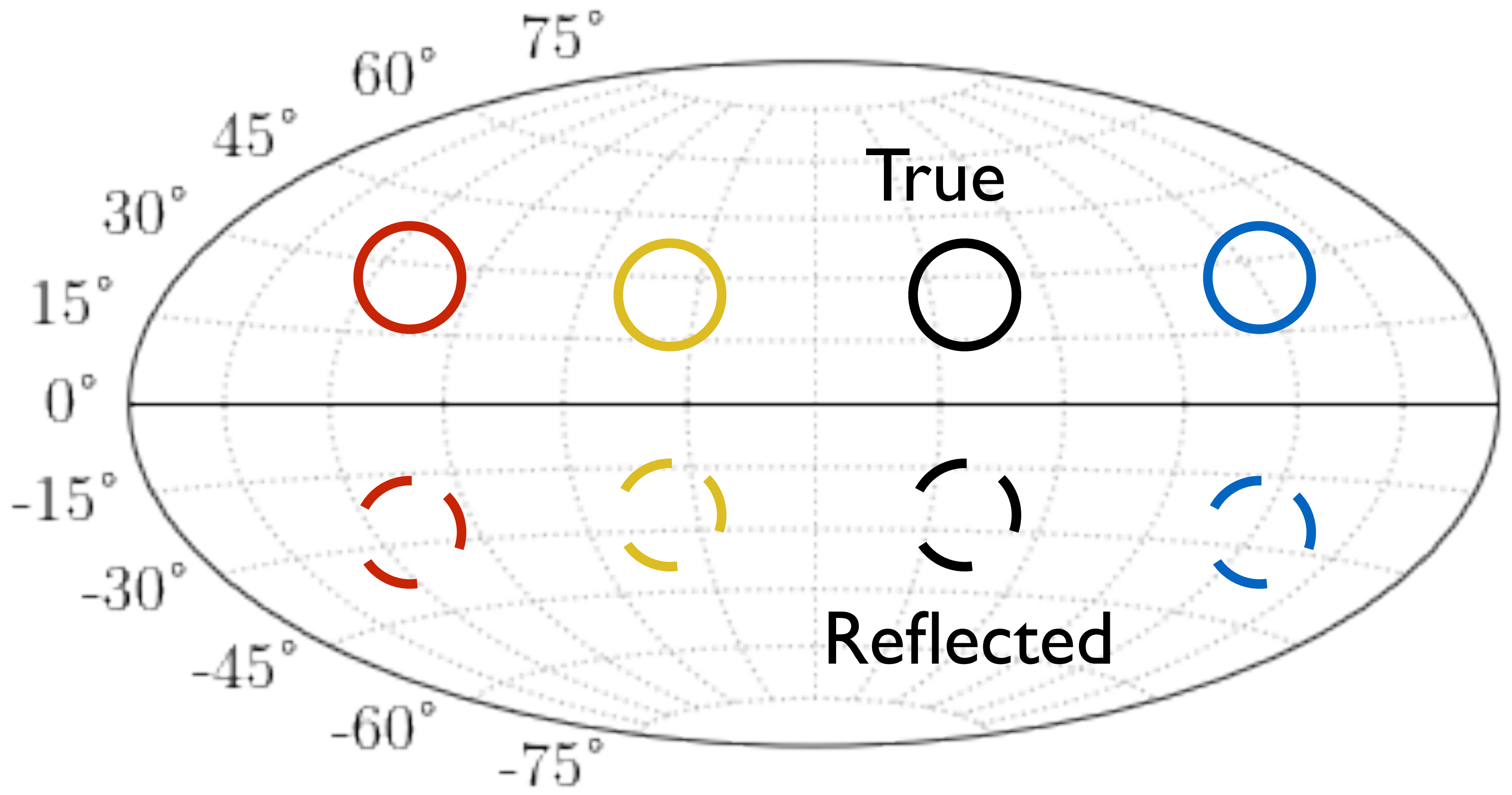
Time and frequency-dependency in transfer functions
Time: motion of LISA on its orbit
Frequency: departure from long-wavelength approx.

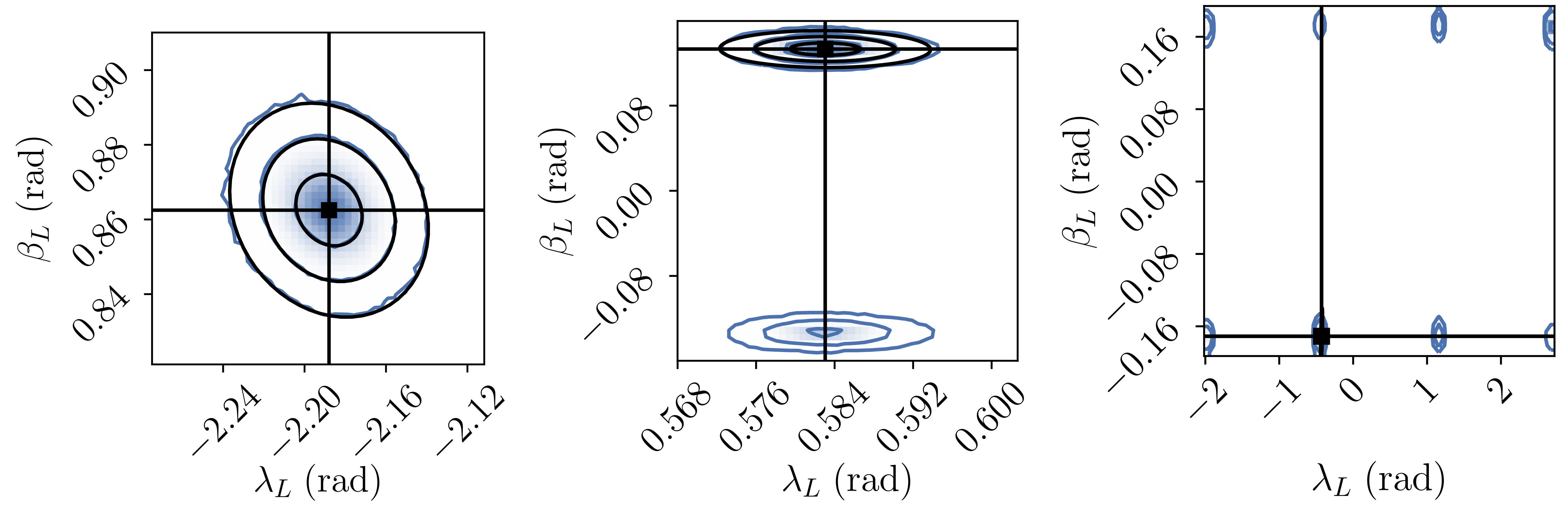
Example FD transfer function



LISA response and multimodality in the sky

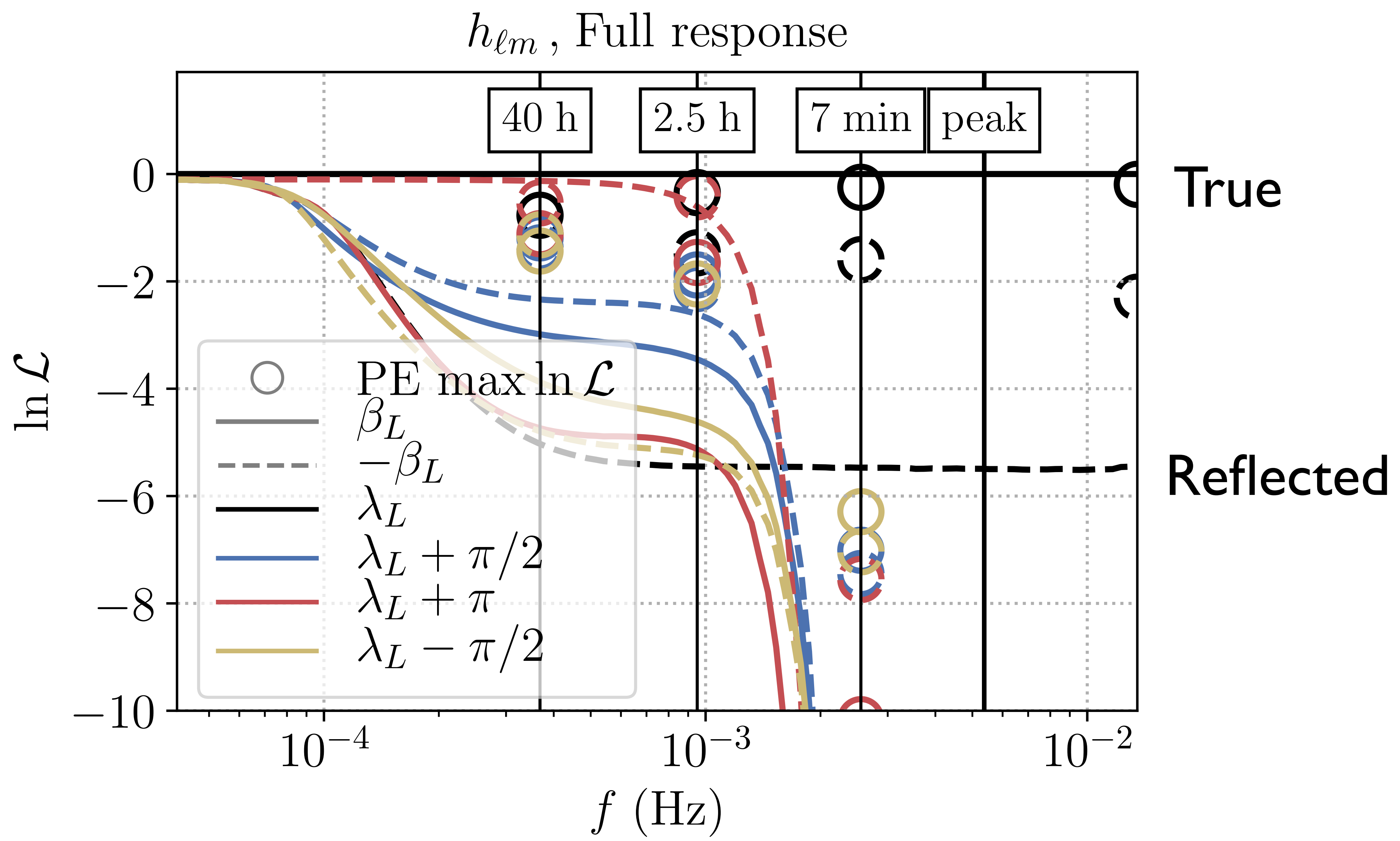
Multimodality pattern:





Degeneracy breaking:

- motion of LISA: eliminates all modes but the antipodal, weak for short high-mass signals
- high-frequency effects in the response: eliminates all modes but the reflected, only at high frequencies



- Multimodality broken by subdominant effects in response (motion, high-f)

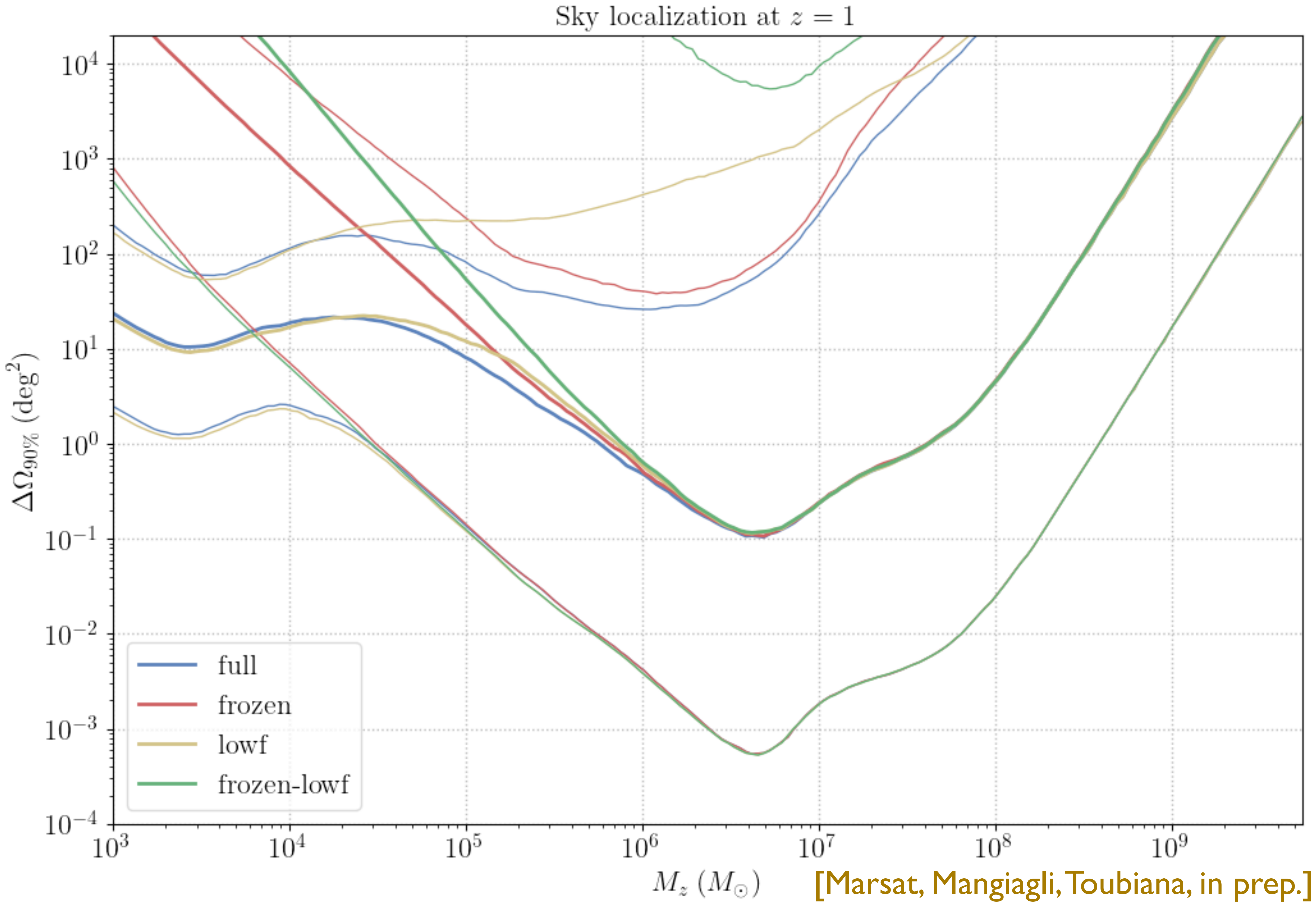
Outline

- MBHBs and LISA response
- **Sky localization: main mode**
- Sky localization: sky degeneracies
- Sky localization: galaxy counts
- Pre-merger sky localization
- Waveform systematics

Fisher localization: impact of response approximation

Analysis settings:

- Fisher matrix localization: sky area of the main mode of the posterior
- Randomization over 1000 orientations, mass ratios, spins
- Change the response model: keep or ignore the motion and high-f effects



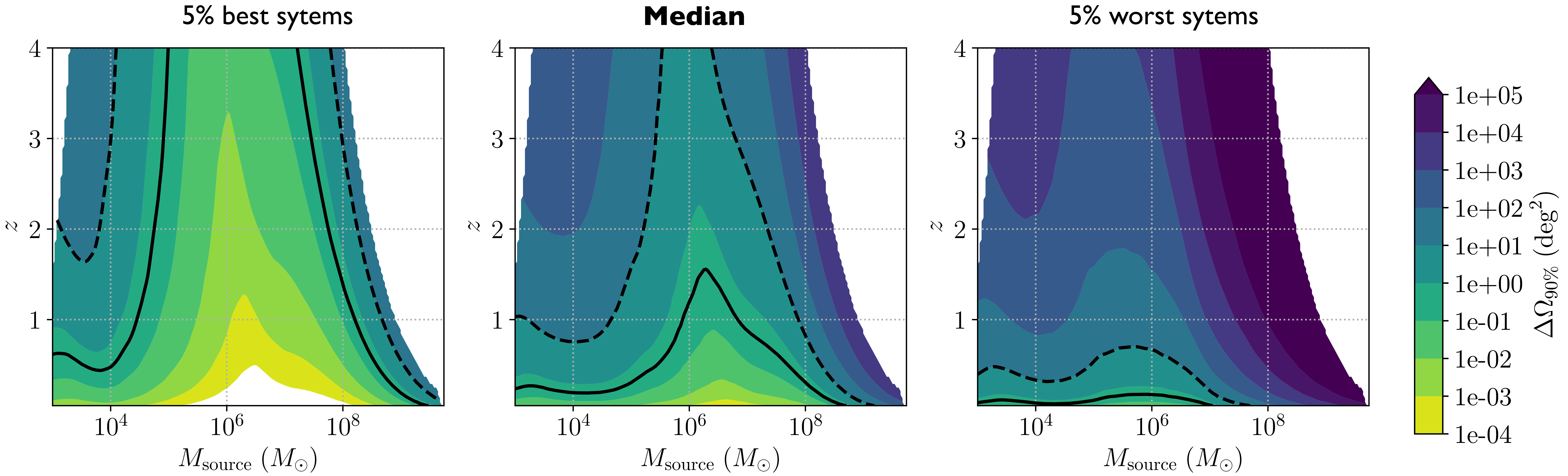
[Marsat, Mangiagli, Toubiana, in prep.]

- 'Pattern function' response is the main source of main-mode localization at high mass, from subdominant HM
- Multimodality broken in turn by subdominant effects in response (motion, high-f)

Large dispersion of main-mode sky localization depending on orientation

- Sky localization at high mass: weak effects, high SNR
- Unlike LVK localization from triangulation, LISA localization potentially vulnerable to systematics

MBHB sky localization at merger



- - 10 sq. deg. : LSST field of view
— 0.4 sq. deg.: Athena Wide Field Imager

See also: [McGee&al2018, Mangiagli&al 2020]

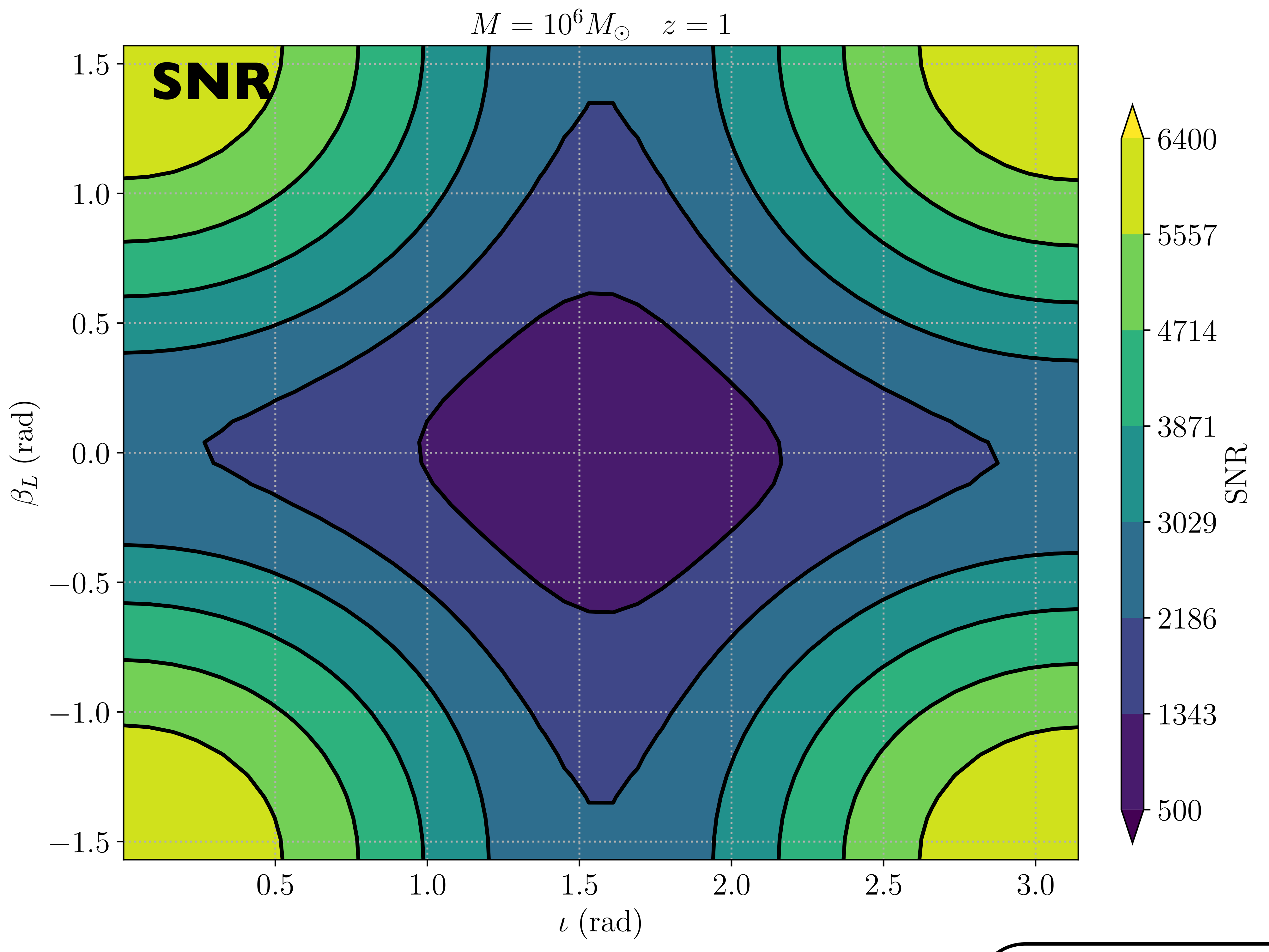
Sky area: which parameters are the most important ?

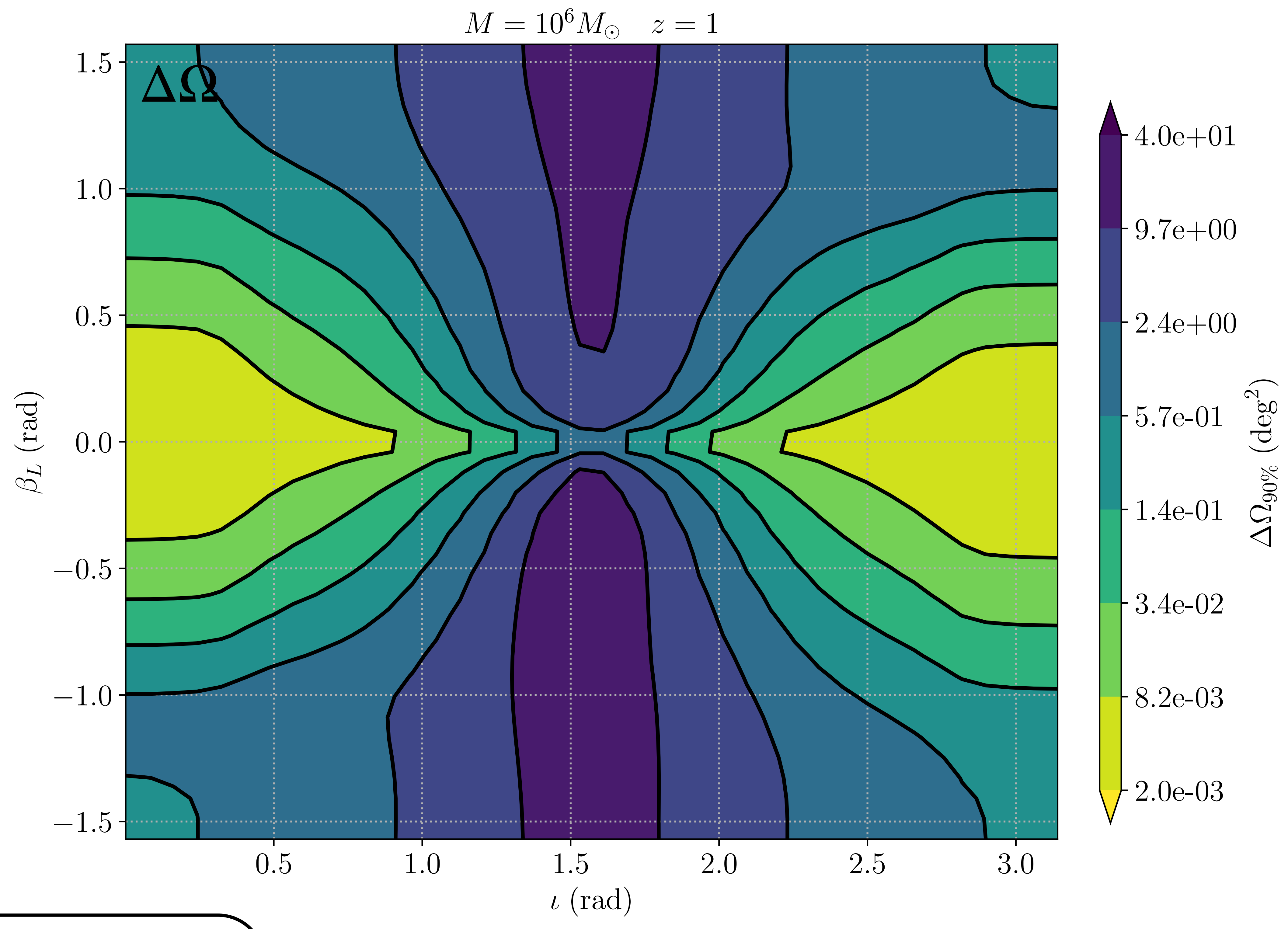
Large dispersion in sky area, ~4 orders of magnitude

What are the dominant parameters determining the sky area error ?

Best case: face-on/face-off, in the plane of LISA

Trend goes against SNR for the latitude!





Dominant parameters to determine sky area:
Inclination
L-frame latitude

Outline

- MBHBs and LISA response
- Sky localization: main mode
- **Sky localization: sky degeneracies**
- Sky localization: galaxy counts
- Pre-merger sky localization
- Waveform systematics

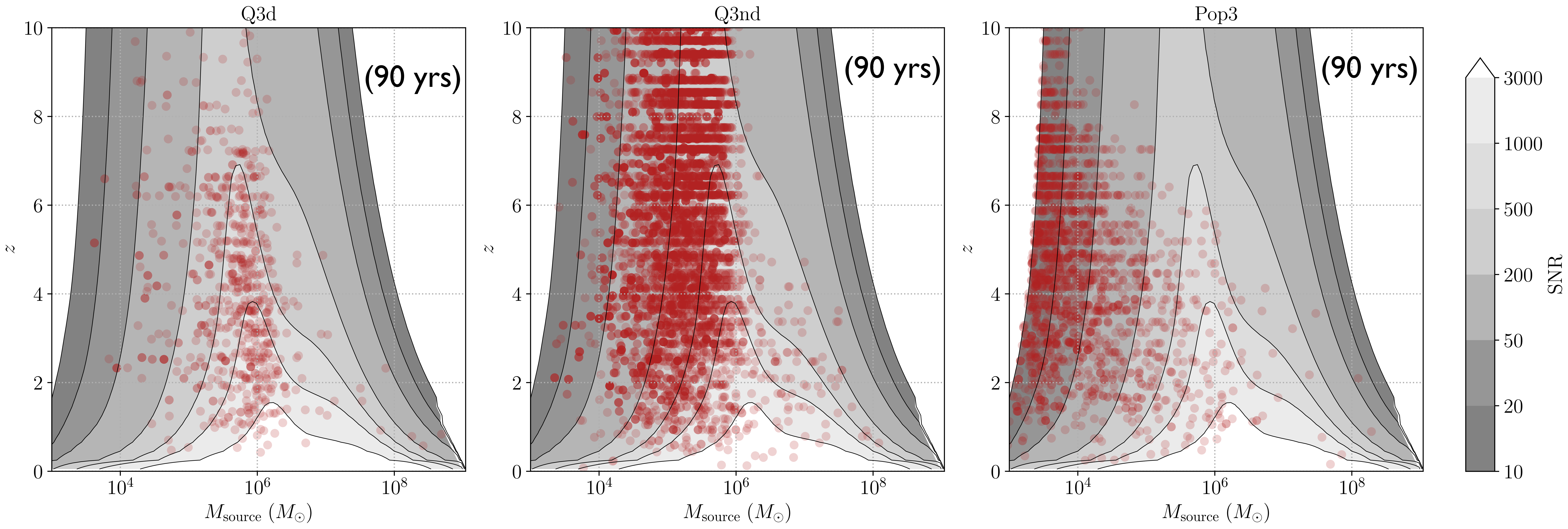
MBHB catalogs: full parameter estimation

Astrophysical models [Barausse 2012]:

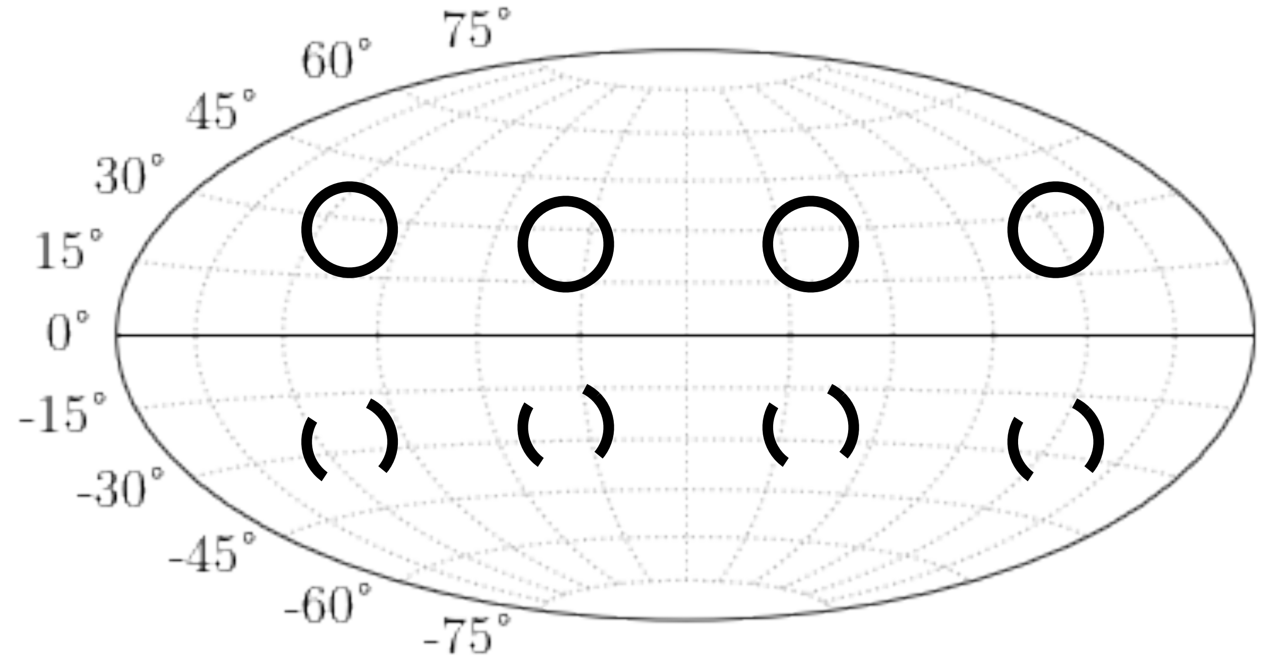
- Heavy seeds - delay (Q3d)
- Heavy seeds - no delay (Q3nd)
- PopIII seeds - delay (Pop3)

LISA detection rates from 90 yrs simulated:

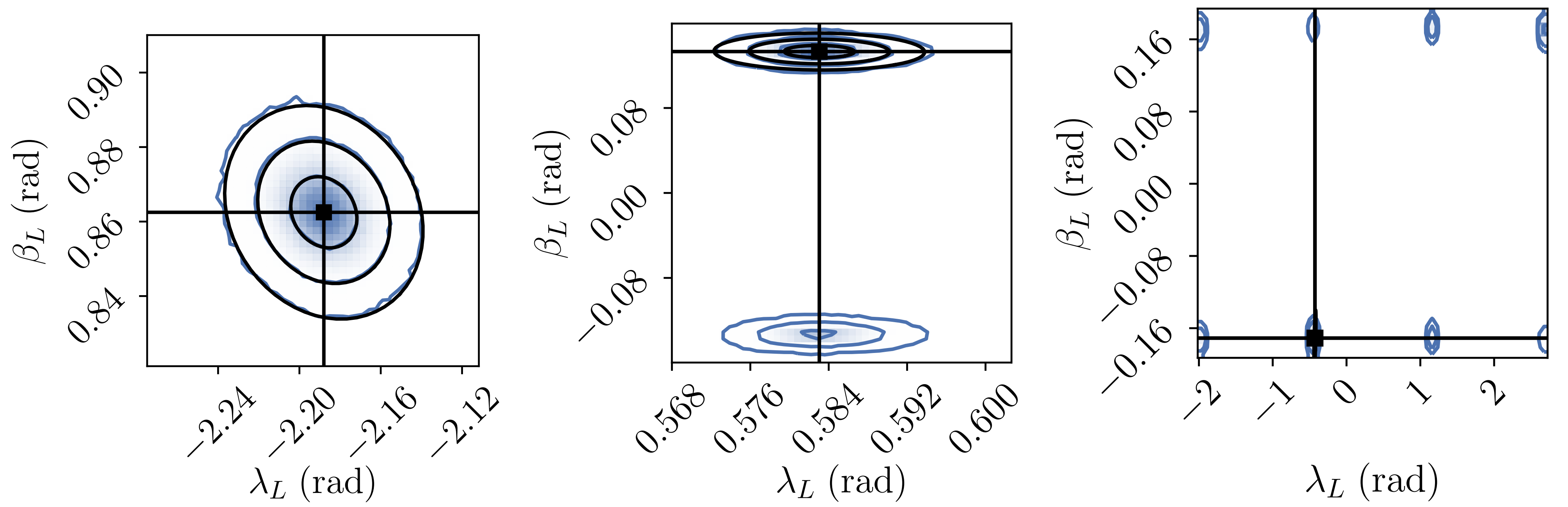
- Q3d: 30 / 4yrs
- Q3nd: 471 / 4 yrs
- Pop3: 129 / 4yrs



MBHB catalogs: sky multimodality

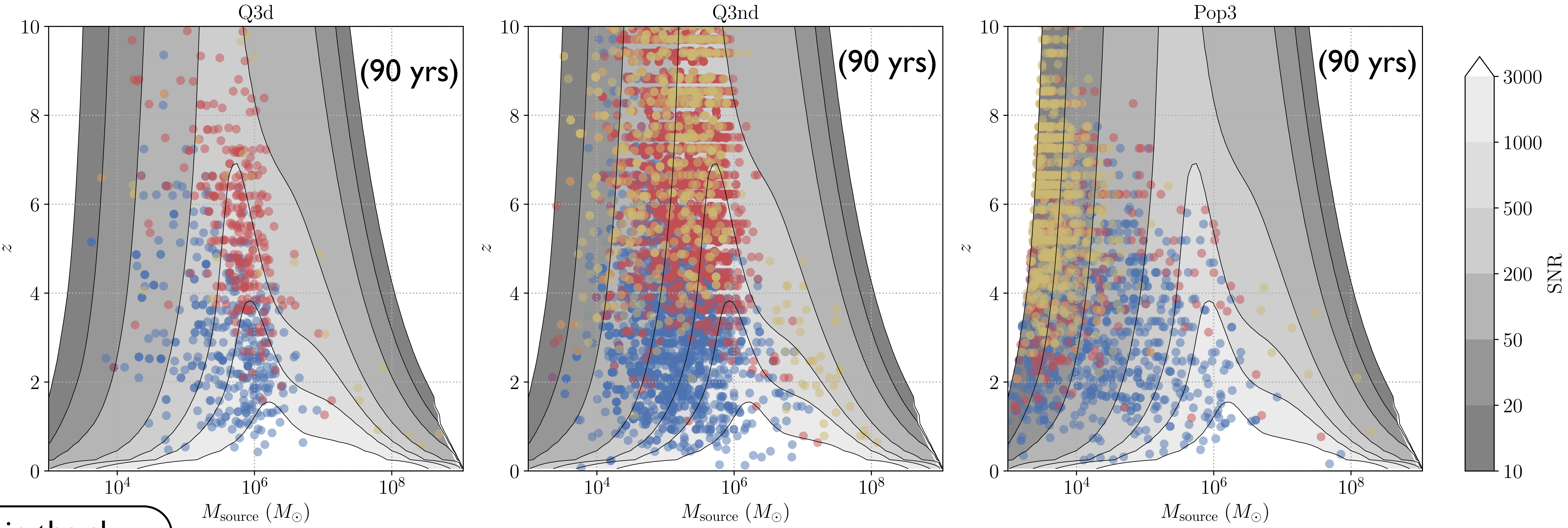


- **Bayesian PE** required to explore multimodal posteriors
- Simulation of 90yrs catalogs
- Custom proposals for degeneracies



- o 1 mode
- o 2 modes
- o >2 modes

Threshold: 5% probability in the sky mode



Multimodality in the sky present, but rare for counterpart candidates post-merger

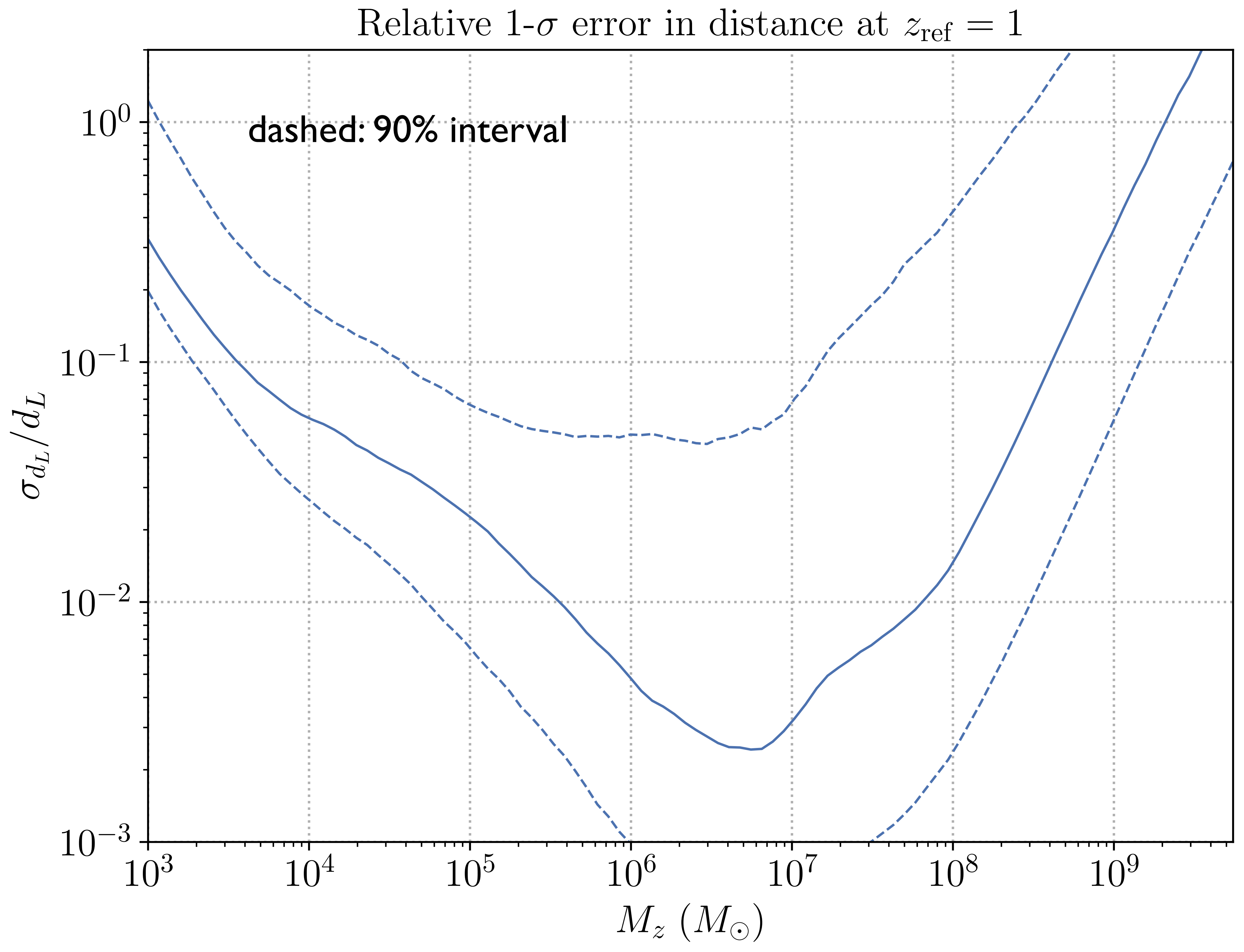
[Mangiagli&al 2022]

Outline

- MBHBs and LISA response
- Sky localization: main mode
- Sky localization: sky degeneracies
- **Sky localization: galaxy counts**
- Pre-merger sky localization
- Waveform systematics

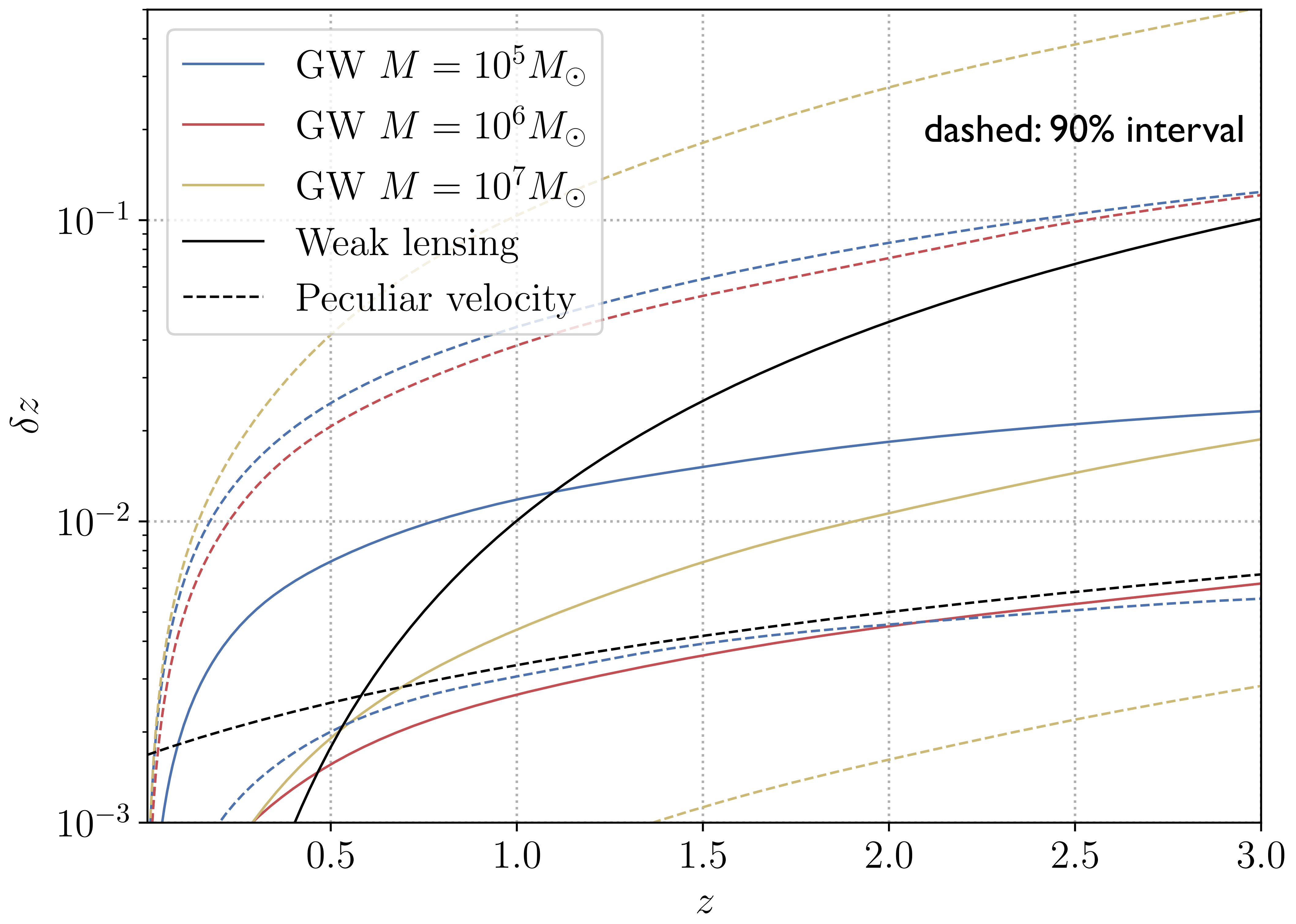
Luminosity distance determination

For distance determination, higher harmonics are **crucial**, breaking the distance-inclination degeneracy.



Large dispersion in distance determination, ~4 orders of magnitude

Looking for a host by converting dL to z, weak lensing and peculiar motions have to be taken into account



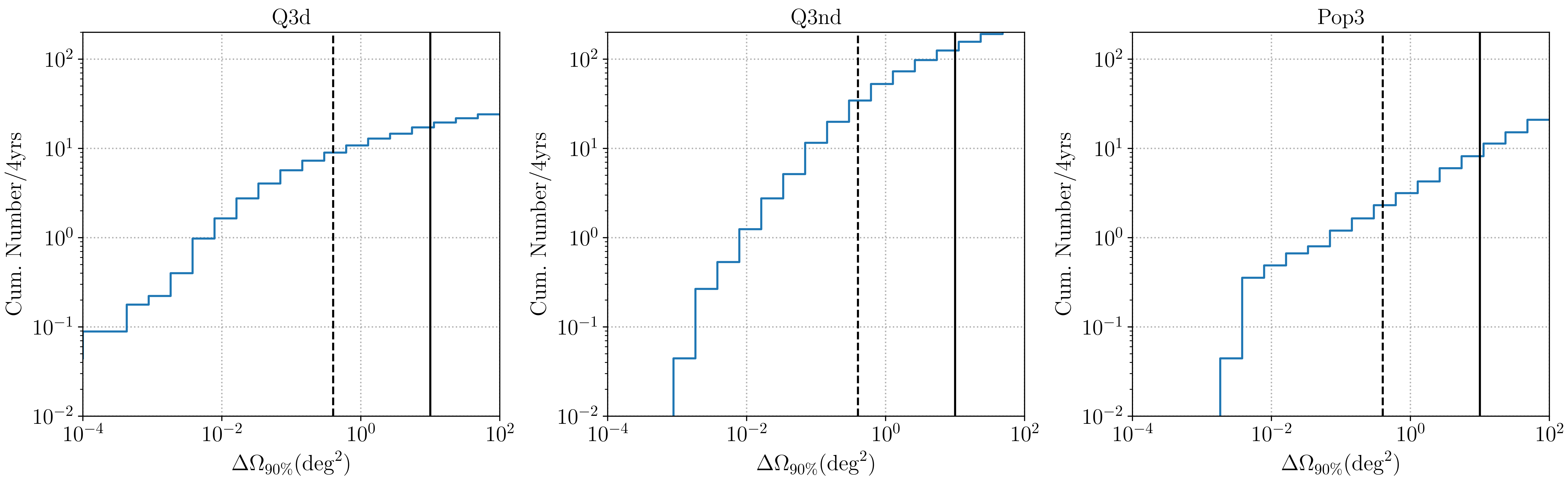
Weak lensing can dominate over the dL error from LISA

MBHB catalogs: sky localisation and galaxy counts

Sky areas

Sky area and error volume computed from Bayesian PE (main mode), with lensing

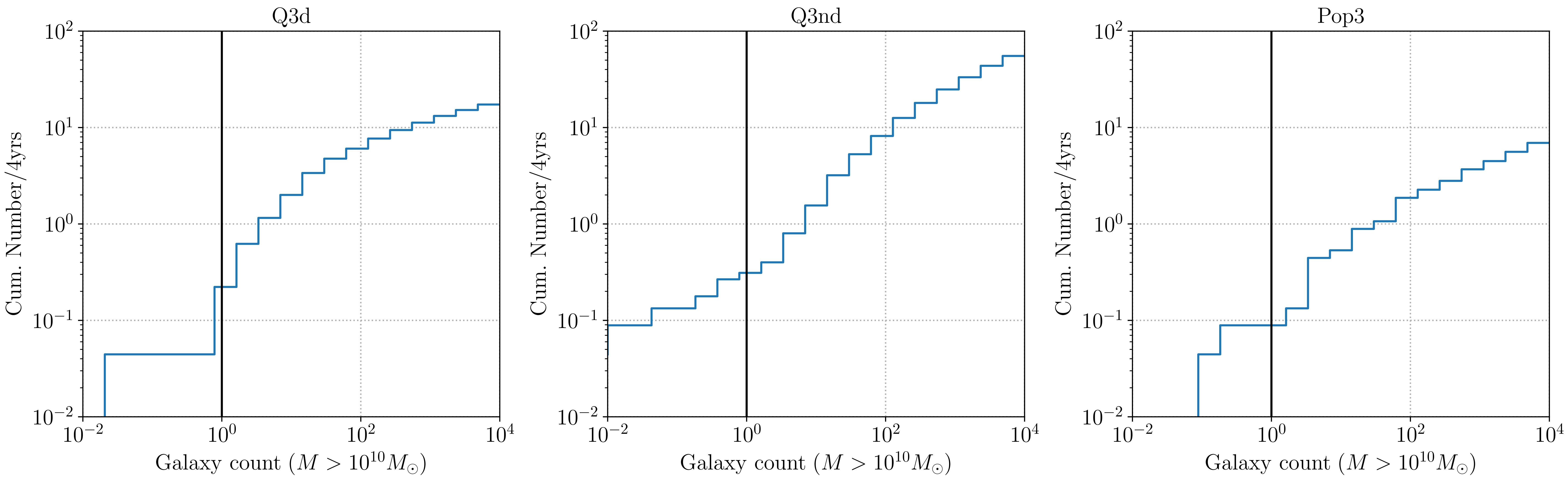
Catalogs: [Barausse 2012]



Galaxy counts

Simplistic, simulated catalog cut in mass with no consideration of completeness, EM emissions…

Simulated galaxy catalog courtesy of [D. Izquierdo-Villalba&al]



Outline

- MBHBs and LISA response
- Sky localization: main mode
- Sky localization: sky degeneracies
- Sky localization: galaxy counts
- **Pre-merger sky localization**
- Waveform systematics

Pre-merger localization

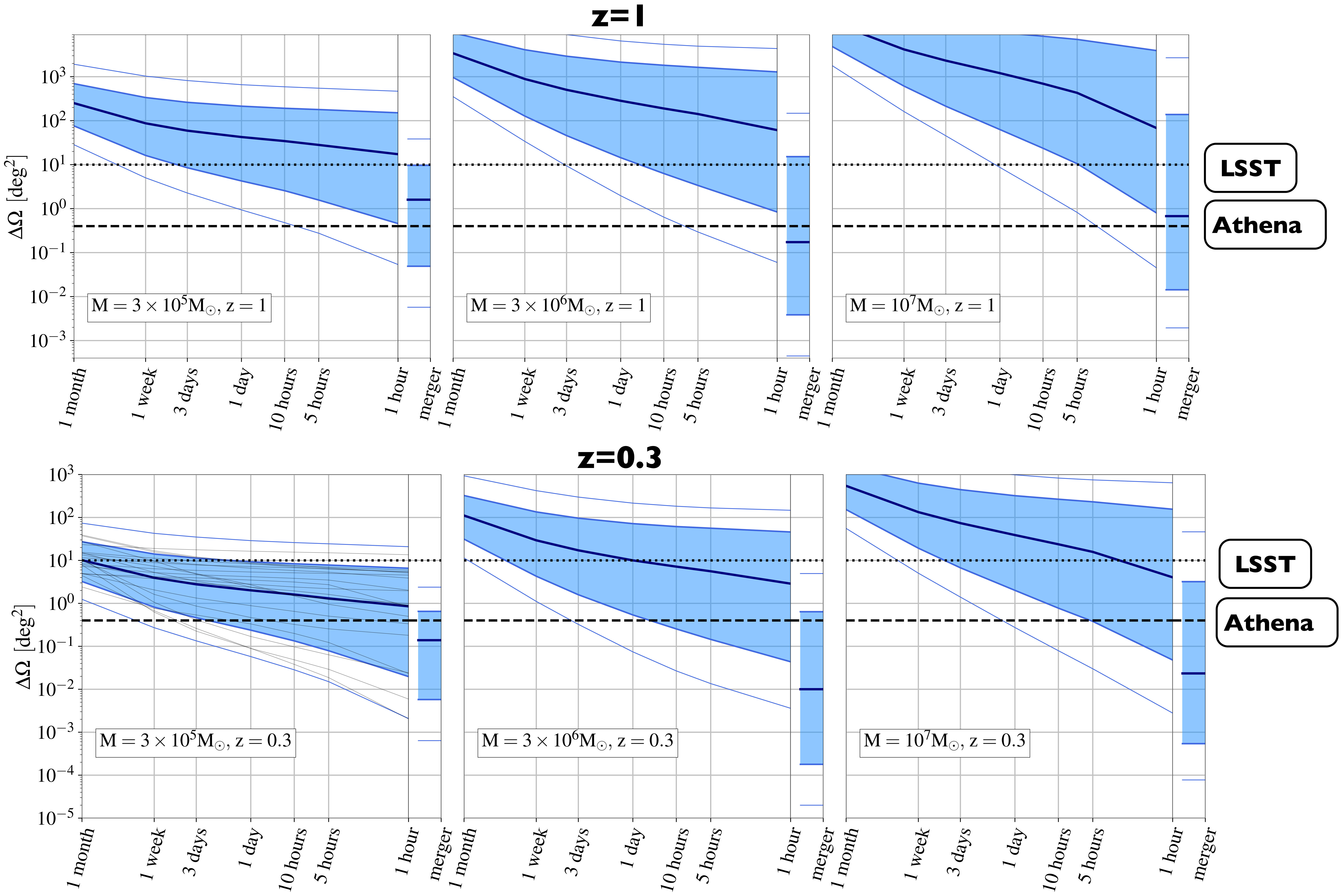
- Fisher matrices: 10000 random parameters
- MCMC: 100 PE runs
- Here, sky area of the main mode

[Piro&al 2022]

[See also Mangiagli&al 2020]

Advance localization challenging, much better post-merger

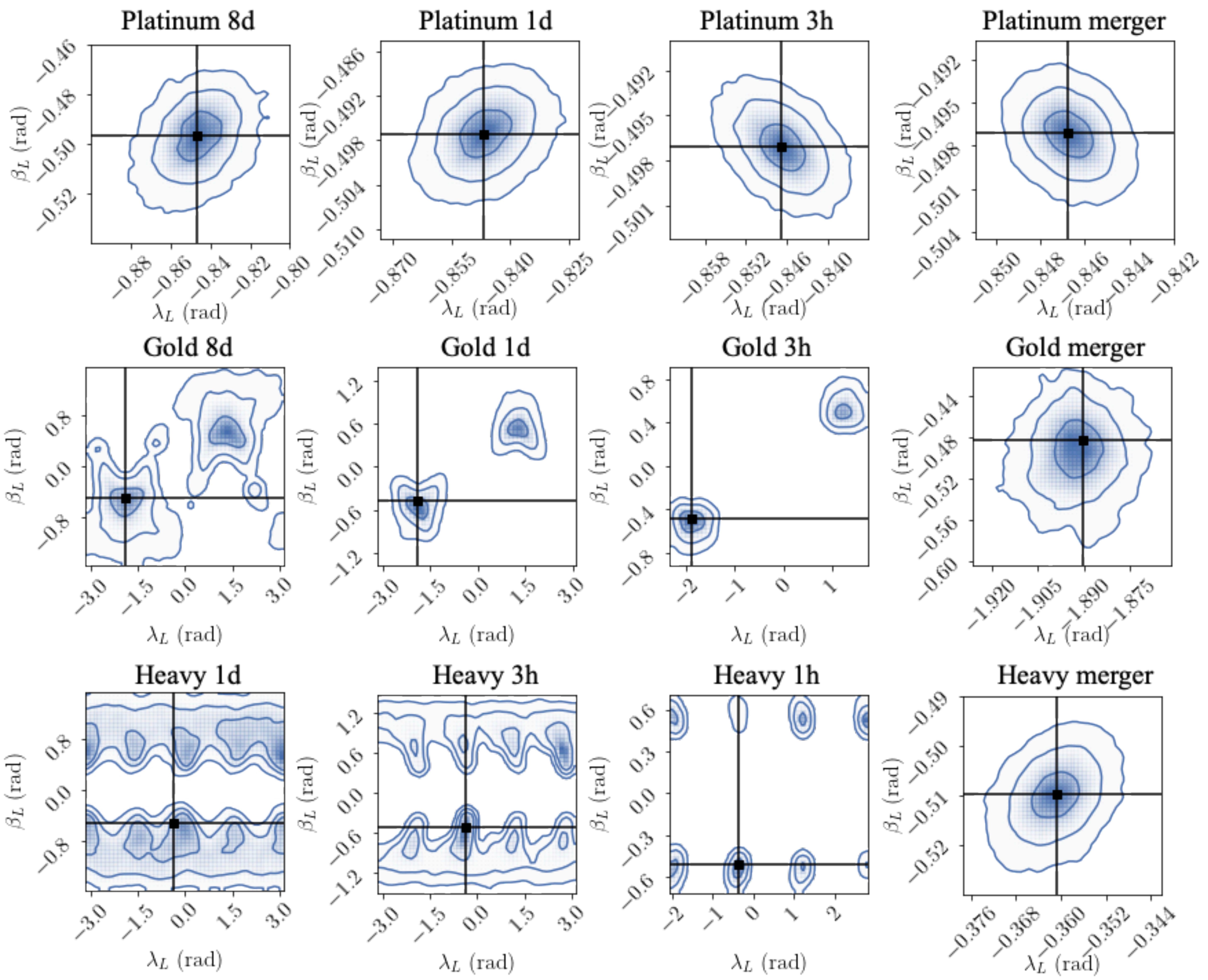
Large dispersion in sky area, ~4 orders of magnitude



Pre-merger localization: degeneracies

Bayesian PE: sky localization cutting at different times

- ‘Platinum’: M3e5, z=0.3
- ‘Gold’: M3e6, z=1
- ‘Heavy’: M1e7, z=1



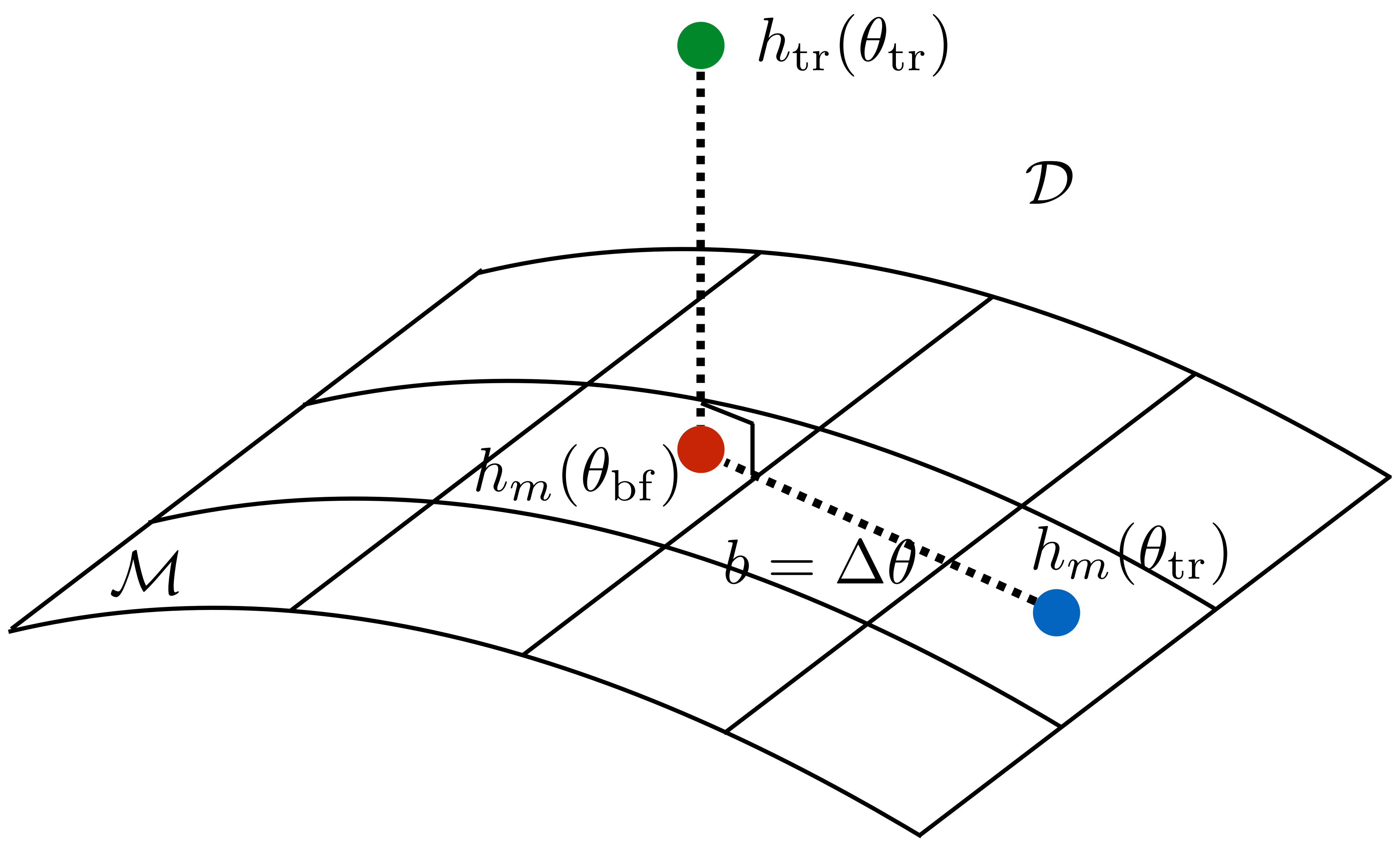
- Wide range of multimodalities dep. on parameters
- Post-merger localization unimodal here

[Piro&al 2022]

Outline

- MBHBs and LISA response
- Sky localization: main mode
- Sky localization: sky degeneracies
- Sky localization: galaxy counts
- Pre-merger sky localization
- **Waveform systematics**

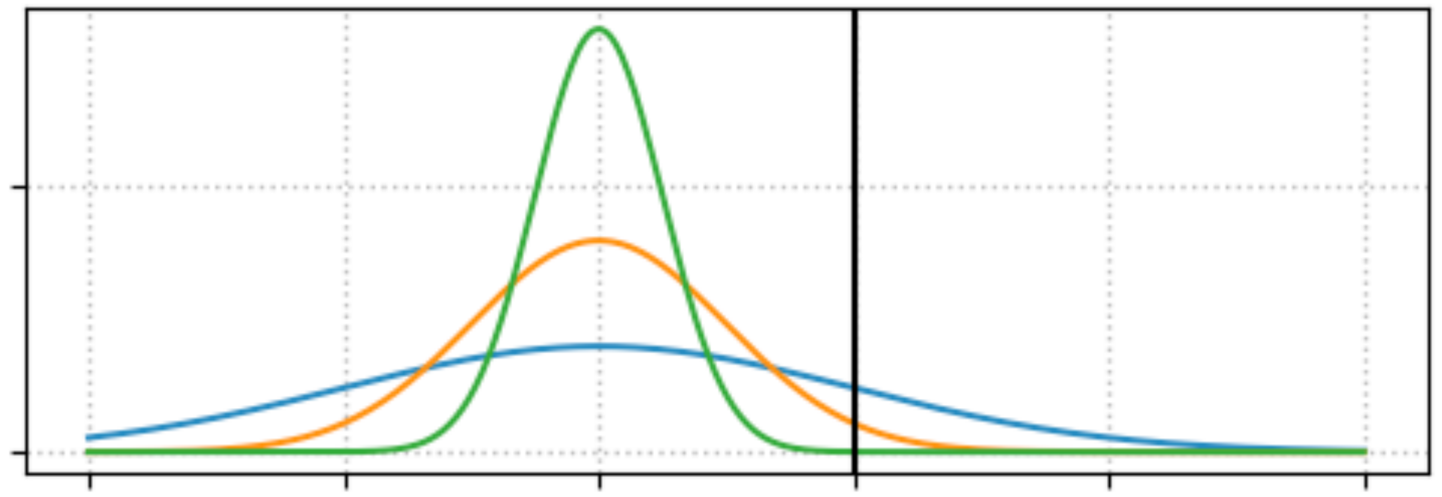
Waveform systematics and parameter estimation



Systematic biases:

Ignoring the effect of the noise, bias given by the **best-fit** parameters on the model signal manifold: $\Delta\theta = \theta_{bf} - \theta_{tr}$

- the **bias** is SNR-independent (optimization problem), but requires to explore the full parameter space [**expensive**]
- the statistical errors scale with SNR



Are current waveform models accurate enough for LISA ?
Can the sky localization be biased ?

Parameter space exploration:

- $M_z = [10^5, 10^6, 10^7] M_\odot$
- $z_{min} = 1$
- $N = 240$ PE runs
- uniform q, χ_1, χ_2
- randomize orientations

Injections:

NRHybSur3dq8

- SXS NR simulations hybridized with long EOB inspirals (covers ~6 months for $M = 10^5 M_\odot$)
- Surrogate interpolant, time-domain

Templates:

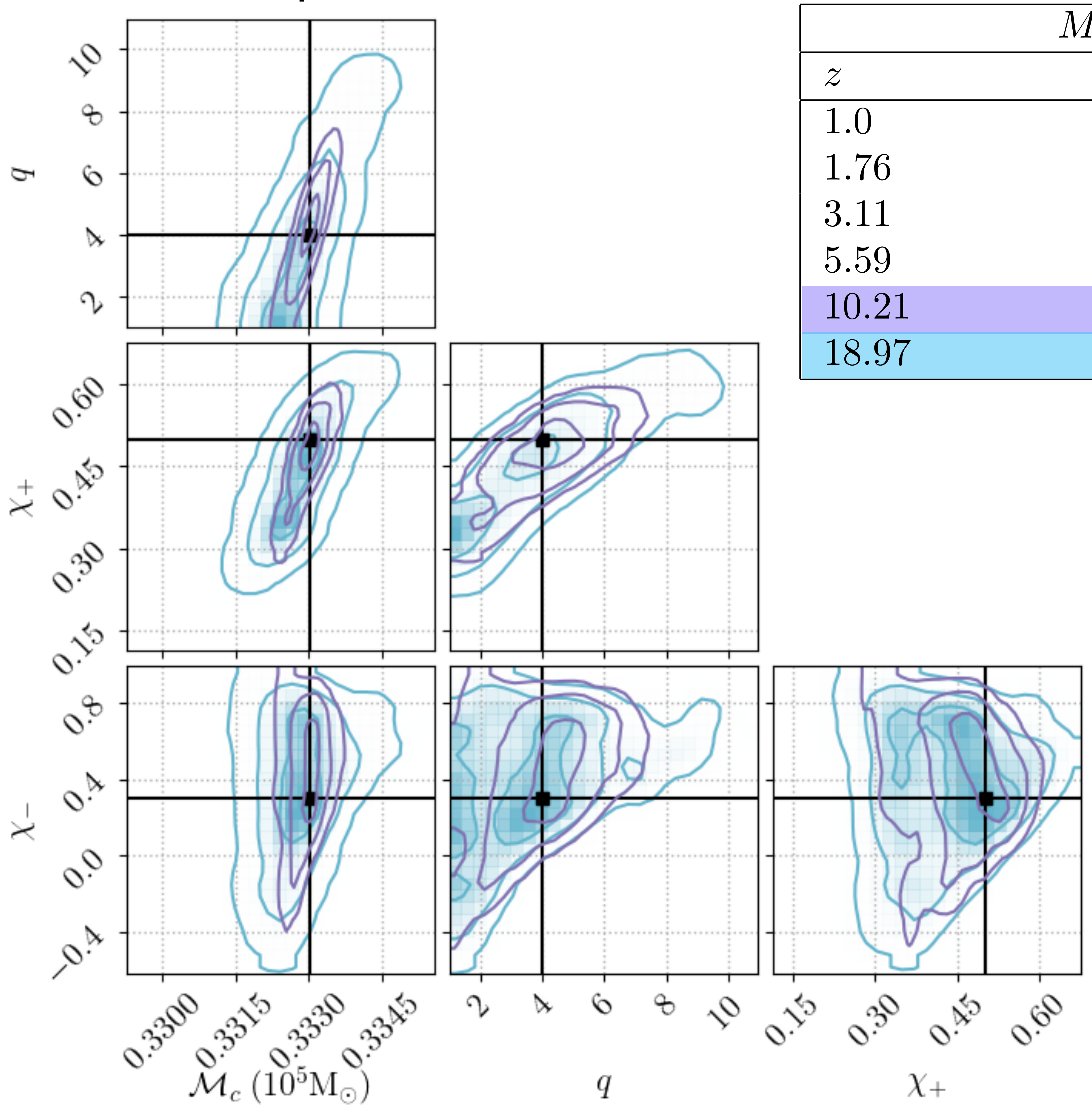
Efficient Fourier-domain models:

- PhenomHM
- PhenomXHM
- SEOBNRv4HM_ROM
- SEOBNRv5HM_ROM

Example Parameter estimation with systematics I

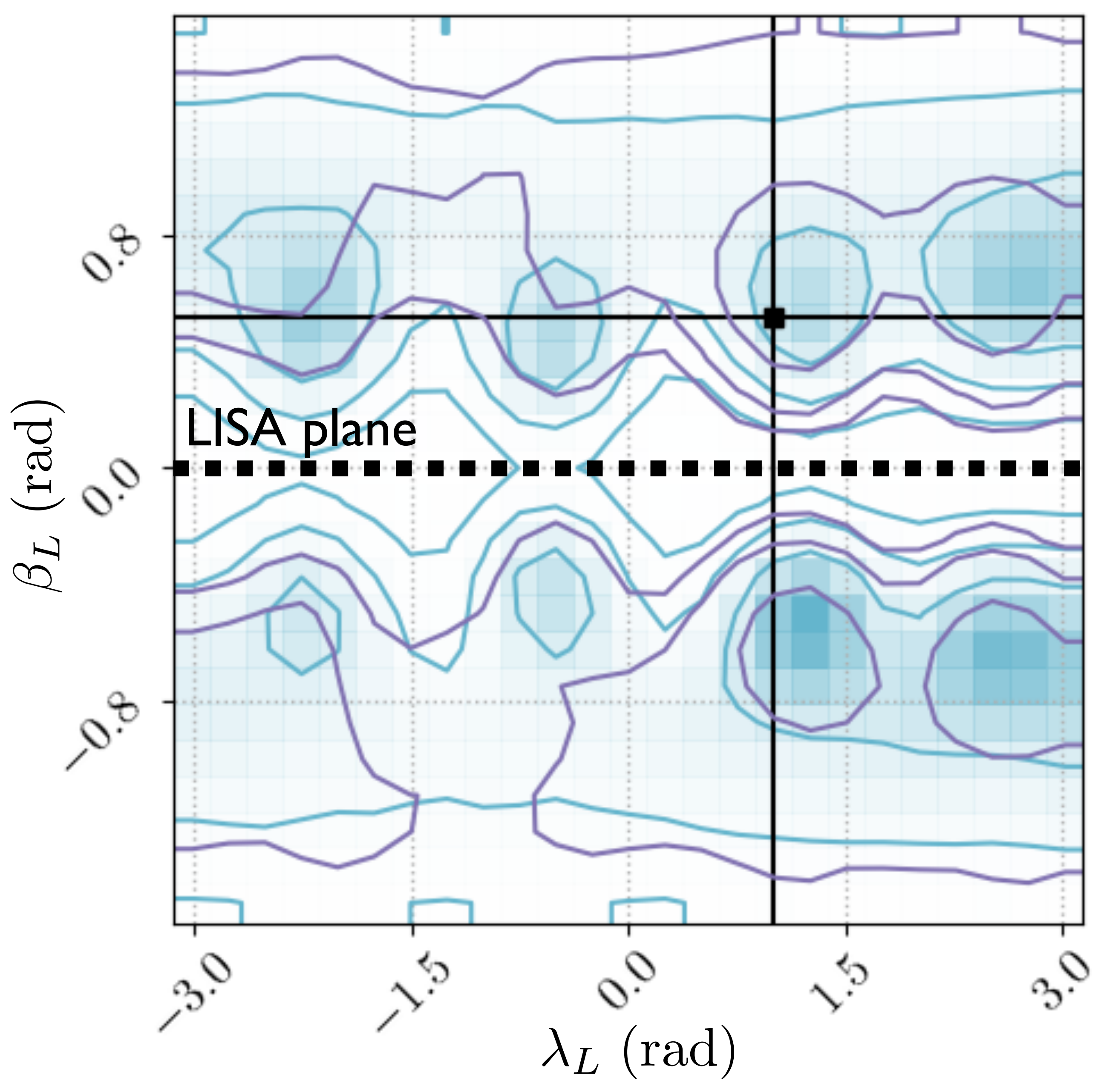
- **Injection:** NRHybSur3dq8 $\{M = 10^5 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^5\ M_\odot$	
z	SNR
1.0	317
1.76	158
3.11	79
5.59	40
10.21	20
18.97	10

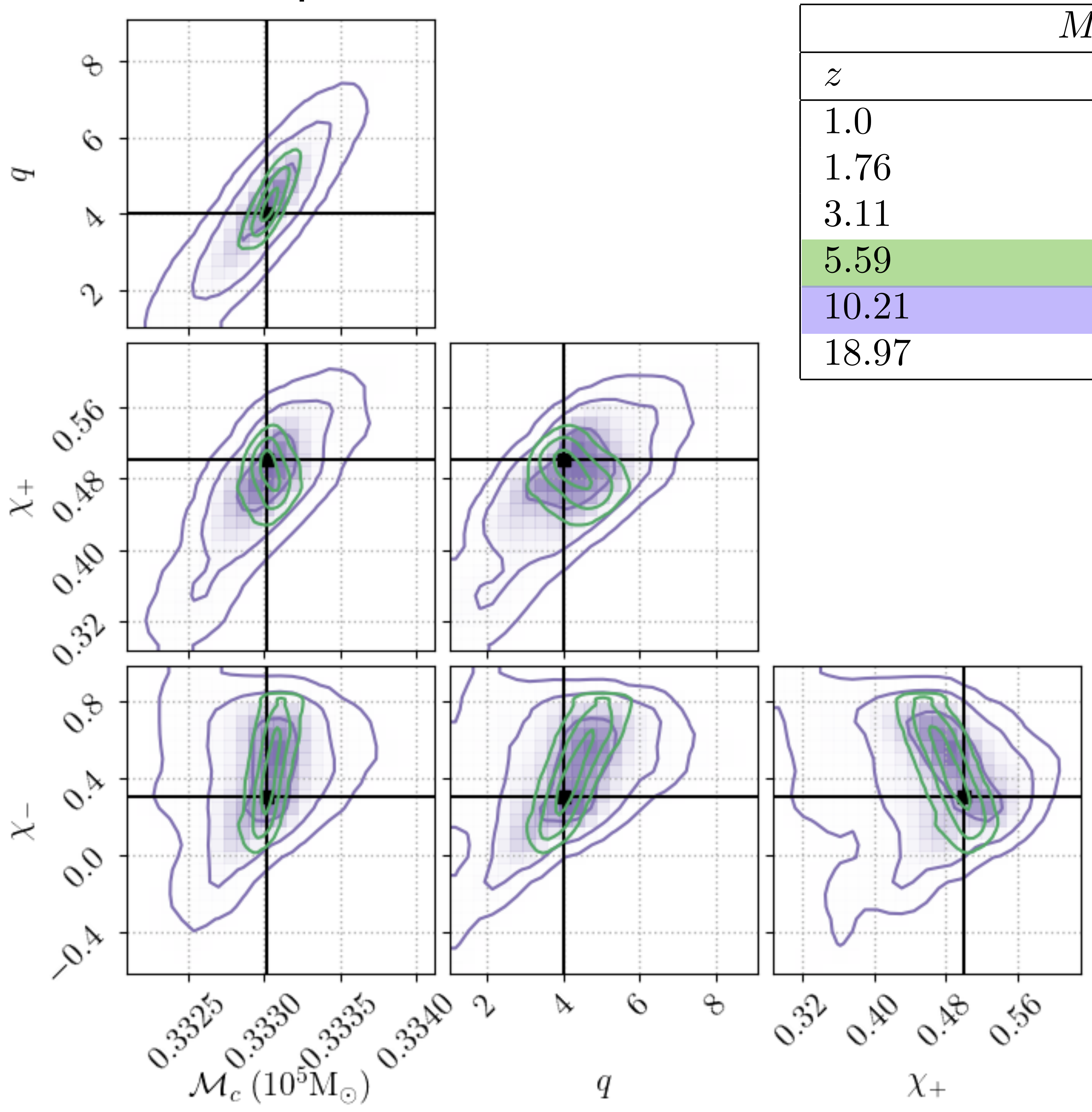
Sky localisation



Example Parameter estimation with systematics I

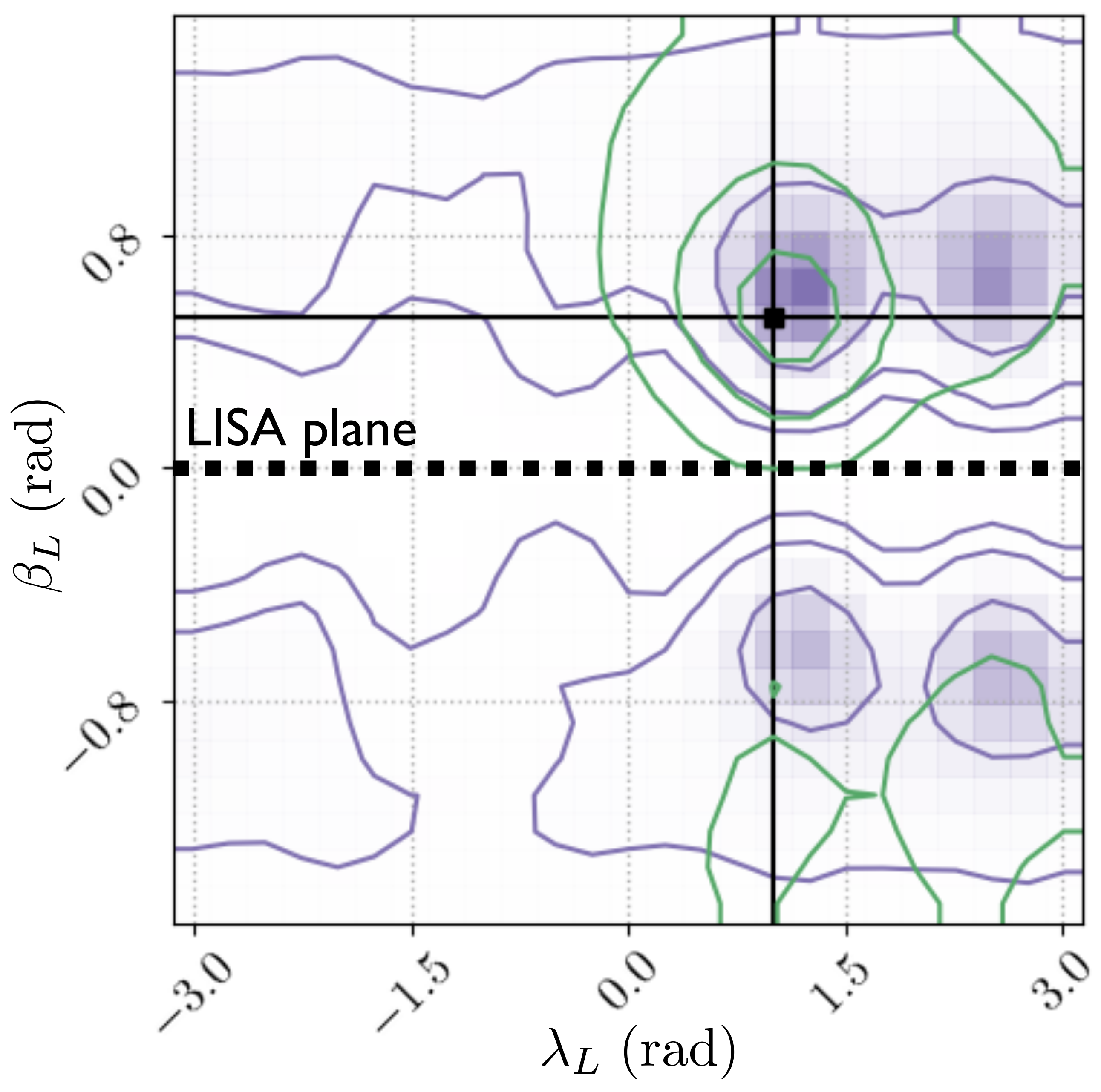
- **Injection:** NRHybSur3dq8 $\{M = 10^5 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^5\ M_\odot$	
z	SNR
1.0	317
1.76	158
3.11	79
5.59	40
10.21	20
18.97	10

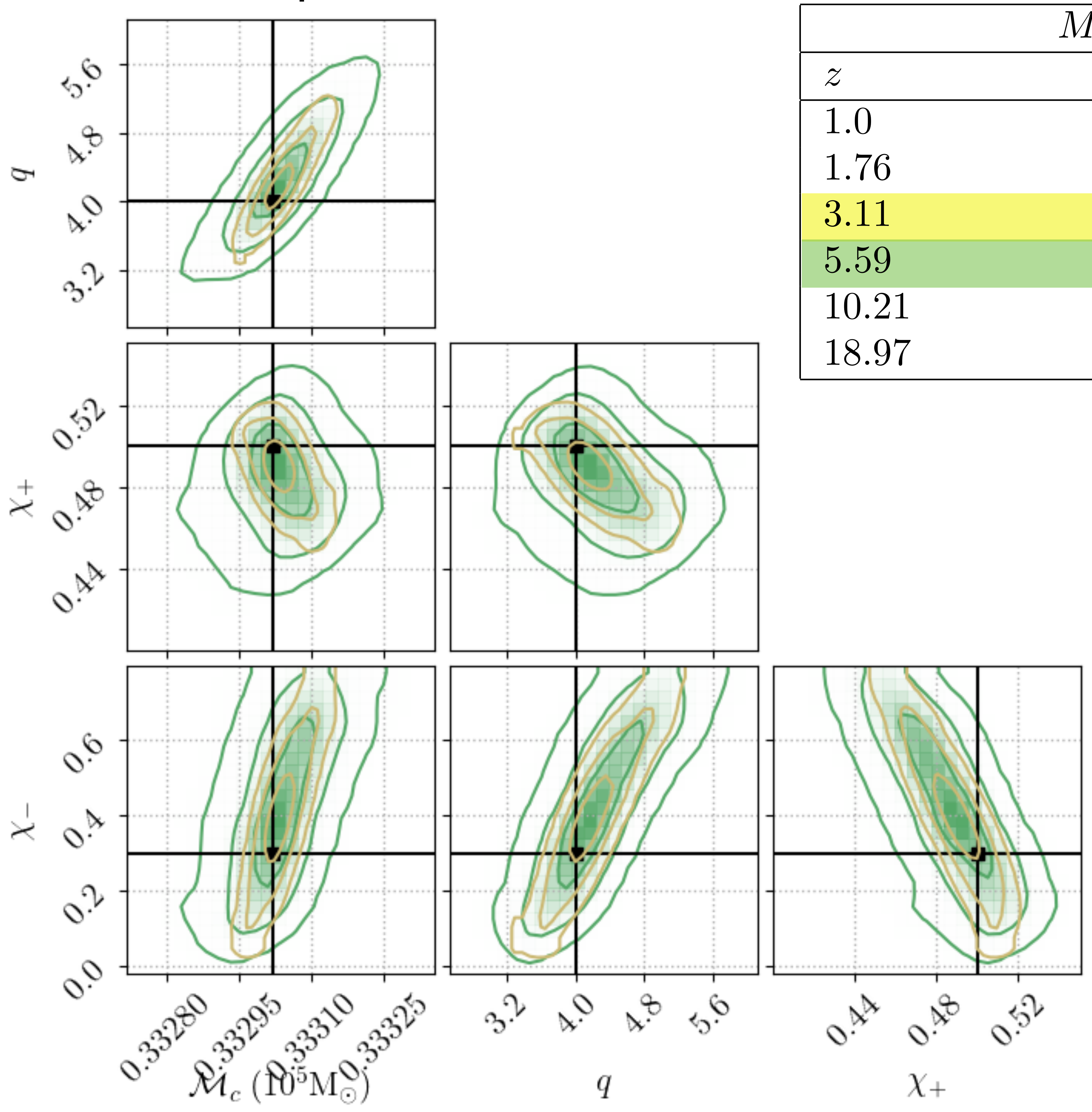
Sky localisation



Example Parameter estimation with systematics I

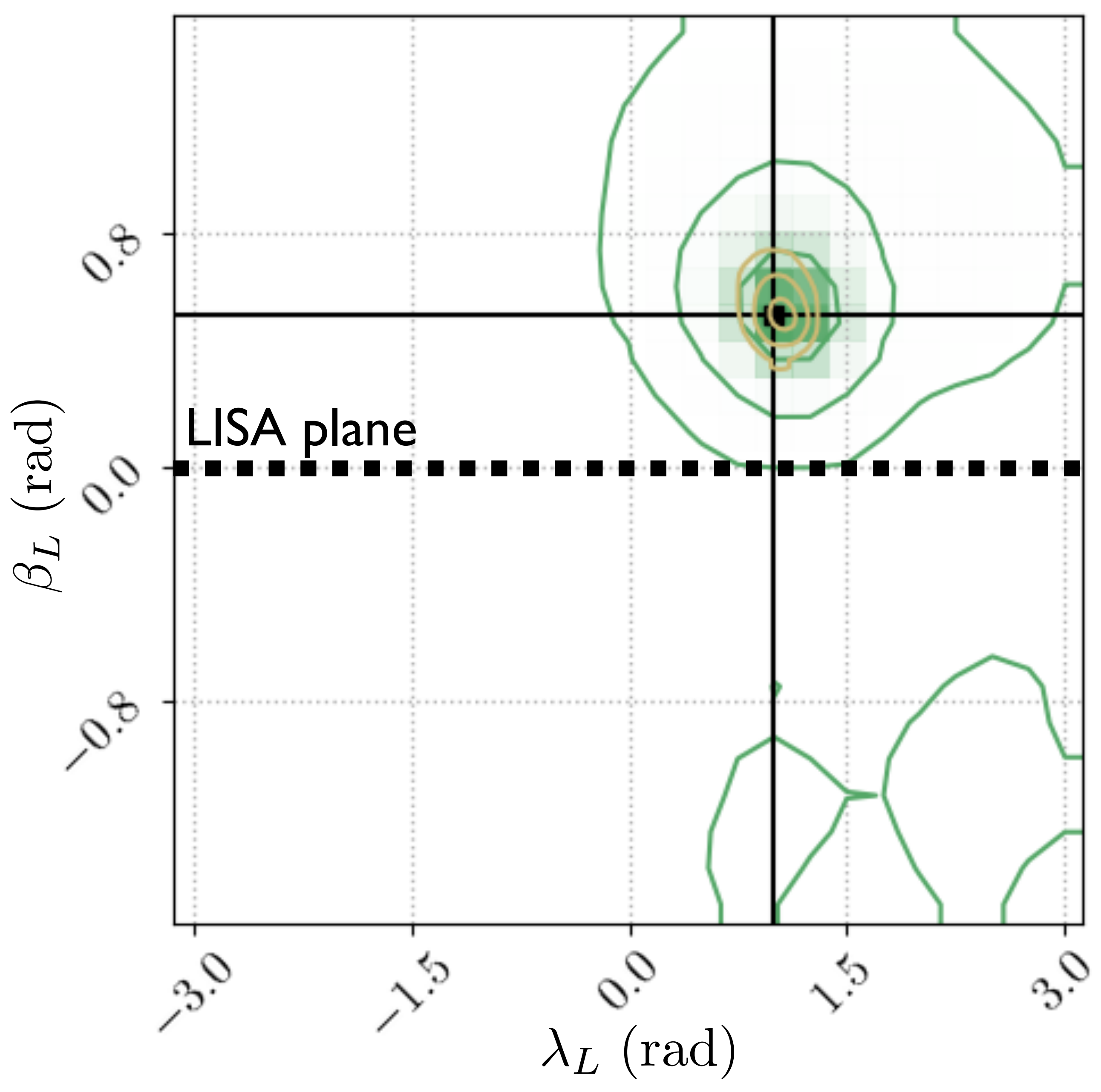
- **Injection:** NRHybSur3dq8 $\{M = 10^5 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^5\ M_\odot$	
z	SNR
1.0	317
1.76	158
3.11	79
5.59	40
10.21	20
18.97	10

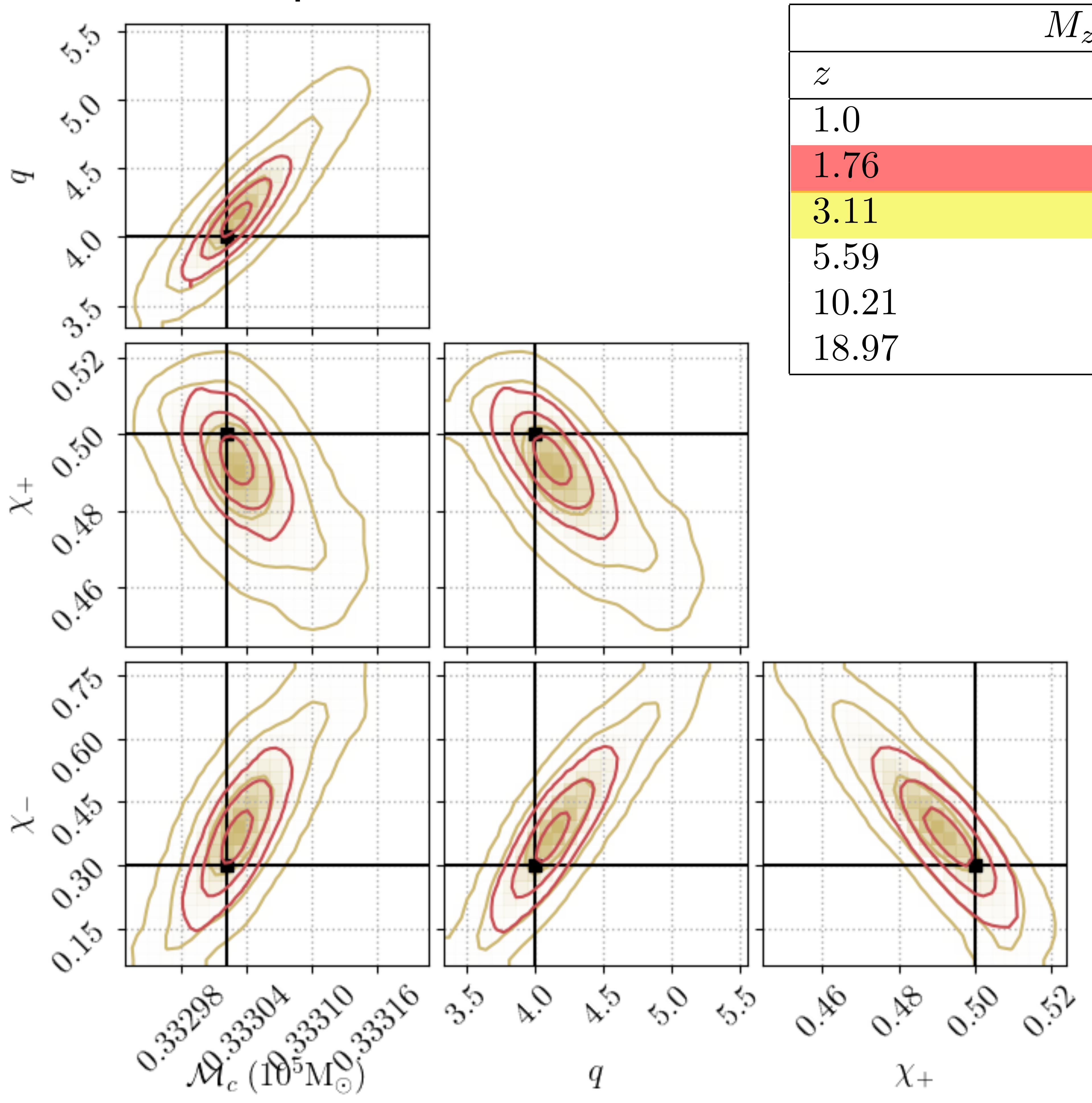
Sky localisation



Example Parameter estimation with systematics I

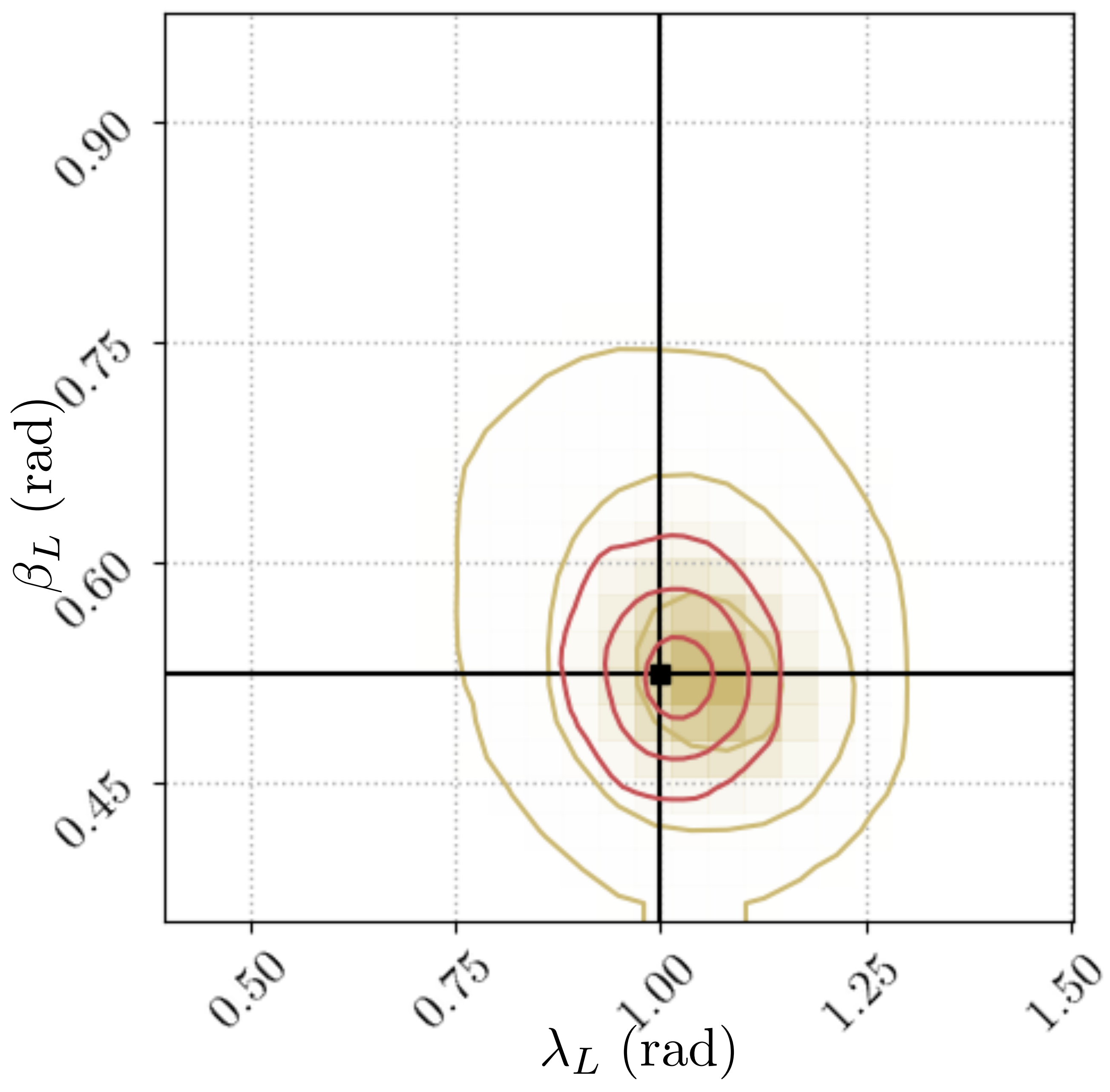
- **Injection:** NRHybSur3dq8 $\{M = 10^5 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^5\ M_\odot$	
z	SNR
1.0	317
1.76	158
3.11	79
5.59	40
10.21	20
18.97	10

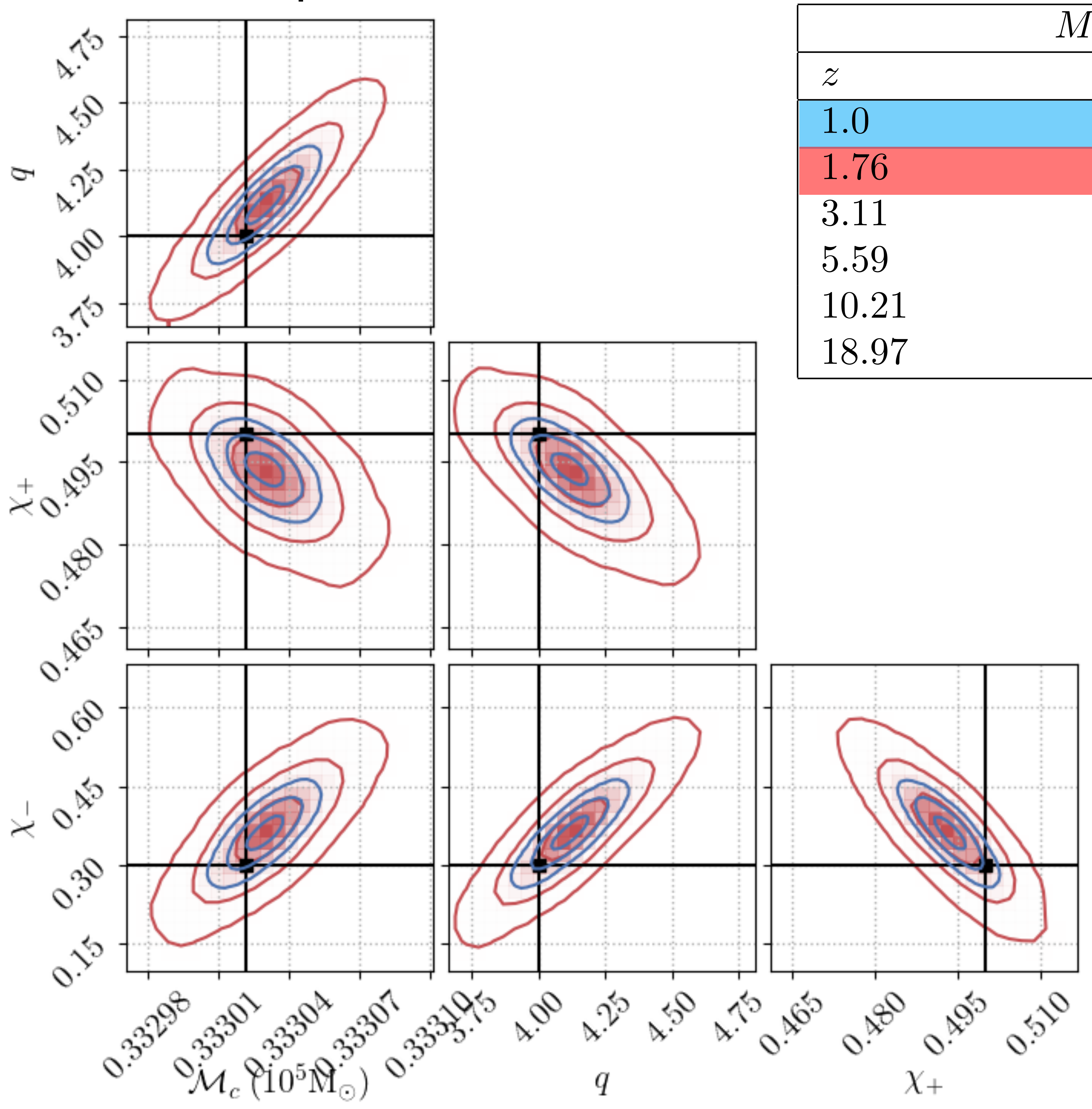
Sky localisation



Example Parameter estimation with systematics I

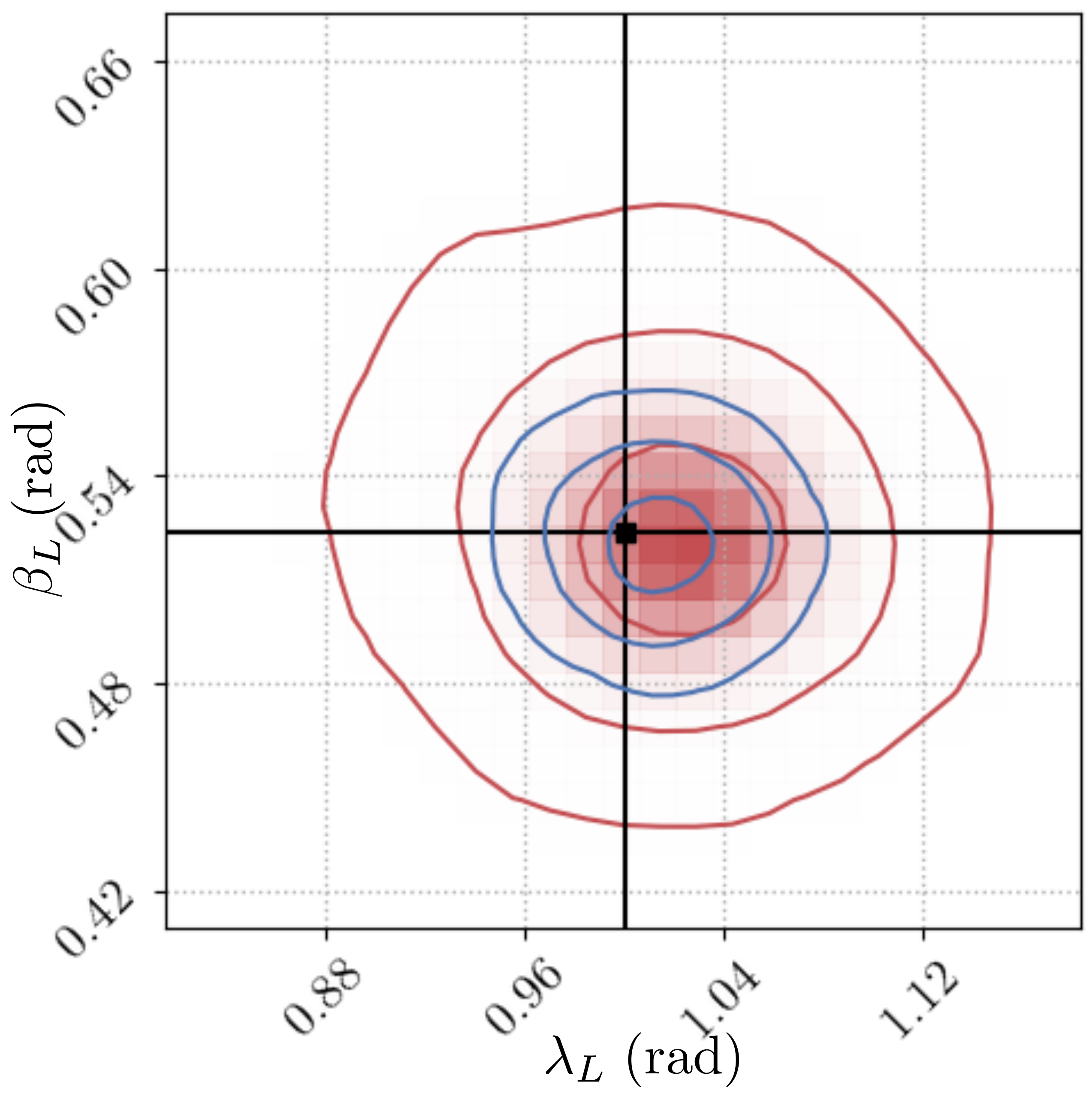
- **Injection:** NRHybSur3dq8 $\{M = 10^5 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^5\ M_\odot$	
z	SNR
1.0	317
1.76	158
3.11	79
5.59	40
10.21	20
18.97	10

Sky localisation

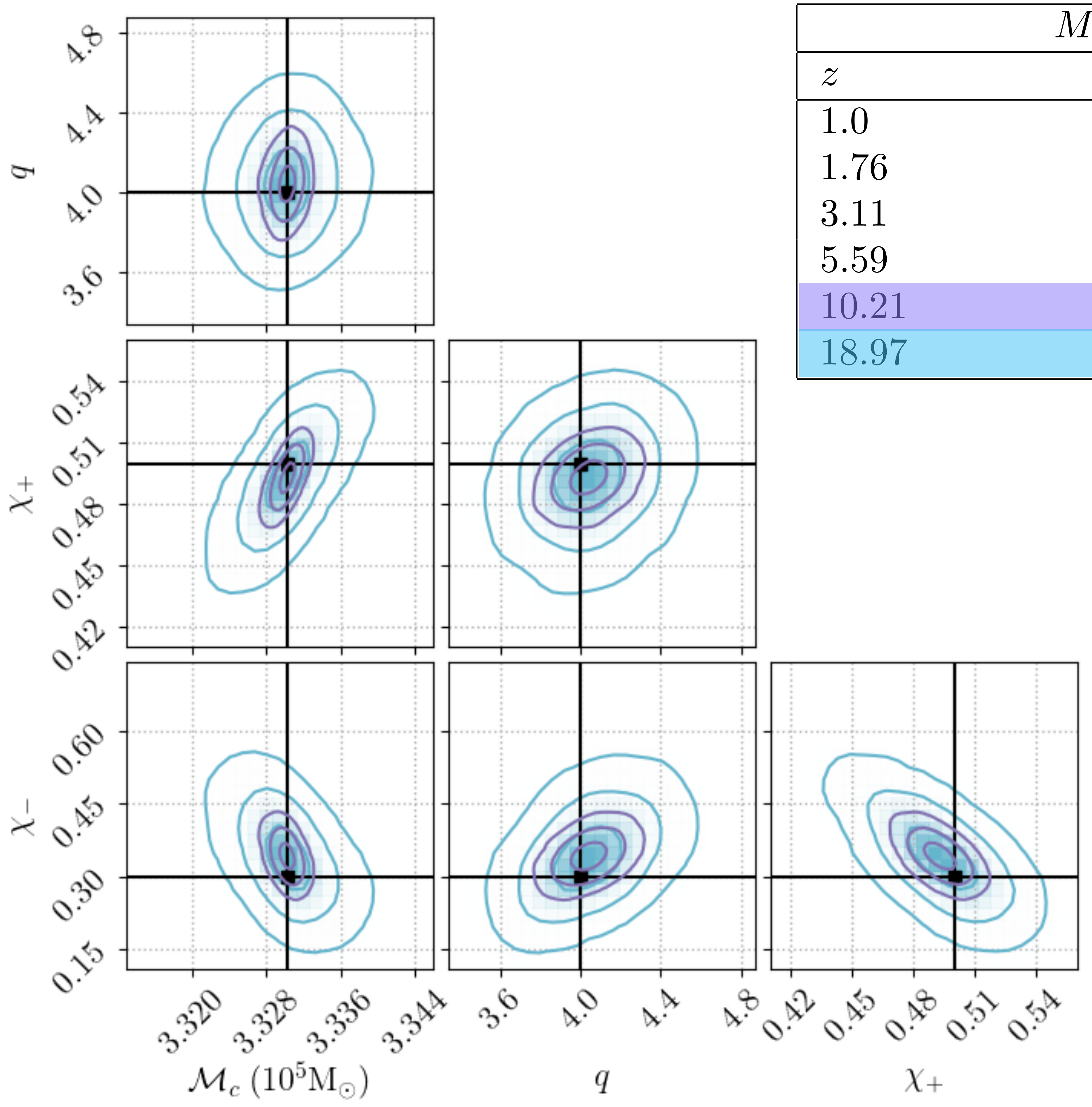


The good:
- converges on the true parameters
- mild bias at $z = 1$, $\mathrm{SNR} = 317$

Example Parameter estimation with systematics II

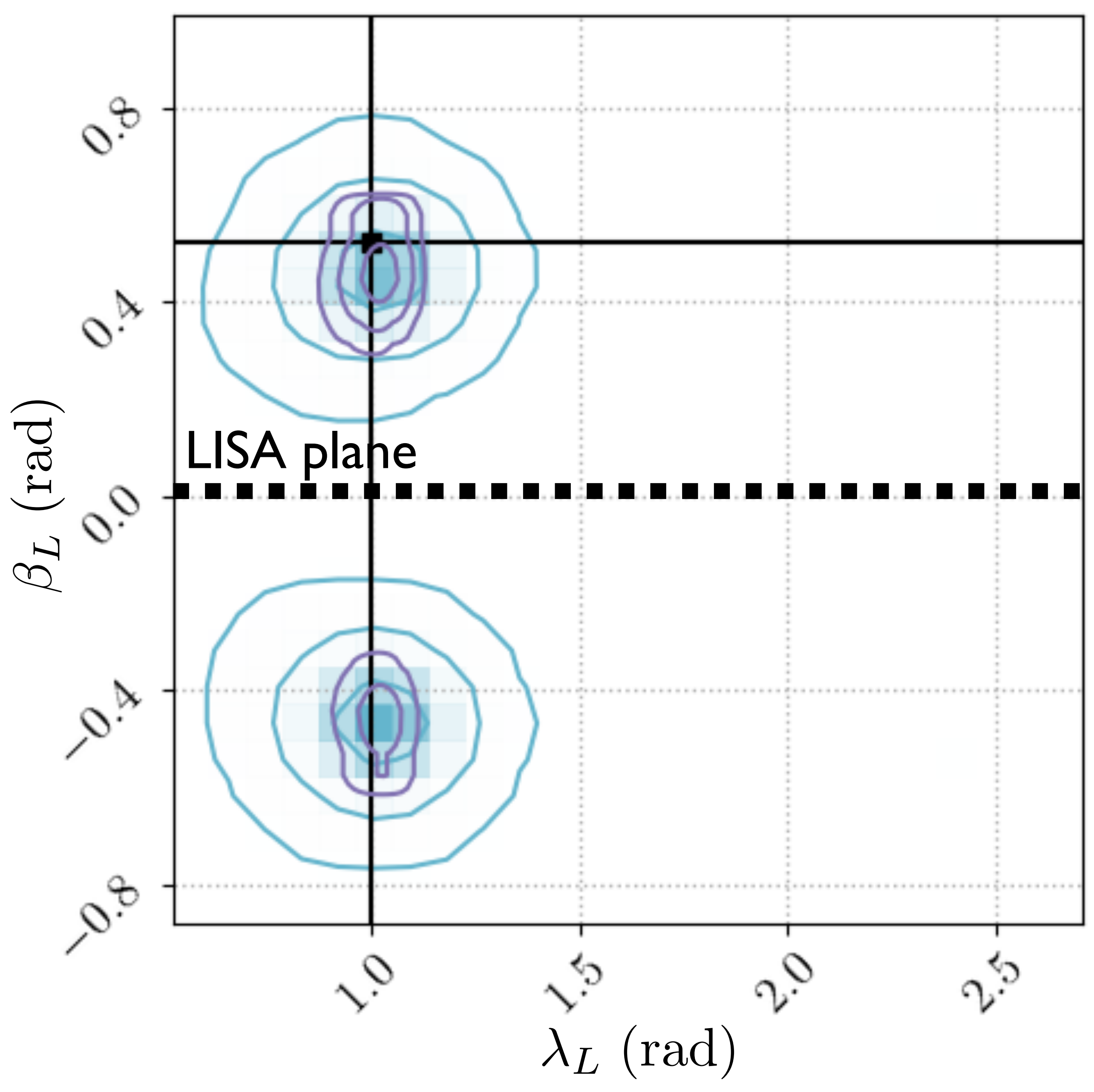
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

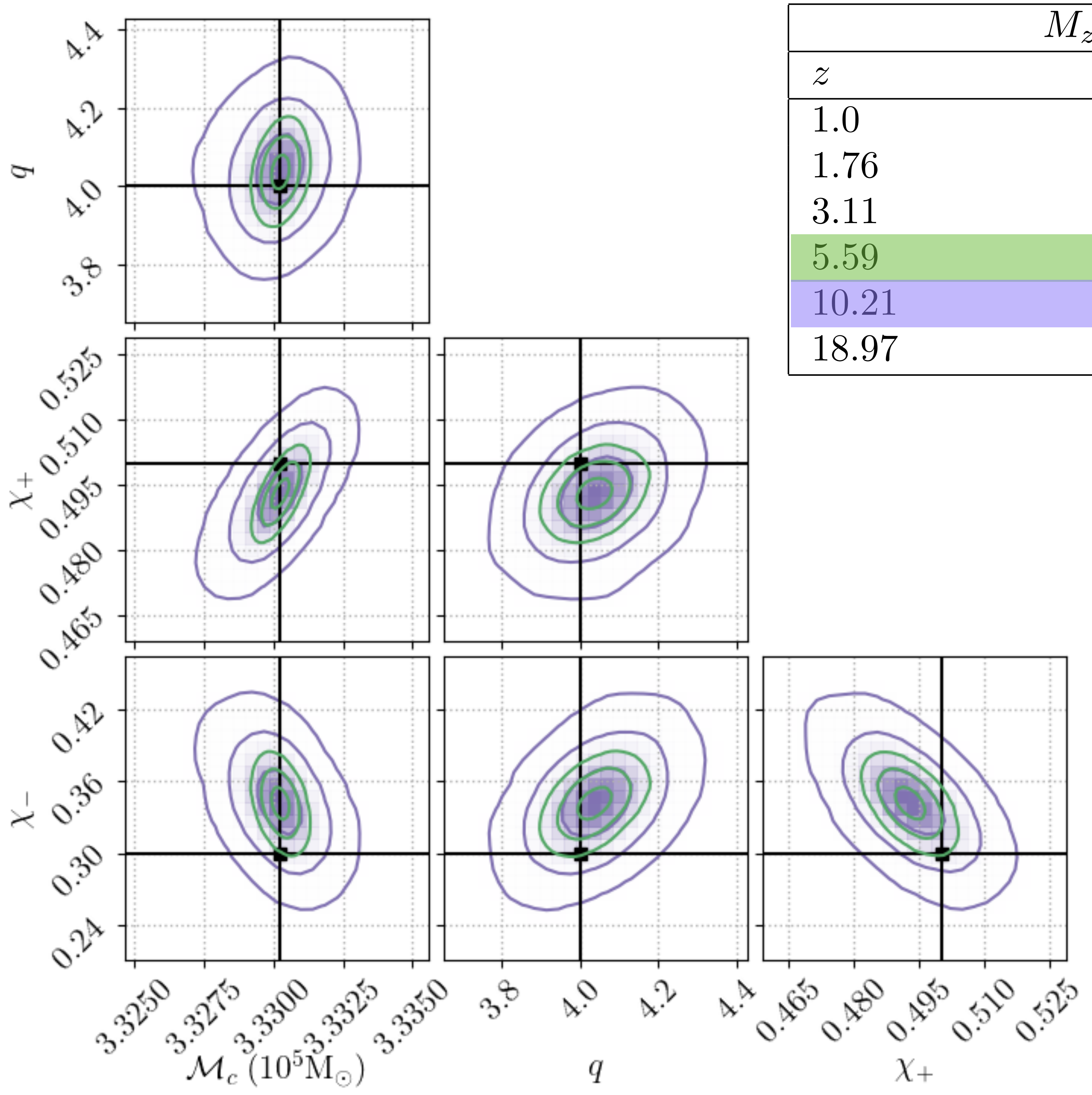
Sky localisation



Example Parameter estimation with systematics II

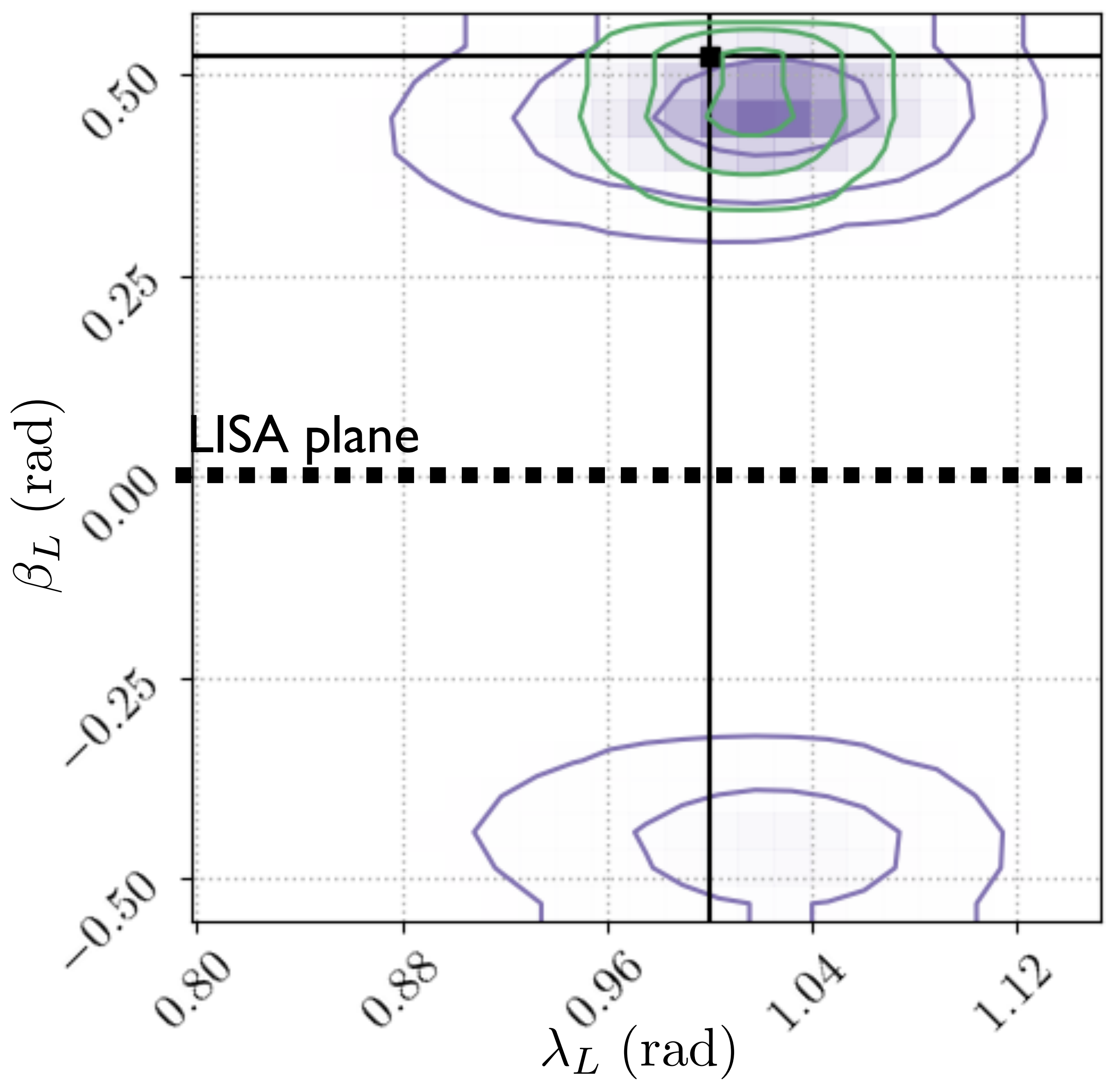
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

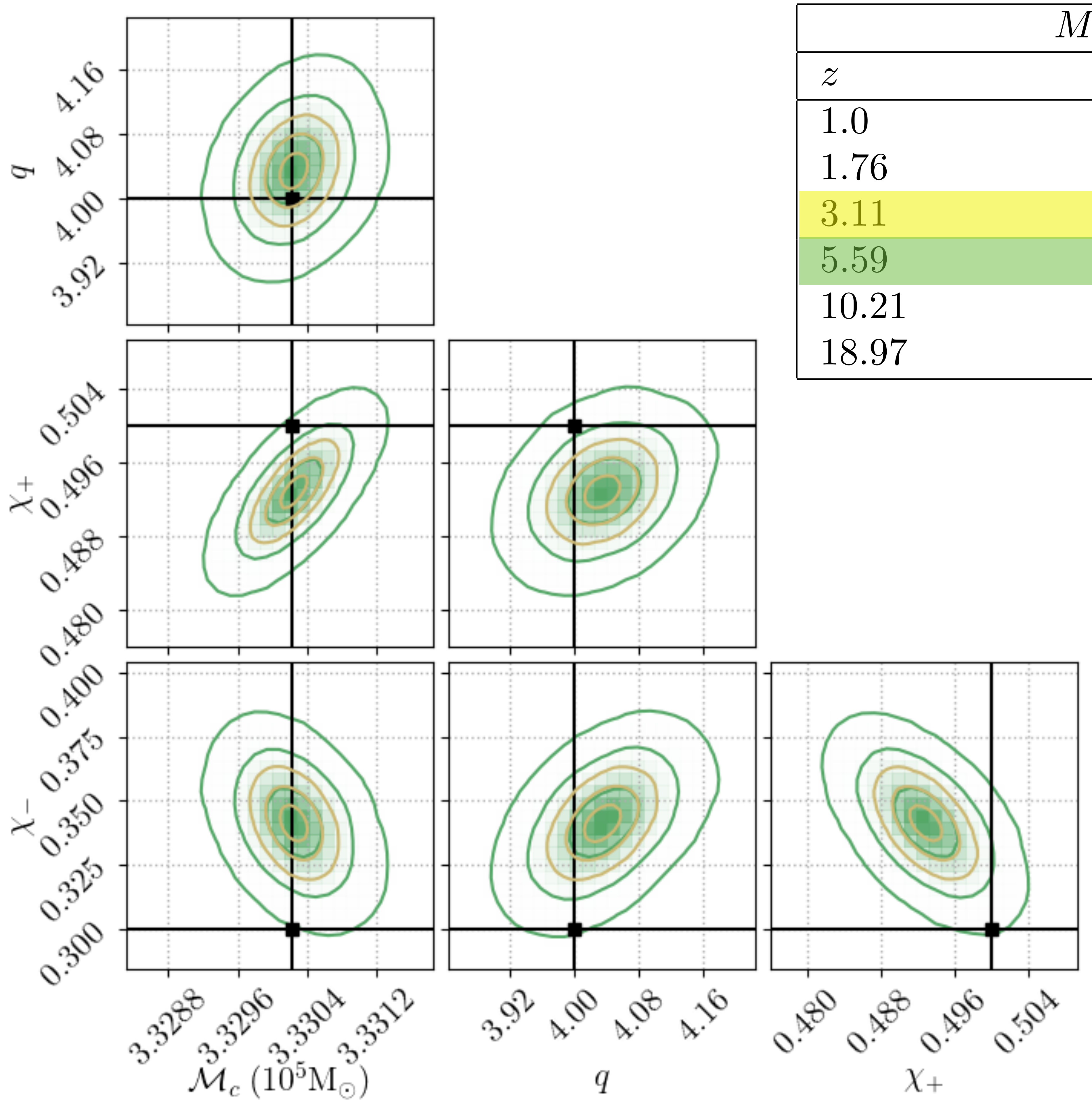
Sky localisation



Example Parameter estimation with systematics II

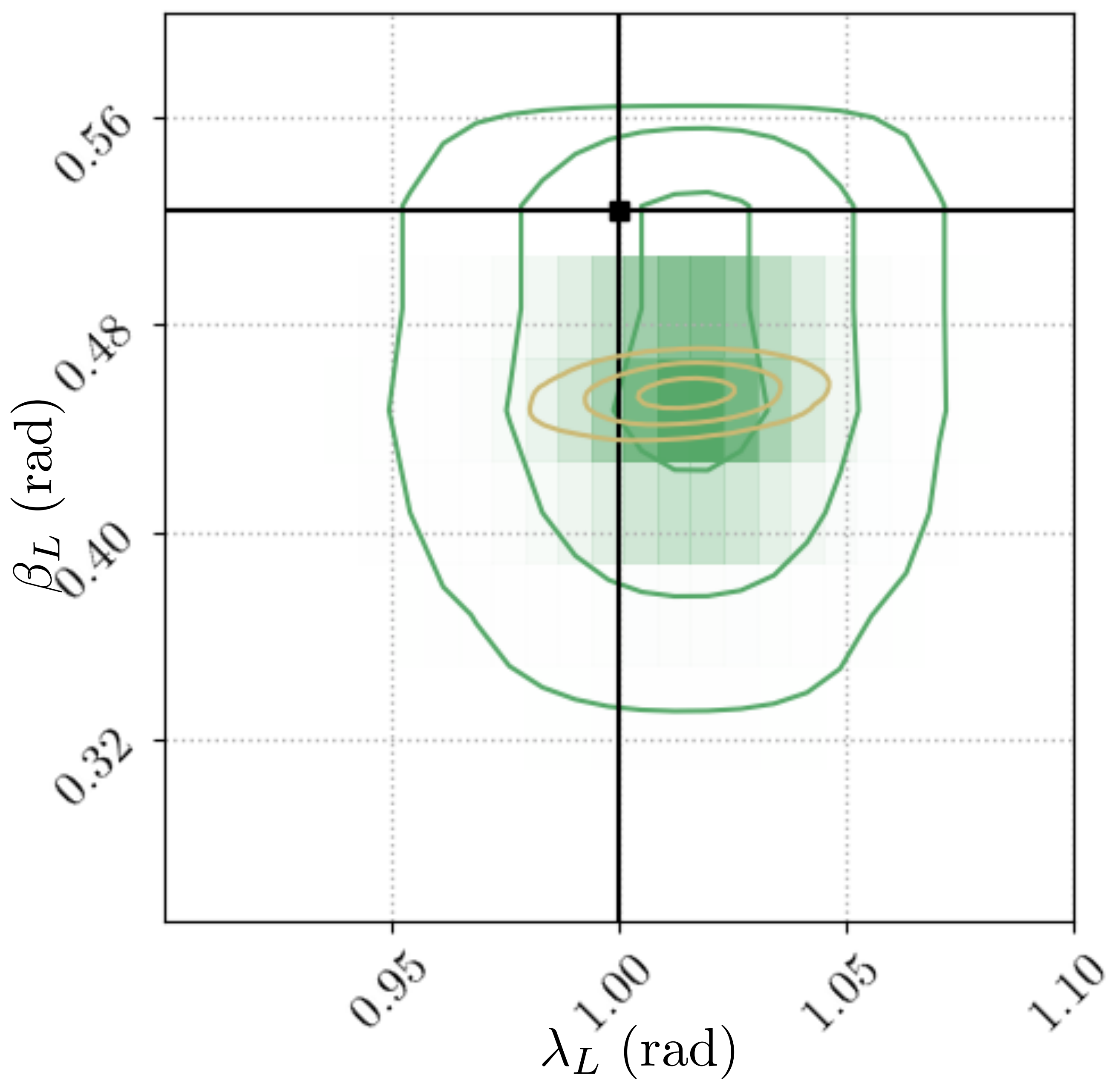
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

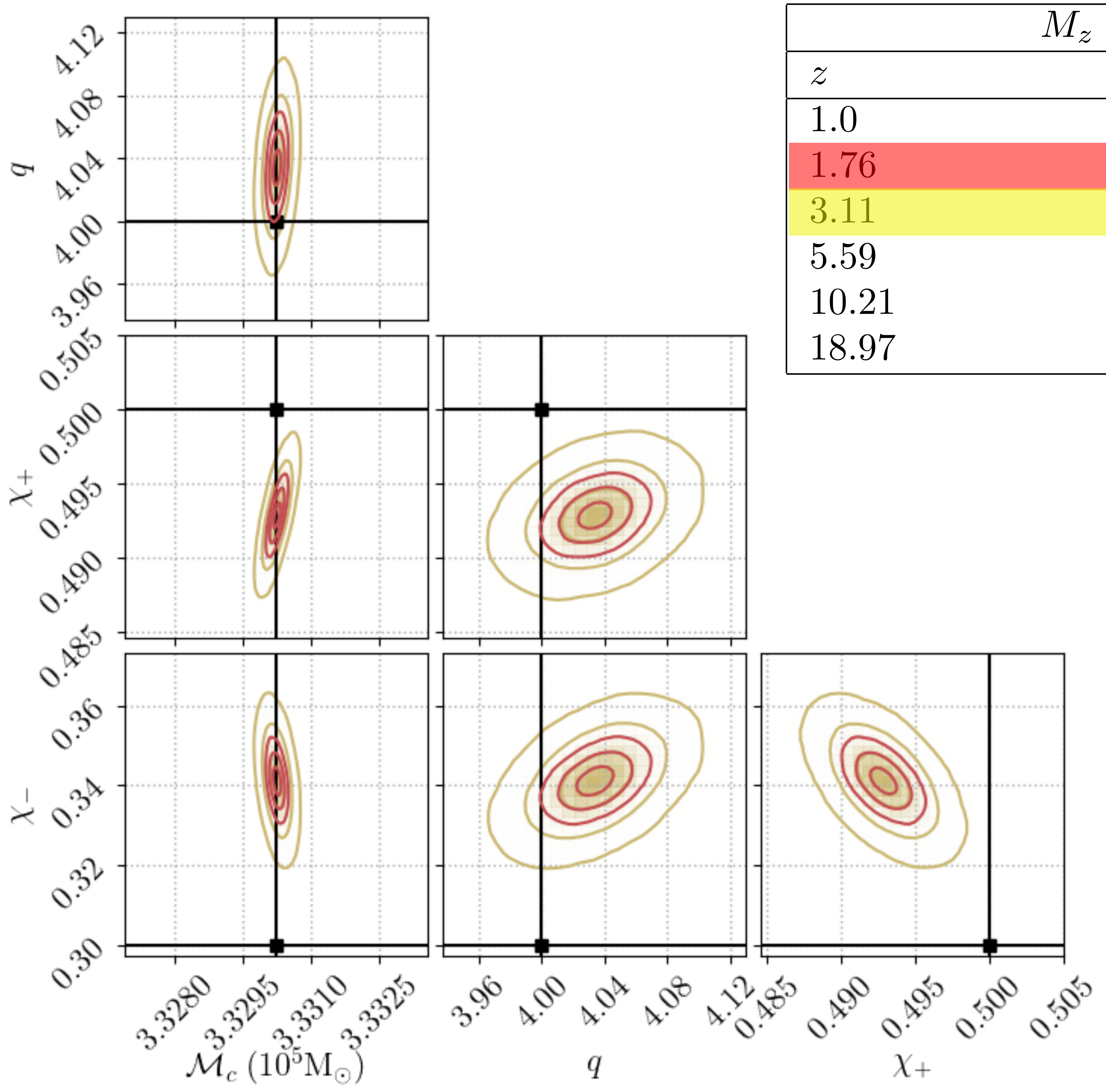
Sky localisation



Example Parameter estimation with systematics II

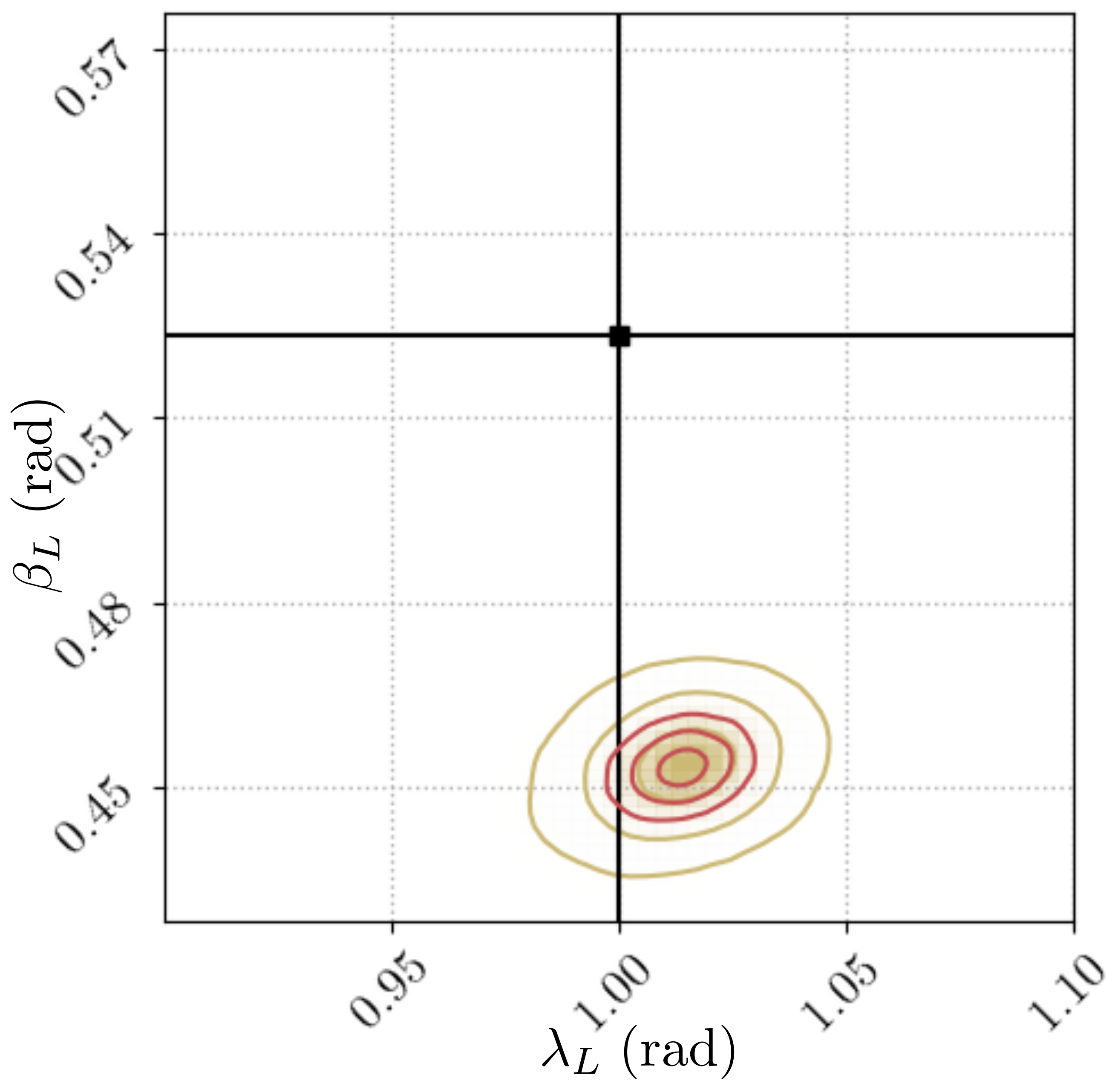
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

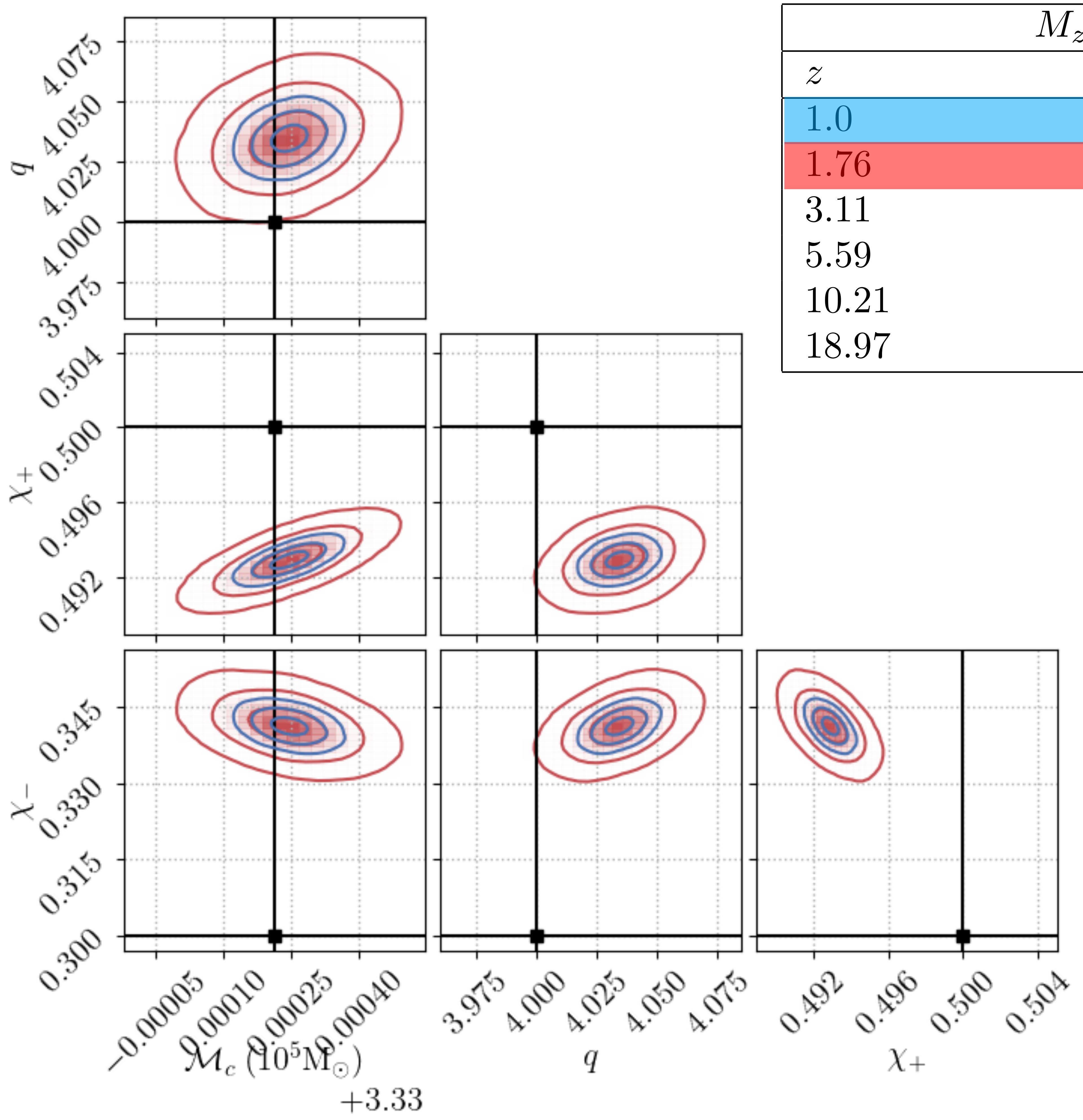
Sky localisation



Example Parameter estimation with systematics II

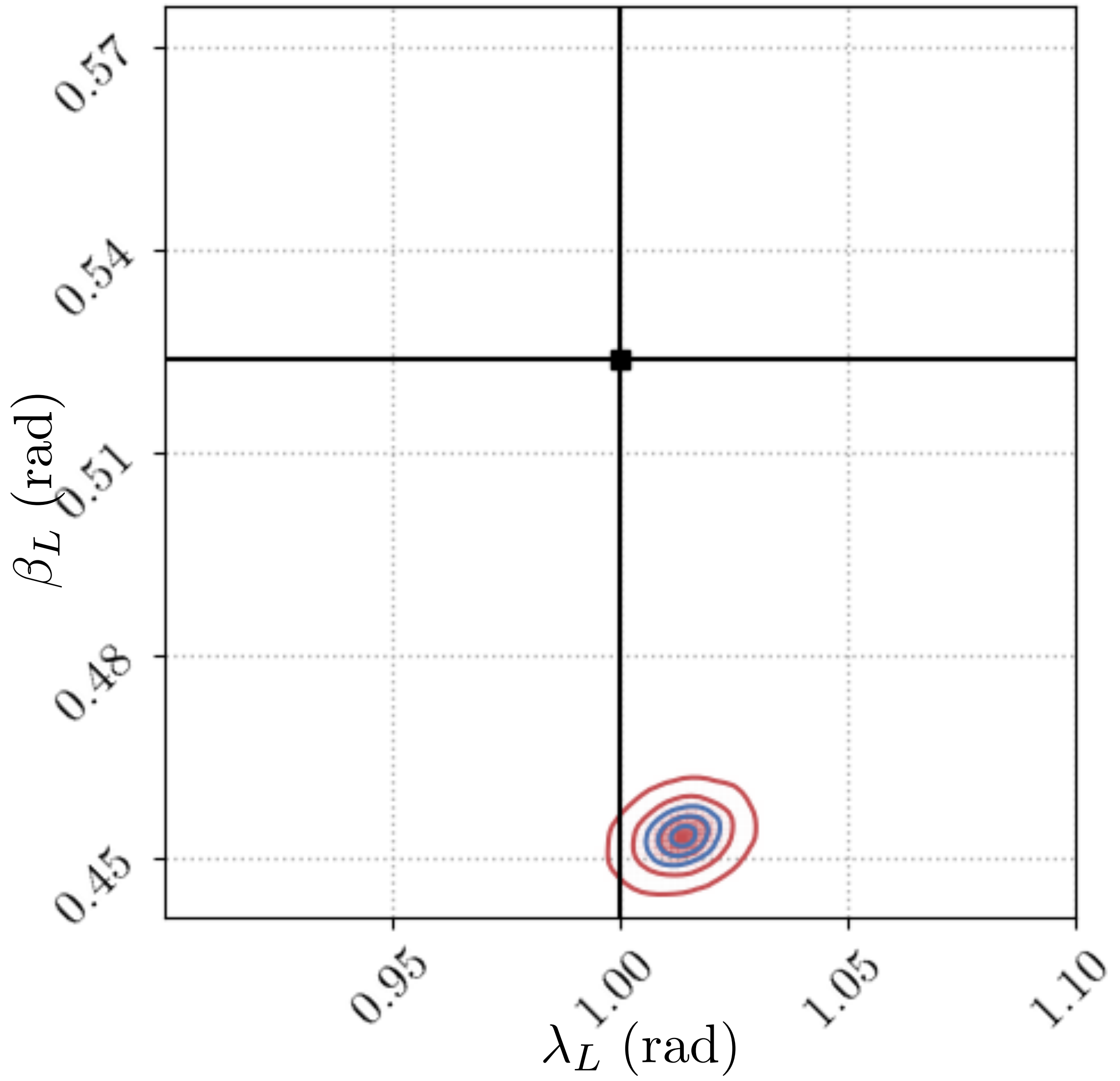
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

Sky localisation

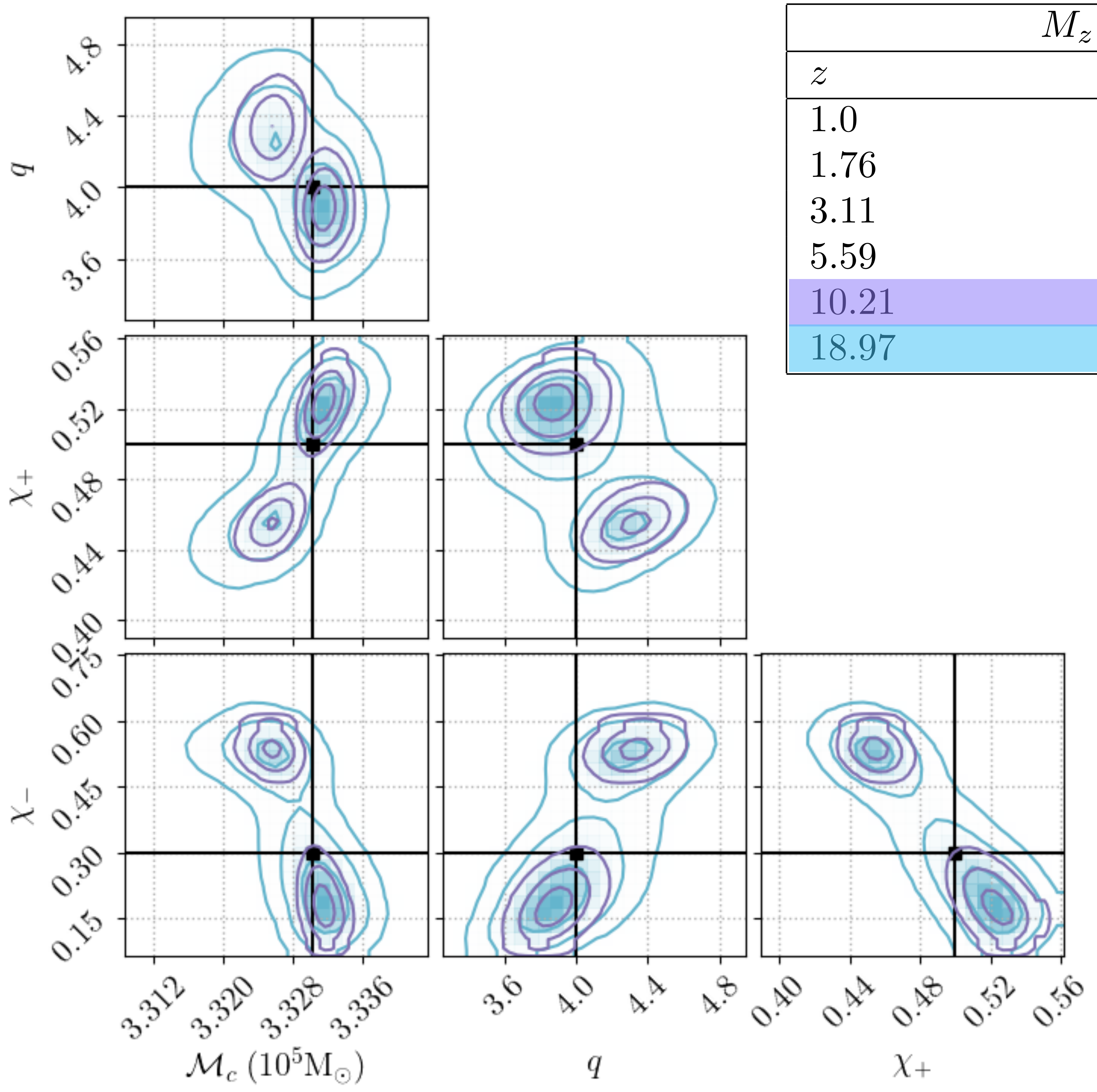


The bad:
- converges 'near' the true parameters
- significant bias at $z = 1$, SNR $= 1907$

Example Parameter estimation with systematics III

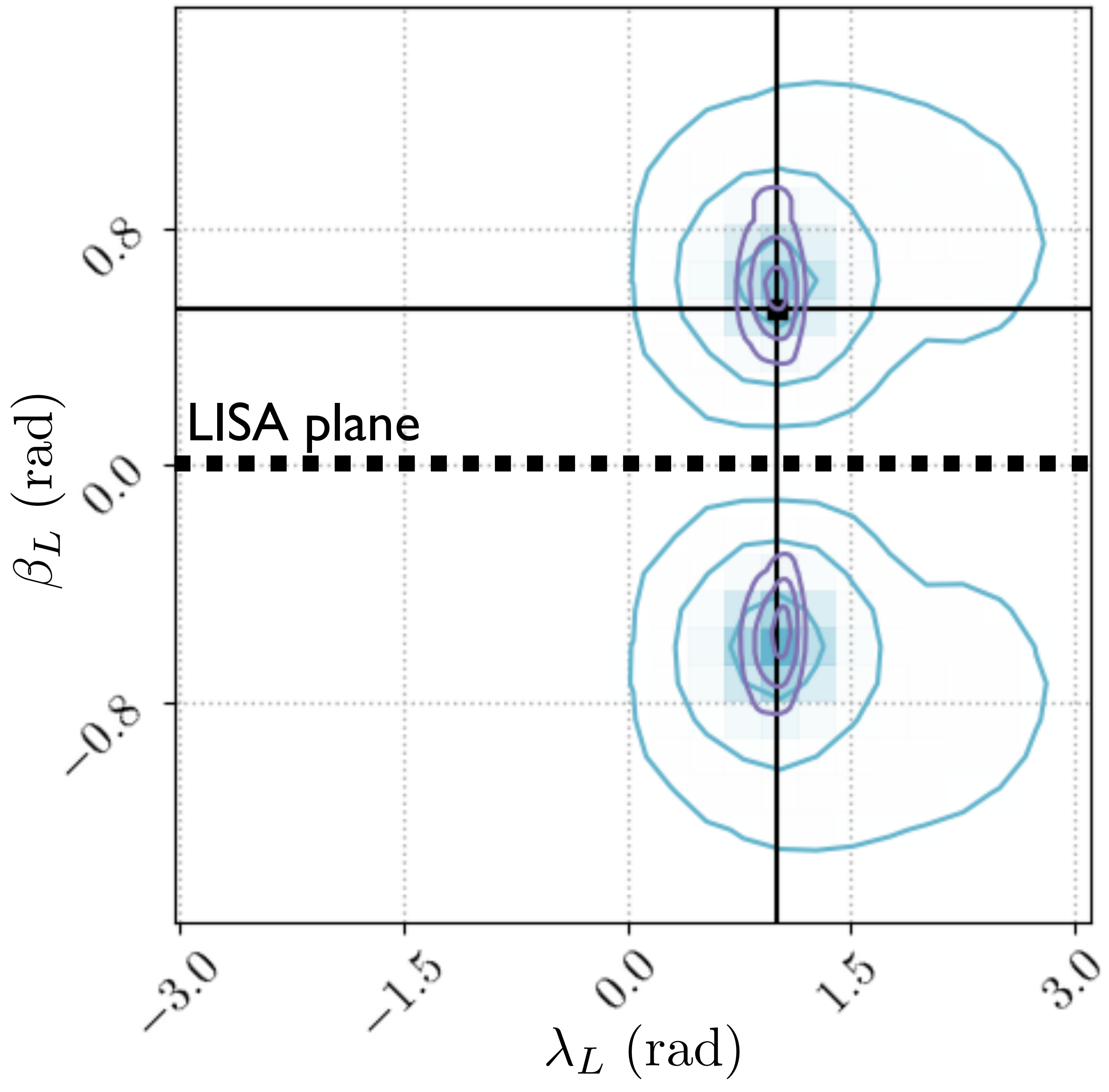
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** SEOBNRv5HM_ROM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

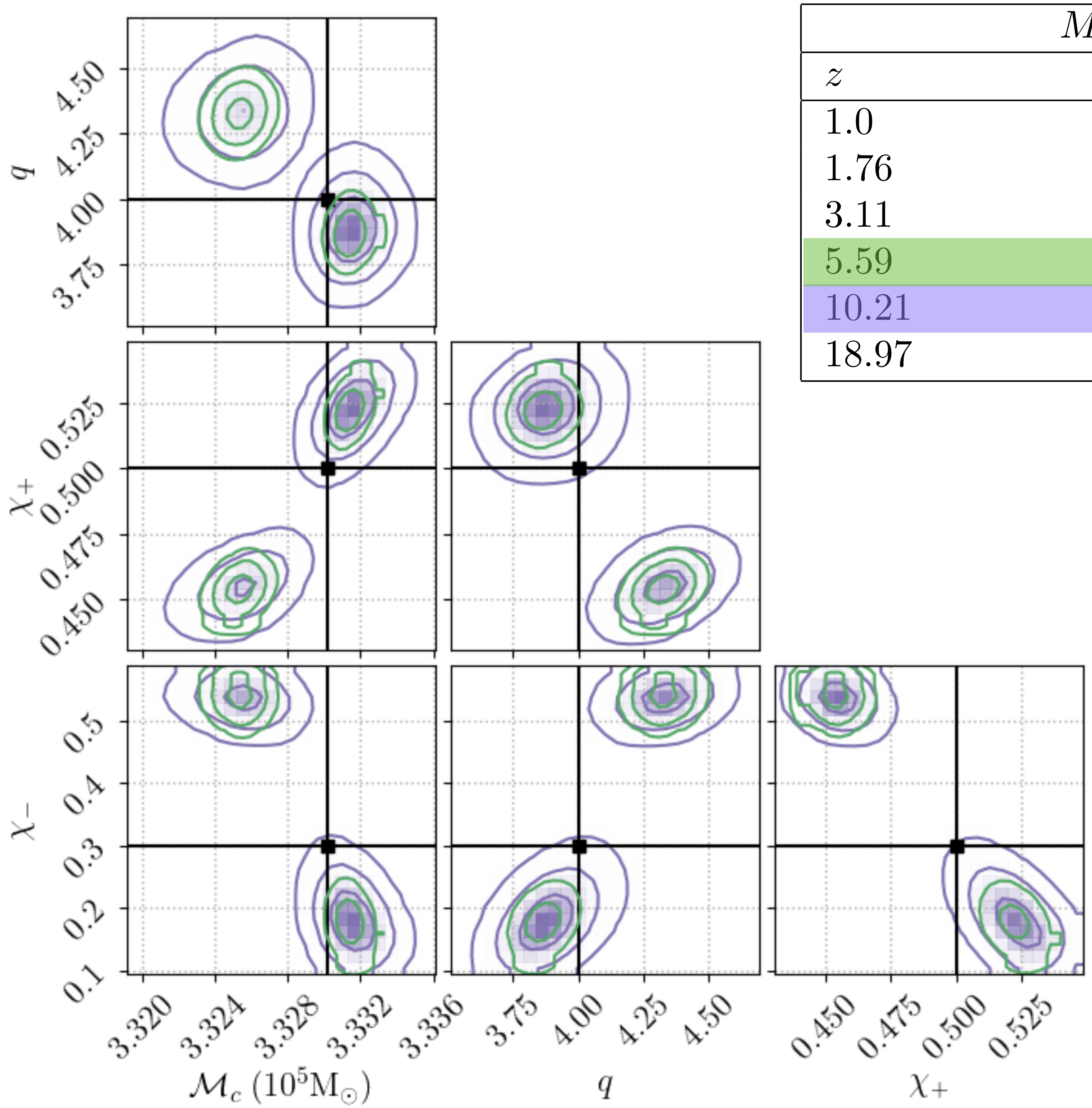
Sky localisation



Example Parameter estimation with systematics III

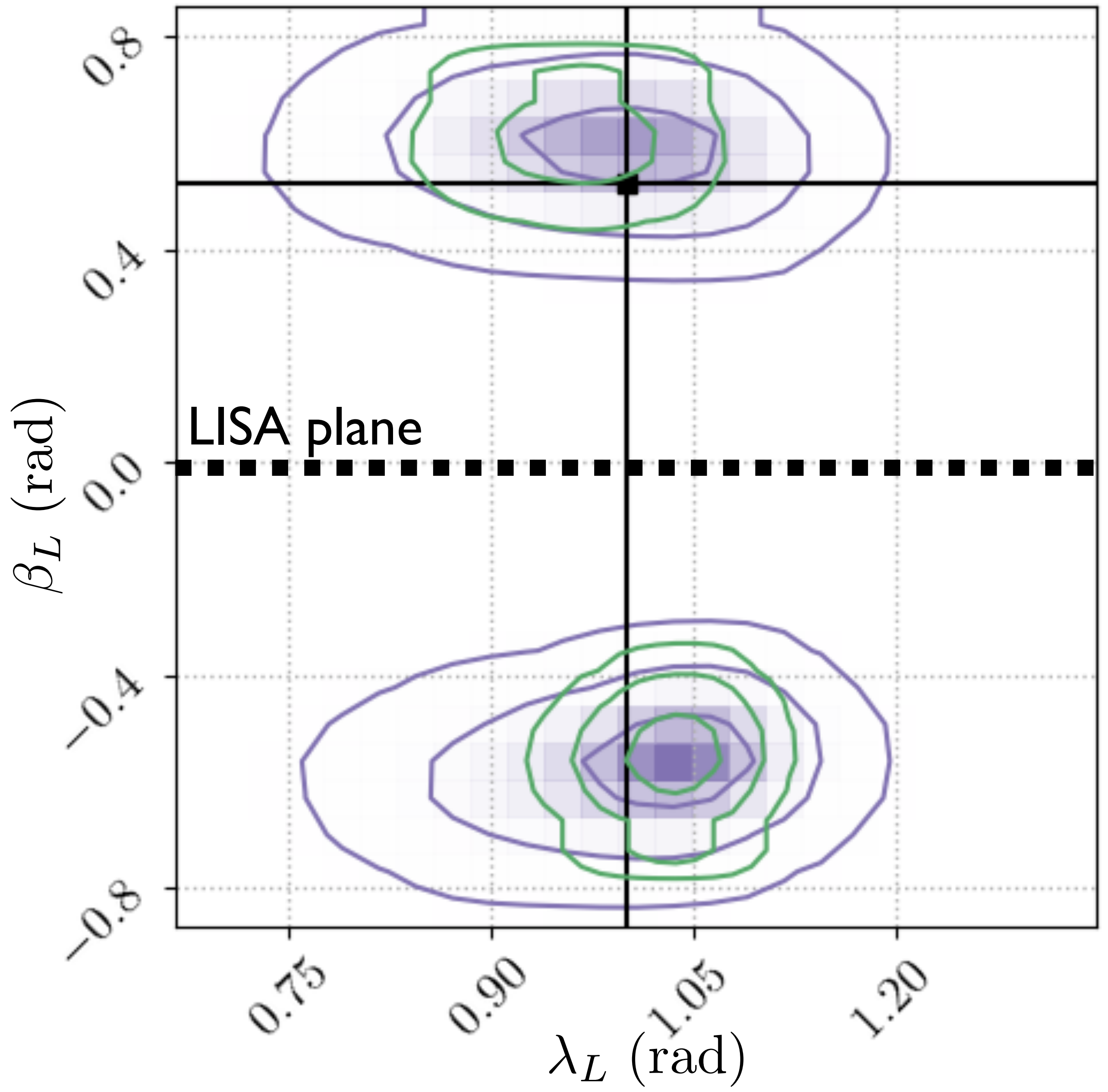
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** SEOBNRv5HM_ROM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

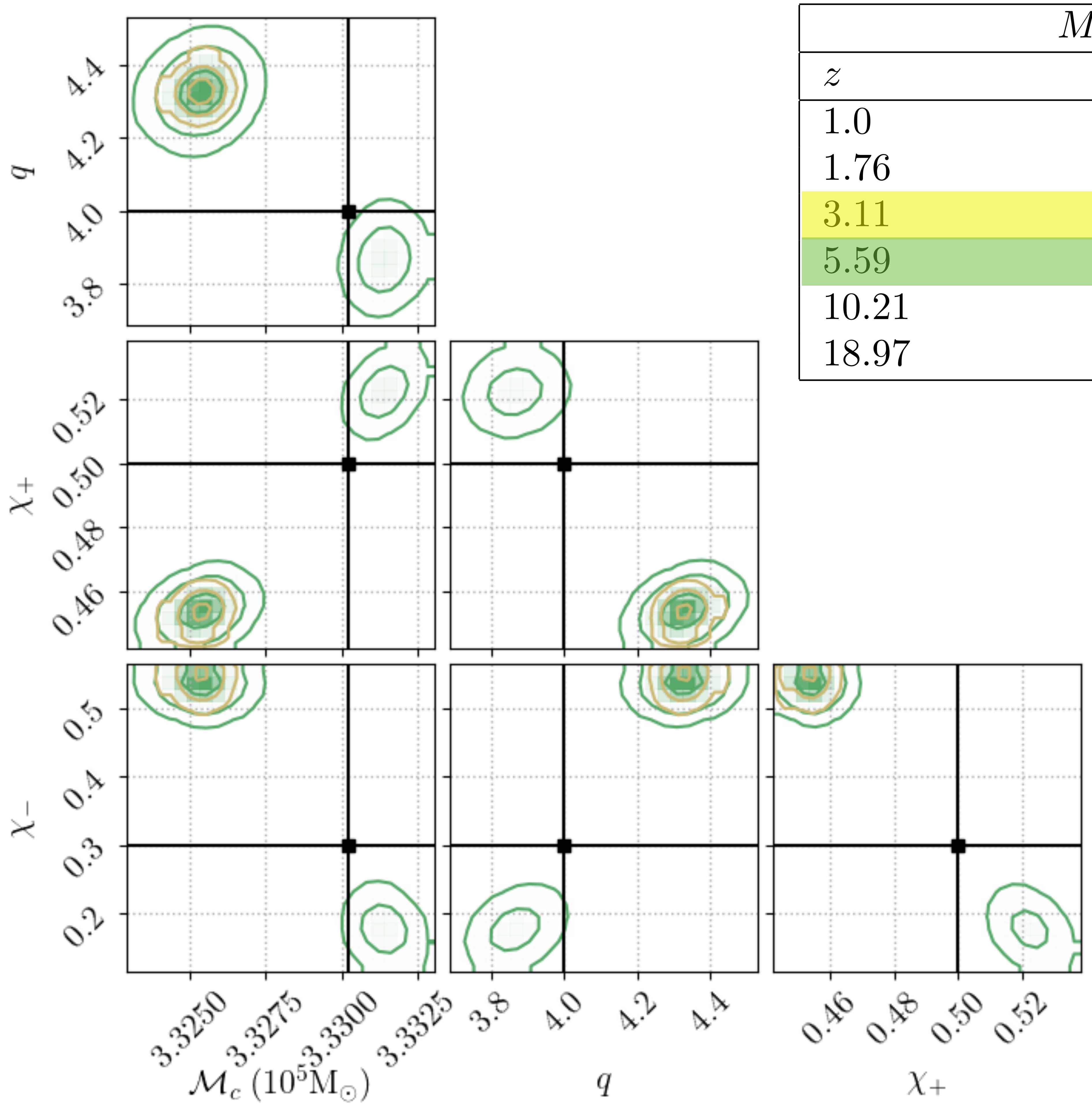
Sky localisation



Example Parameter estimation with systematics III

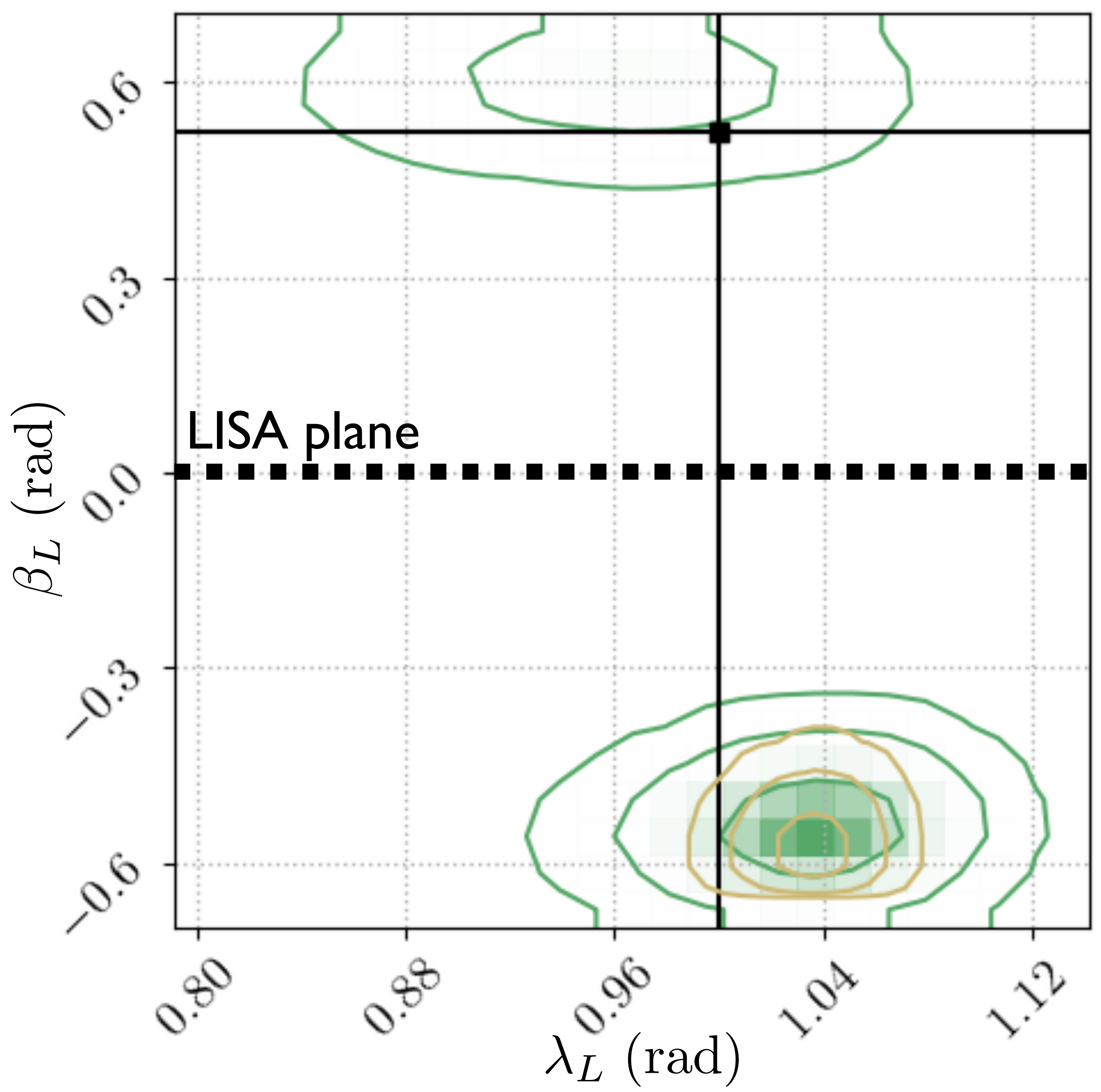
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** SEOBNRv5HM_ROM

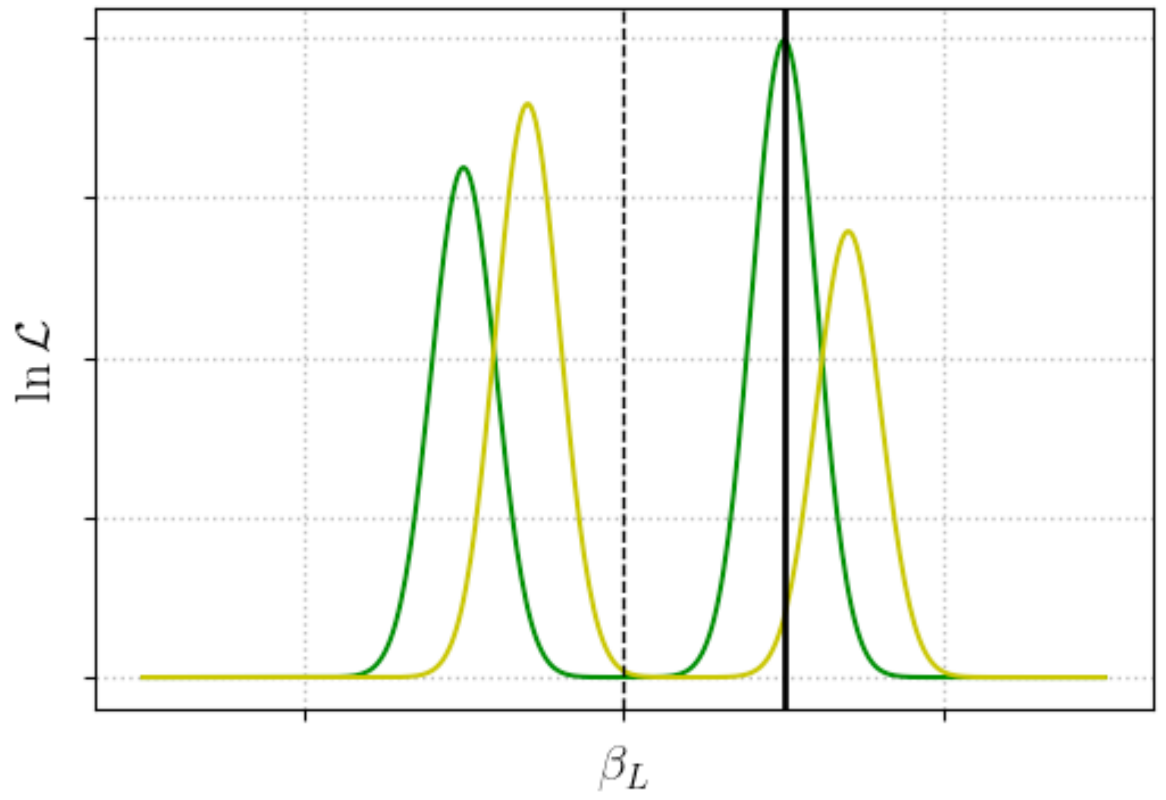
Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

Sky localisation

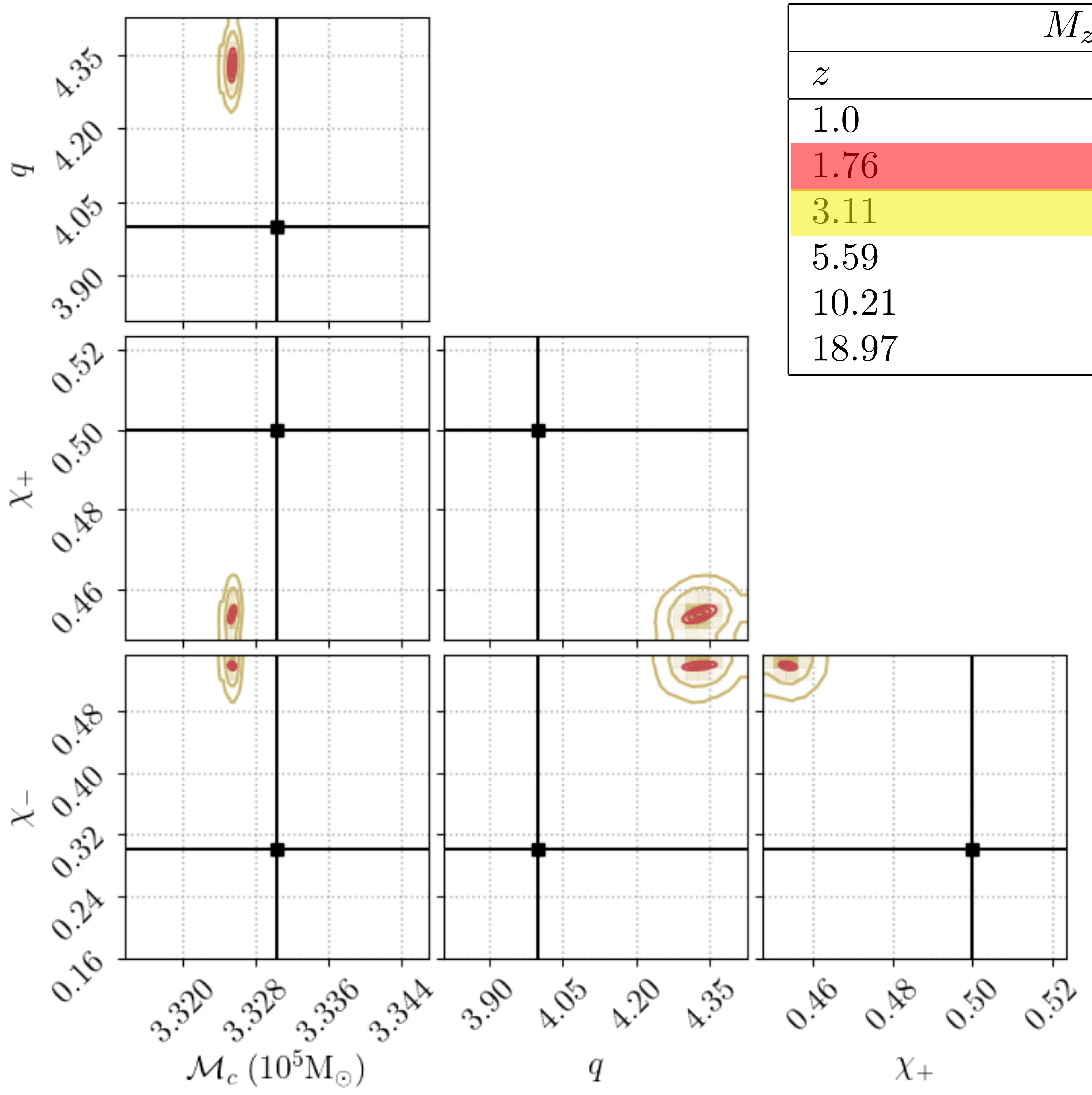




Example Parameter estimation with systematics III

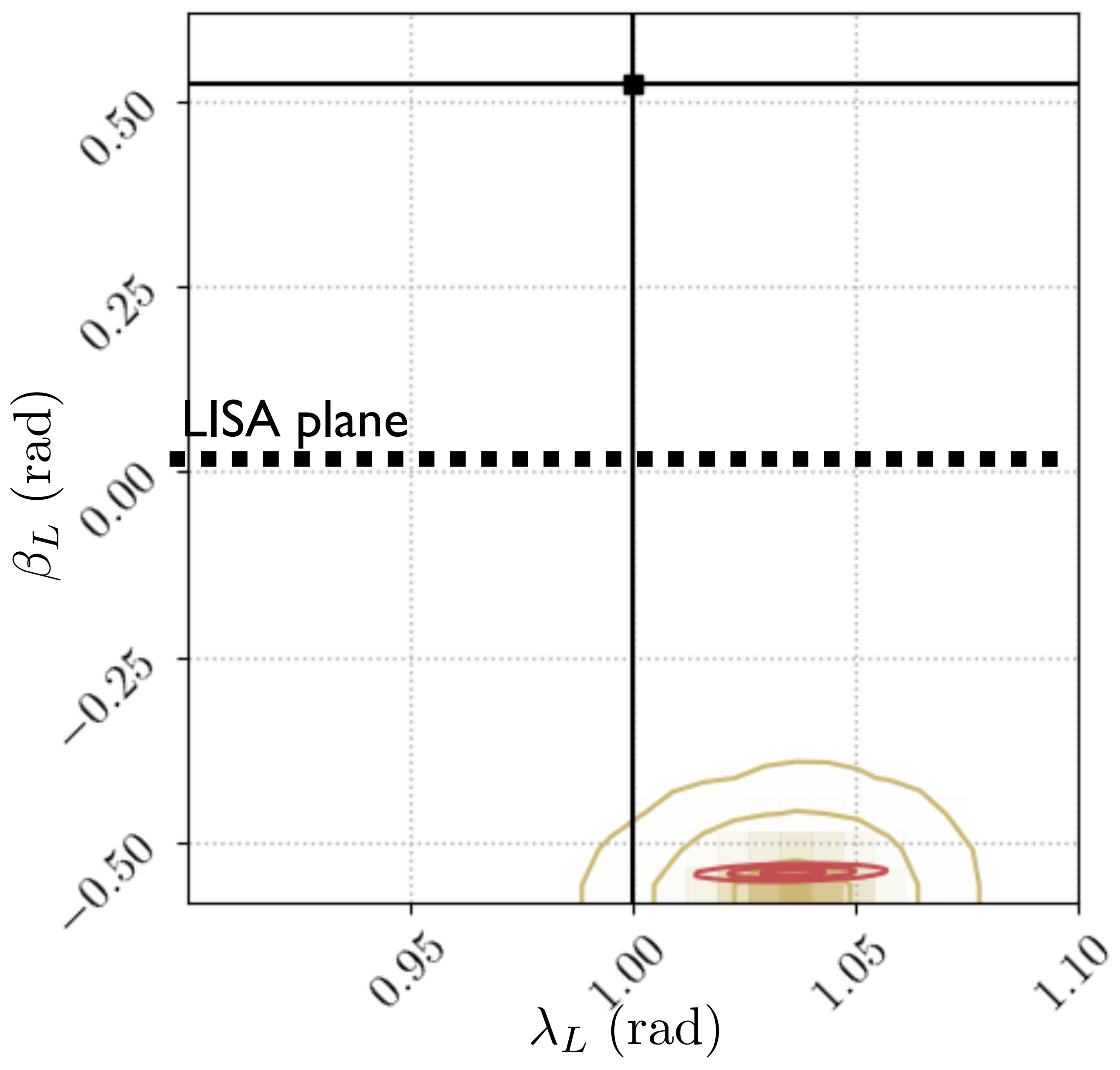
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** SEOBNRv5HM_ROM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

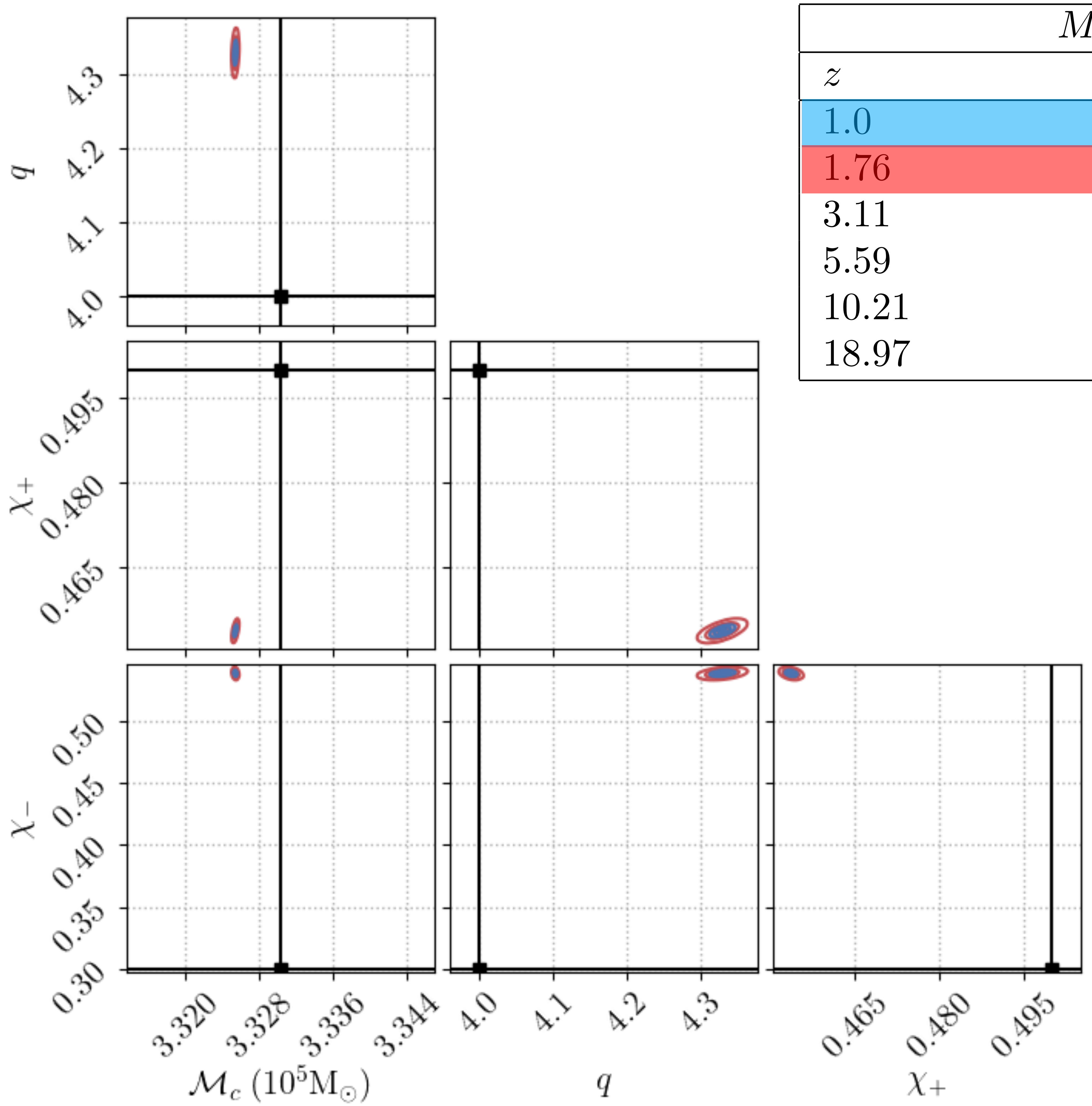
Sky localisation



Example Parameter estimation with systematics III

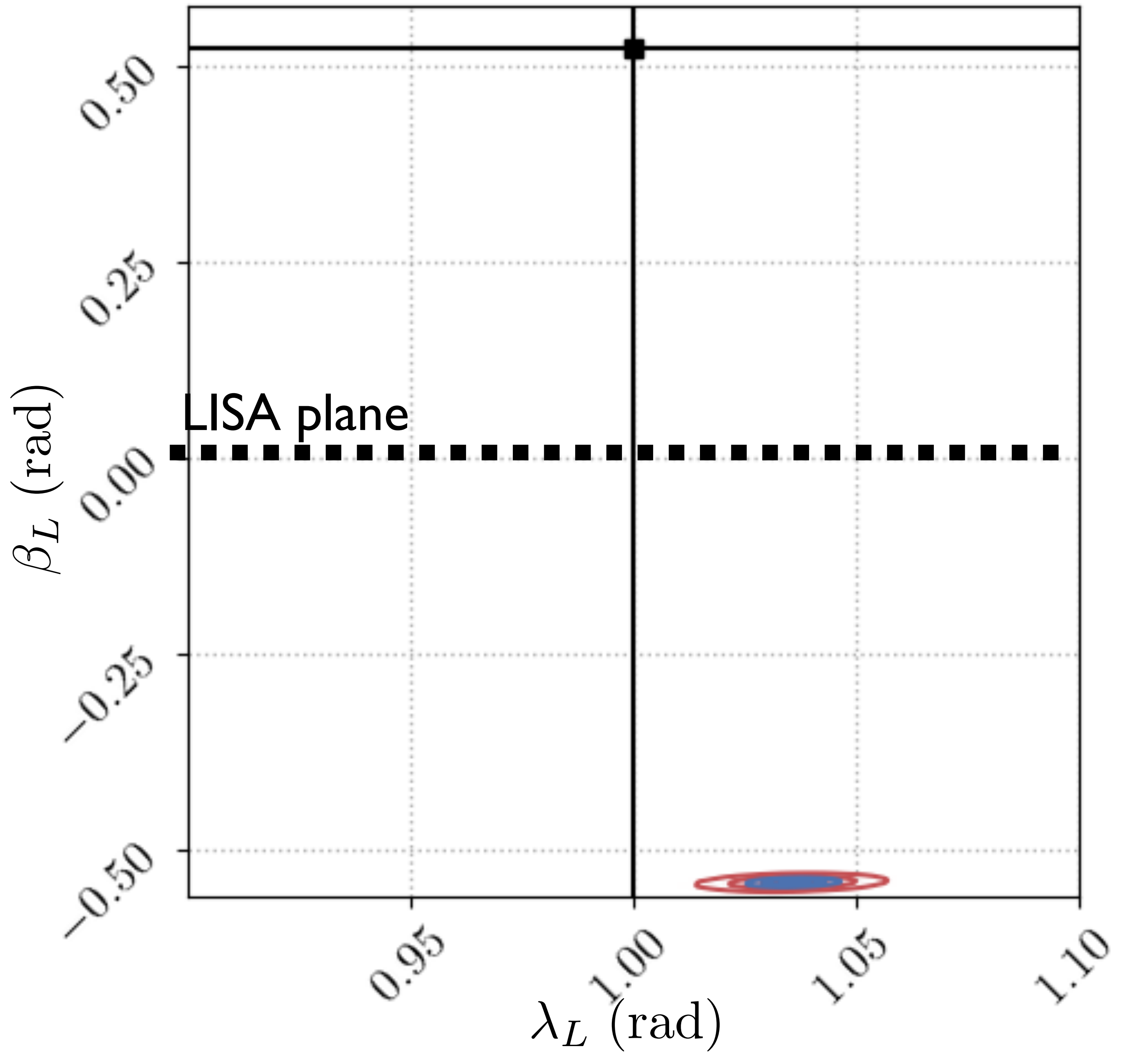
- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** SEOBNRv5HM_ROM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

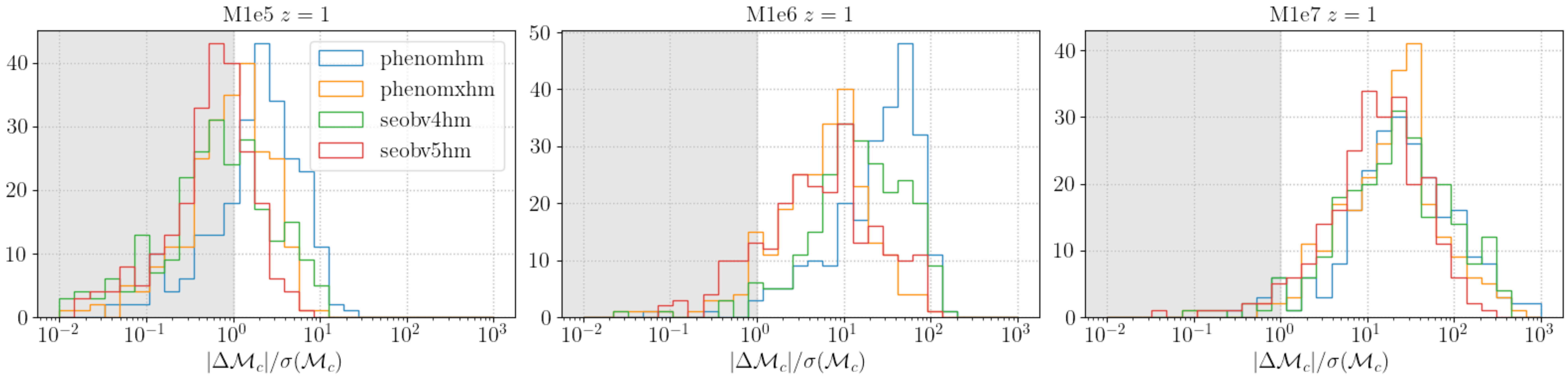
Sky localisation



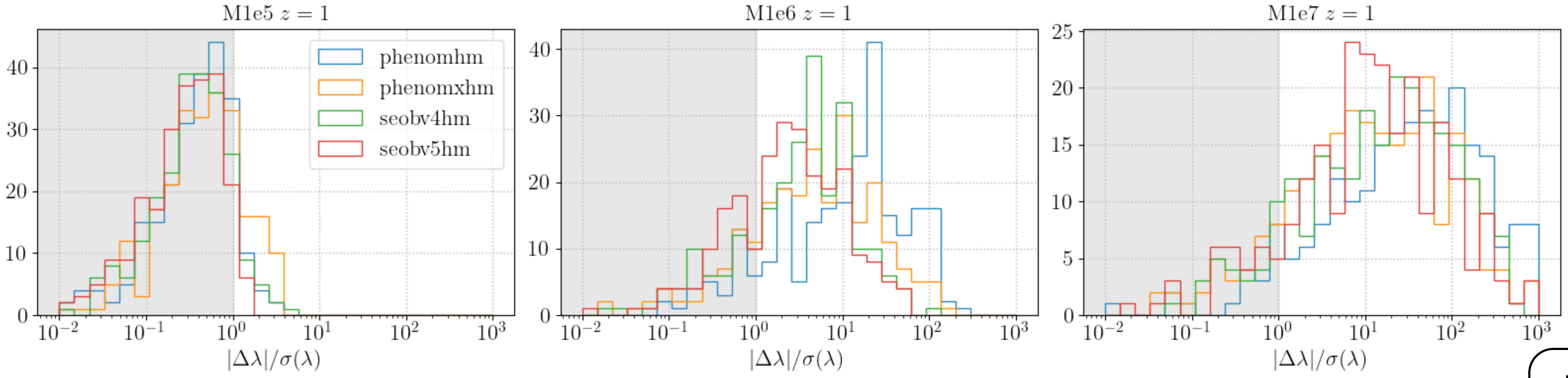
The ugly:
- converges to the **wrong sky mode**
- important bias at $z = 1$, SNR $= 1907$

Statistical significance of biases: intrinsic parameters

Bias in chirp mass:



Bias in longitude (on corrected skymode):



Wrong skymode recovered:

phenomxhm	
$M_z (M_\odot)$	% wrong
10^5	0 %
10^6	10 %
10^7	54 %

Large biases at high mass
Wrong skymodes common

Conclusion and outlook

Highlights

- LISA localisation capabilities for MBHBs crucial for multimessenger science
- MBHB signals are merger-dominated, post-merger localisation can be very good
- Main mode localization: from pattern response at high mass, inclination/latitude dominated
- Pre-merger localisation can be challenging, except for the very best events
- Degeneracies in the sky position can occur, worse pre-merger
- Systematics: possibly strong for high-mass signals, can also mislead towards the wrong sky mode

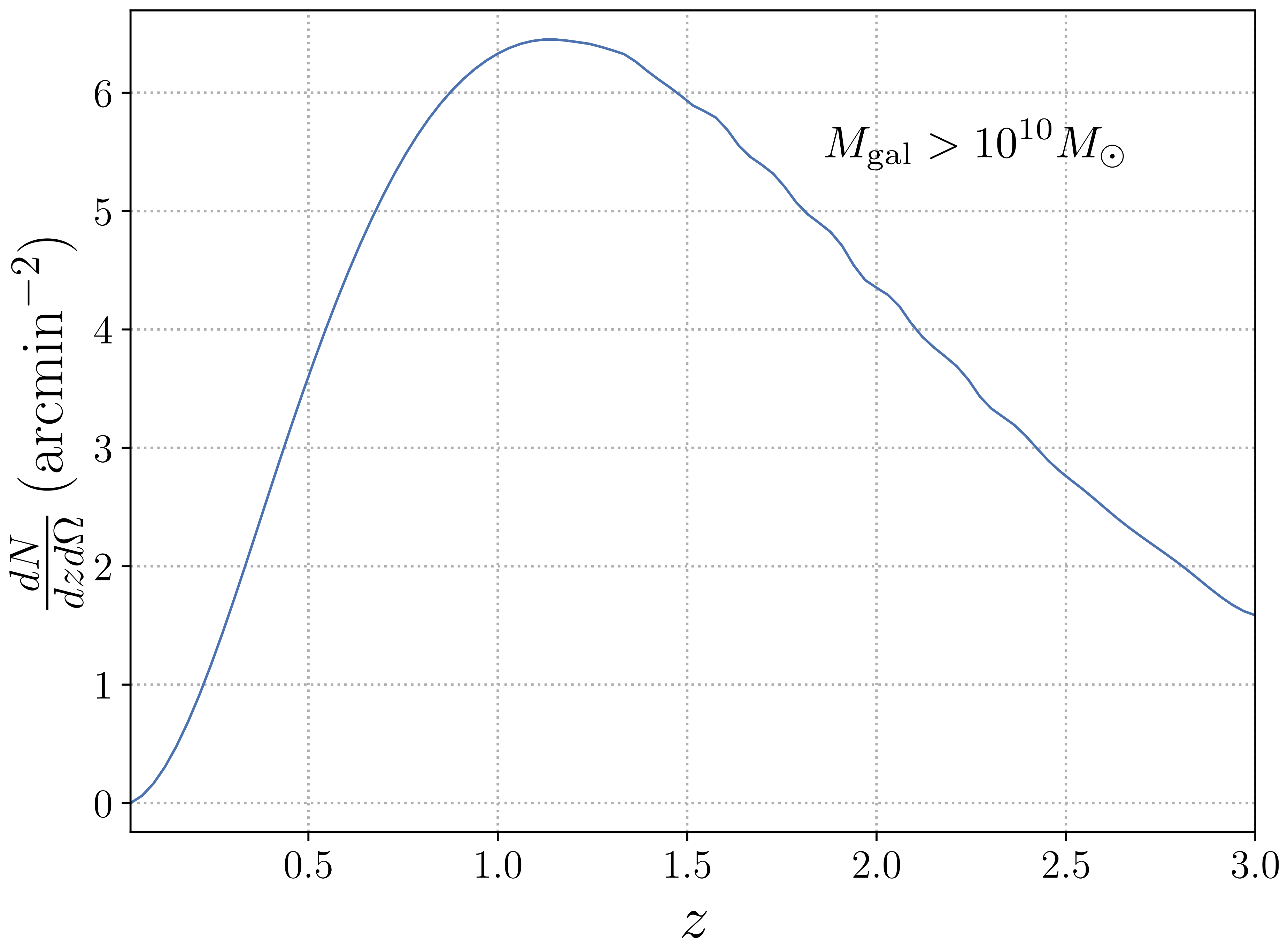
Outlook

- More realistic waveforms: precession and eccentricity
- More realistic analysis: proper time-domain analysis, superposition of multiple signals, realistic noise, data gaps, glitches…

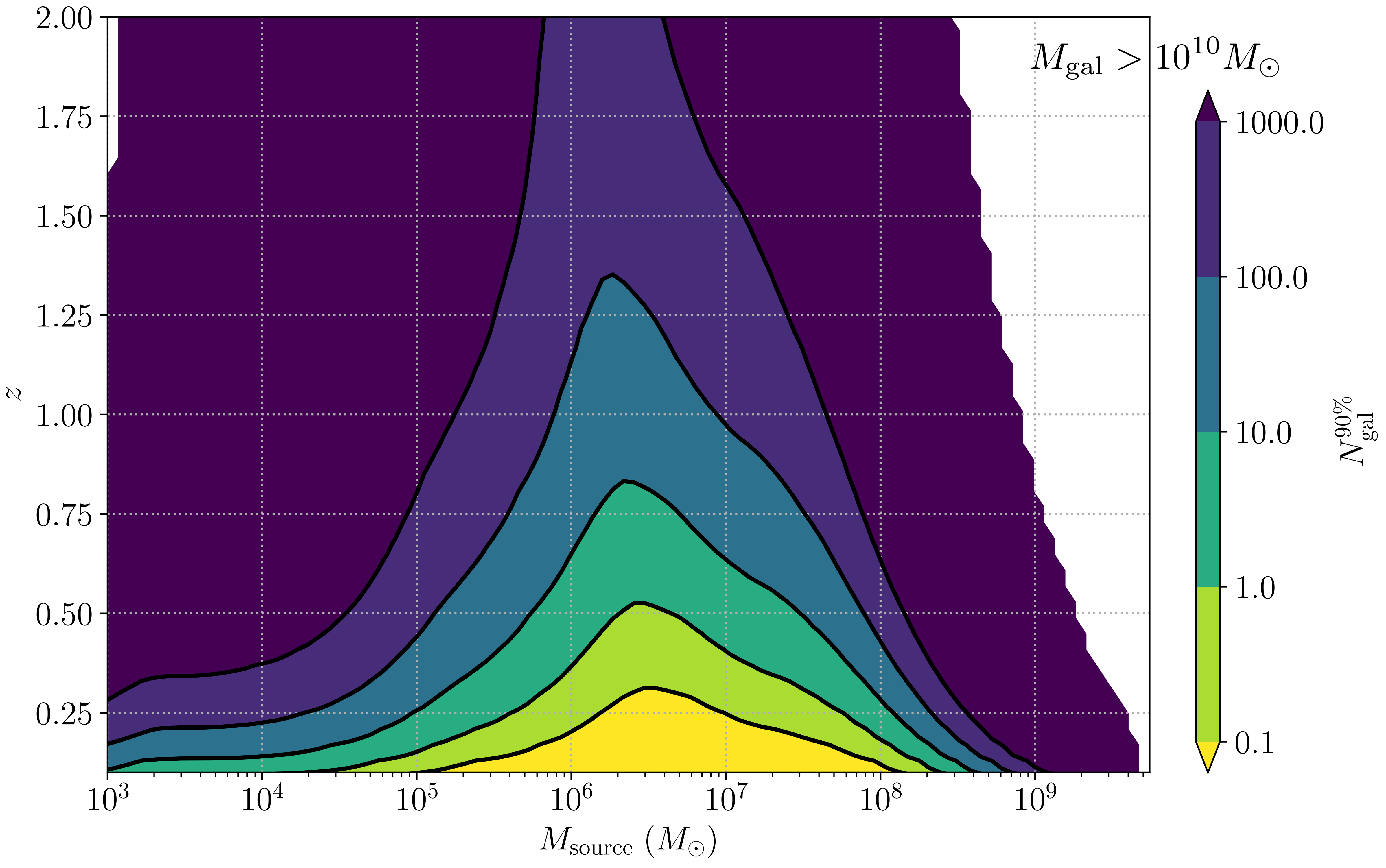
Galaxy counts in the LISA + weak lensing error box

Simplistic estimate: simulated catalog cut in mass with no consideration of completeness, EM emissions…

Simulated catalog courtesy of [D. Izquierdo-Villalba&al]



Weak lensing and peculiar motions have to be taken into account



[see also Lops&al 2022]

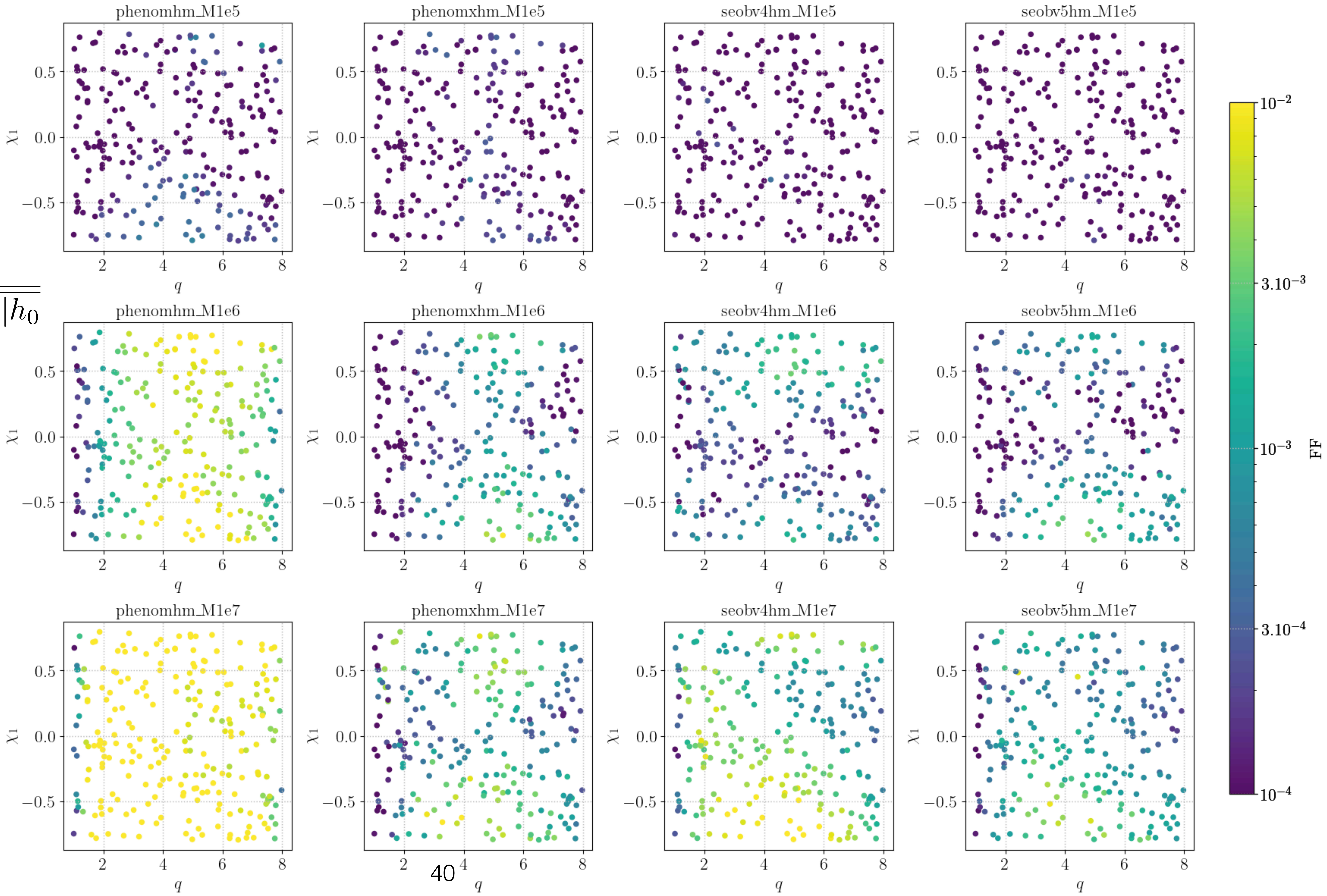
More detailed study needed about the content of the error box !

Fitting factors in parameter space

Fitting factor, computed at best-fit params (optimization over all parameters):

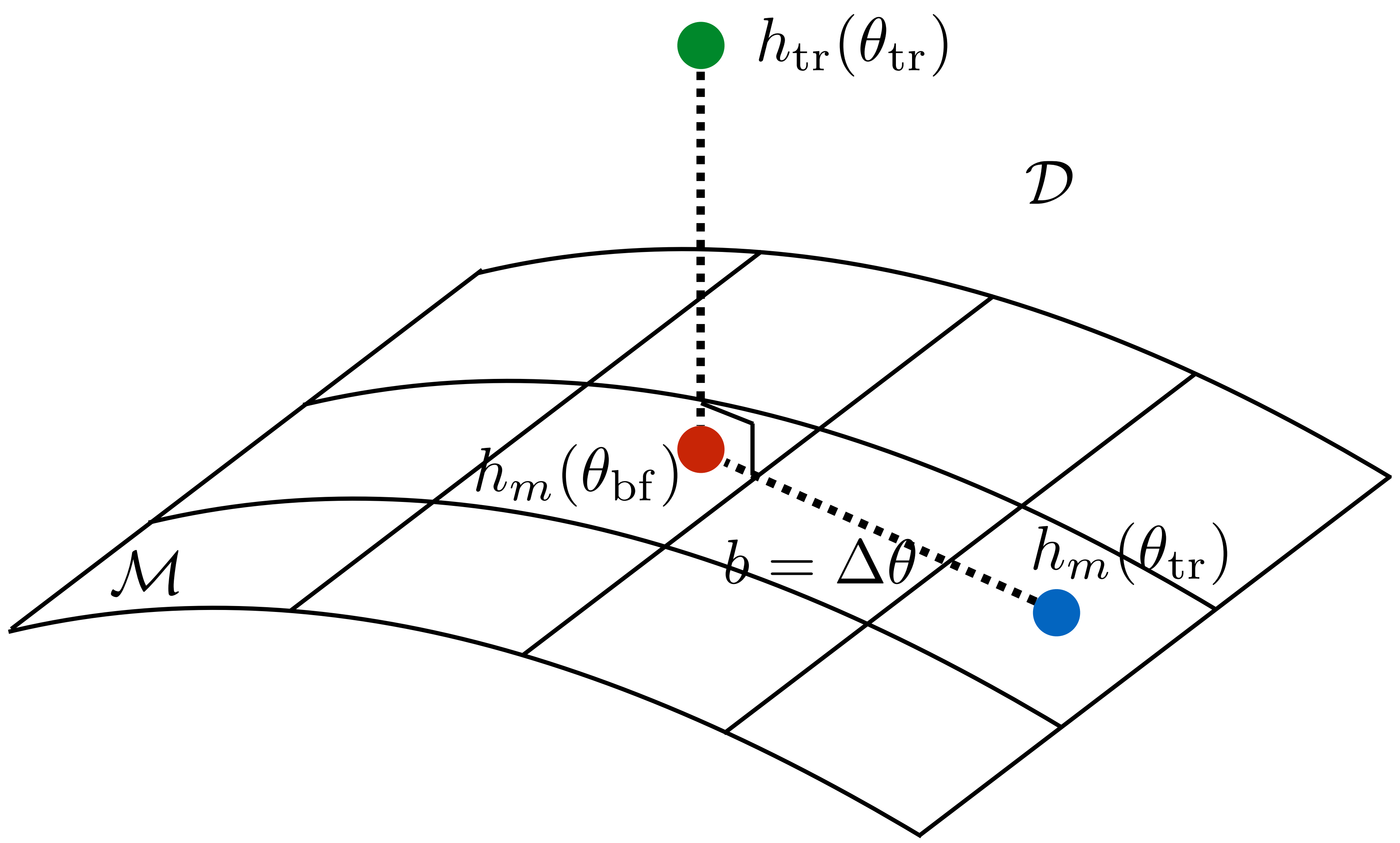
$$\mathrm{FF} = 1 - \max_{\boldsymbol{\theta}} \frac{(h_m|h_0)}{\sqrt{(h_m|h_m)}\sqrt{h_0|h_0}}$$

$$\ln \mathcal{L}_{\max} = -\mathrm{SNR}^2 \times \mathrm{FF}$$



Trend: strong dependence on M
Trend: larger errors at large spins
Trend in q ?

Waveform systematics and parameter estimation



Mismatch (unfaithfulness):

Mismatch, optimization over time/phase/polarization:

$$\mathrm{MM} = 1 - \max_{t,\varphi,\psi,\dots} \frac{(h_m|h_{\rm tr})}{\sqrt{(h_m|h_m)}\sqrt{(h_{\rm tr}|h_{\rm tr})}}$$

- Computed locally **[fast]**
- SNR-independent
- Different versions: single-detector optimized over sky, combining $h_+, h_\times$

Indistinguishability criterion:

[Lindblom&al 2008]
[Chatziioannou&al 2019]
[Toubiana-Gair 2024]

$$\ln \mathcal{L}(\theta) = -\frac{1}{2}(h(\theta) - h_{\rm tr}|h(\theta) - h_{\rm tr})$$

$$\ln \mathcal{L}(\theta_{\rm bf}) \sim \ln \mathcal{L}(\theta_{1-\sigma})$$

$$\mathrm{MM} < \frac{D}{2}\frac{1}{\mathrm{SNR}^2}$$

- Constant D: dimension, approximate
- Scaling SNR^2 robust

Linearized biases (Cutler-Vallisneri):

[Flanagan-Hughes 1997]
[Cutler-Vallisneri 2007]

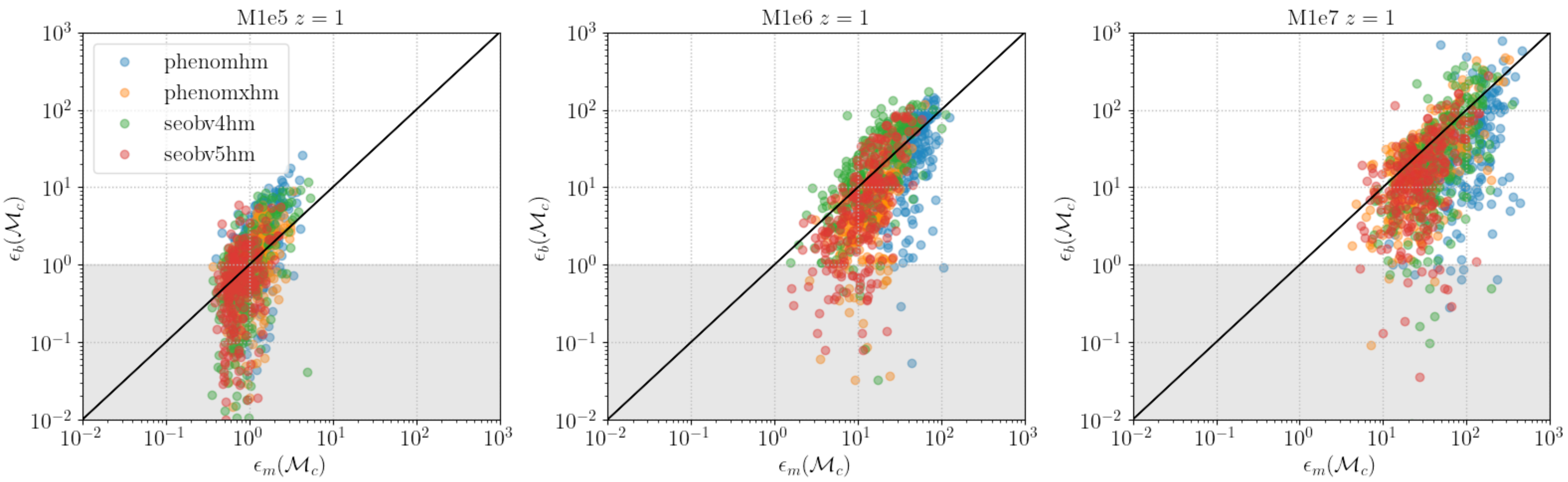
In the linear signal approximation, estimation of bias **[fast]**:

$$F_{ij} = (\partial_i h|\partial_j h)$$

$$\Delta\theta_i = F^{-1}_{ij}(\partial_j h|\delta h)$$

Can we assess biases with efficient tools ?

Linking mismatches and biases



From indistinguishability criterion:

$$\mathrm{MM} < \frac{D}{2}\frac{1}{\mathrm{SNR}^2}$$

$$\epsilon_m = \sqrt{\frac{2}{D}\mathrm{SNR}^2\mathrm{MM}}$$

$\epsilon_m > 1$ means that the mismatch is large enough to indicate a significant bias

From bias measured in PE:

$$\epsilon_b = \frac{\Delta\theta}{\sigma(\theta)}$$

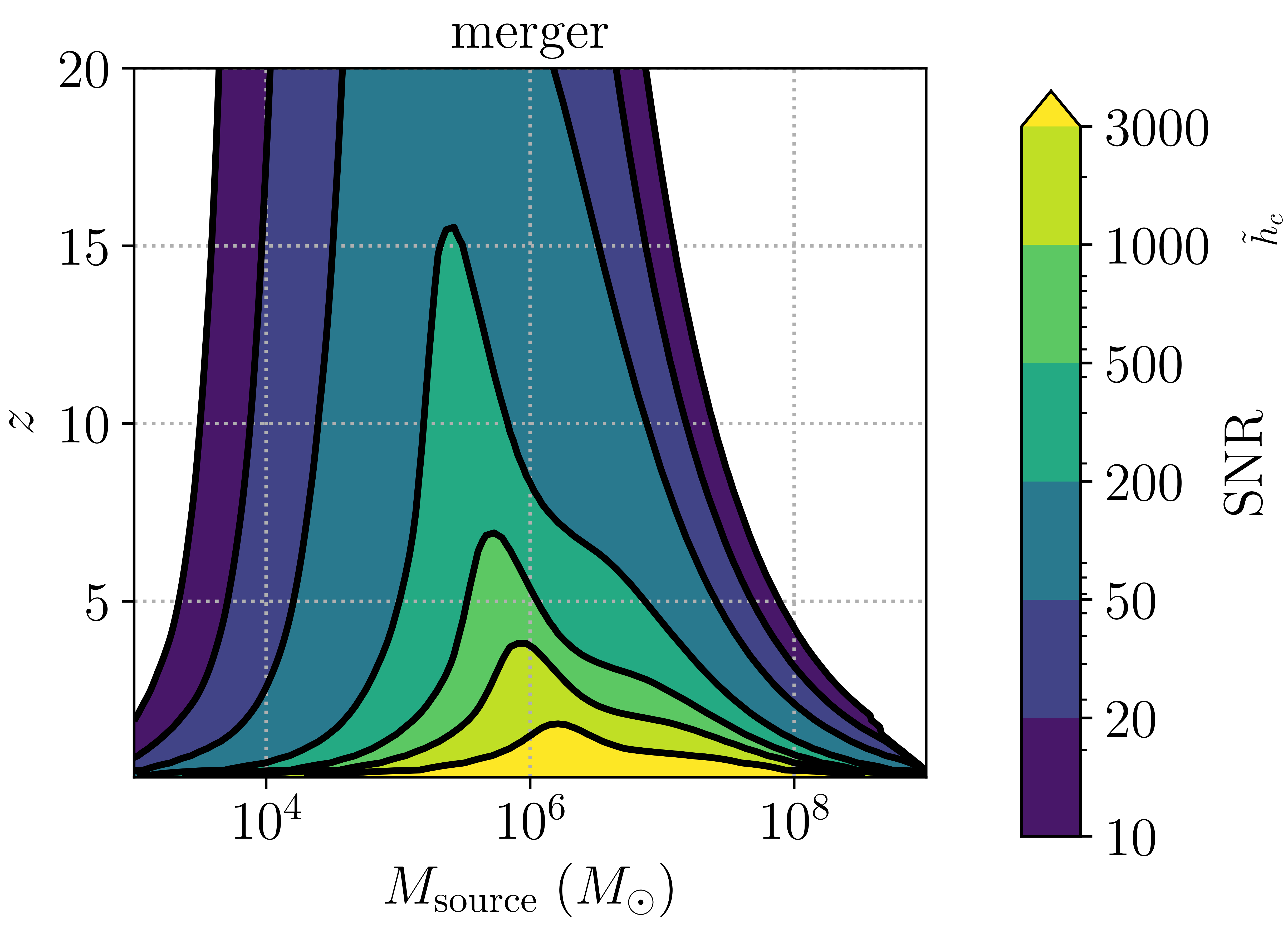
$\epsilon_b > 1$ indicates means that PE measures a significant bias

Both $\epsilon_b,\ \epsilon_m \propto \mathrm{SNR}$

Relation between mismatch and bias unclear

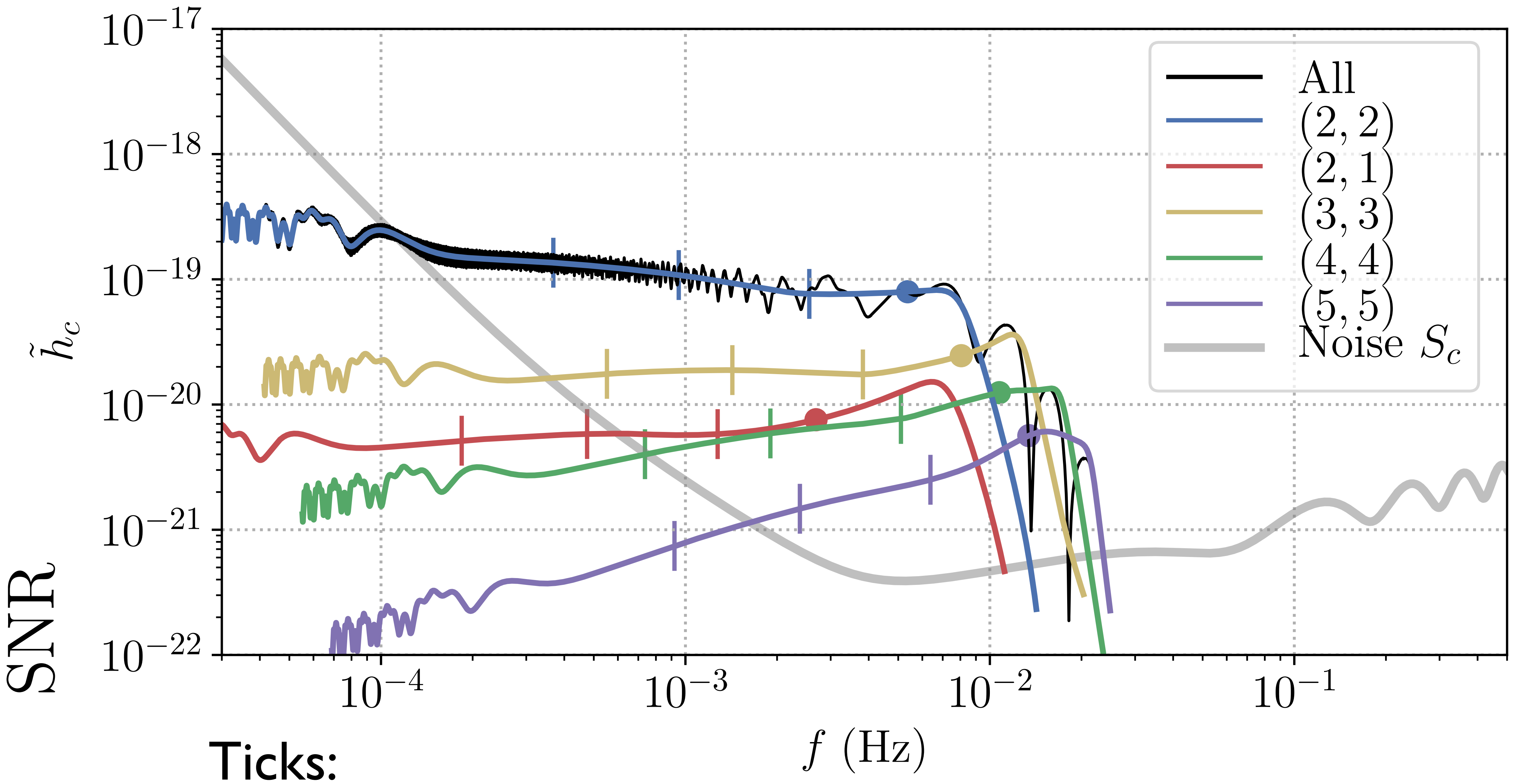
Massive black hole binaries

MBHB SNR contours post-merger



extremely loud signals…

Example MBHB GW signal with higher harmonics



Ticks:

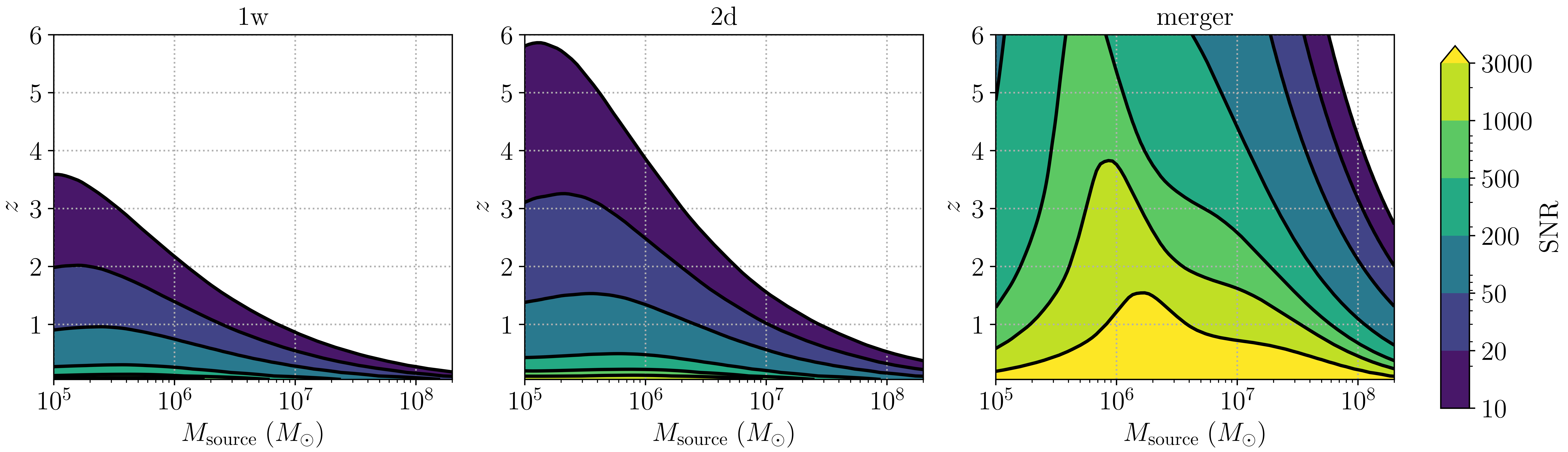
- SNR/64 (40h)
- SNR/16 (2.5h)
- SNR/4 (7min)
- merger

Higher harmonics strong at merger (break degeneracies)

Data analysis simulations still missing precession, eccentricity

MBHB signals are merger-dominated in SNR

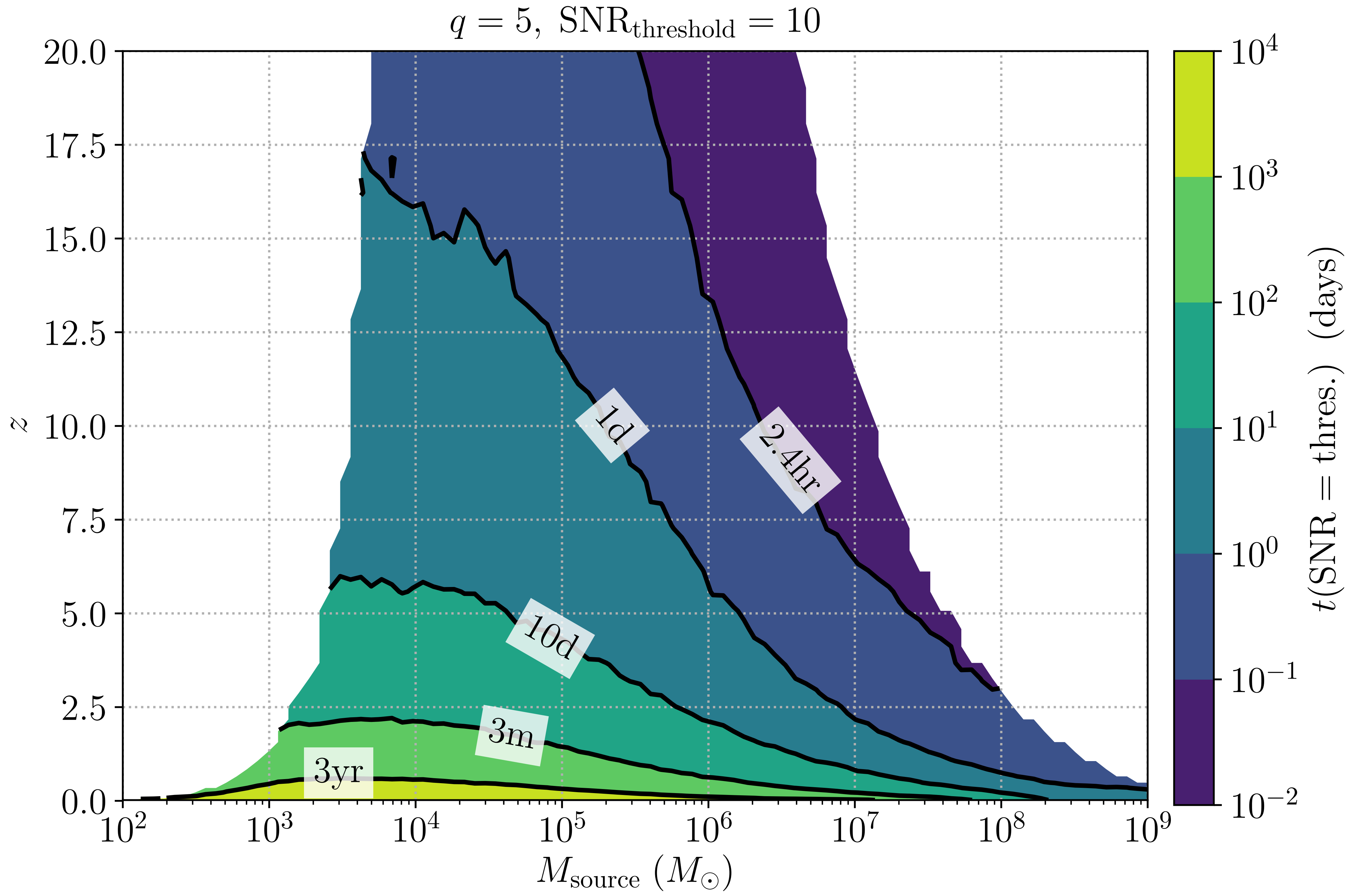
MBHB SNR contours pre-merger and post-merger



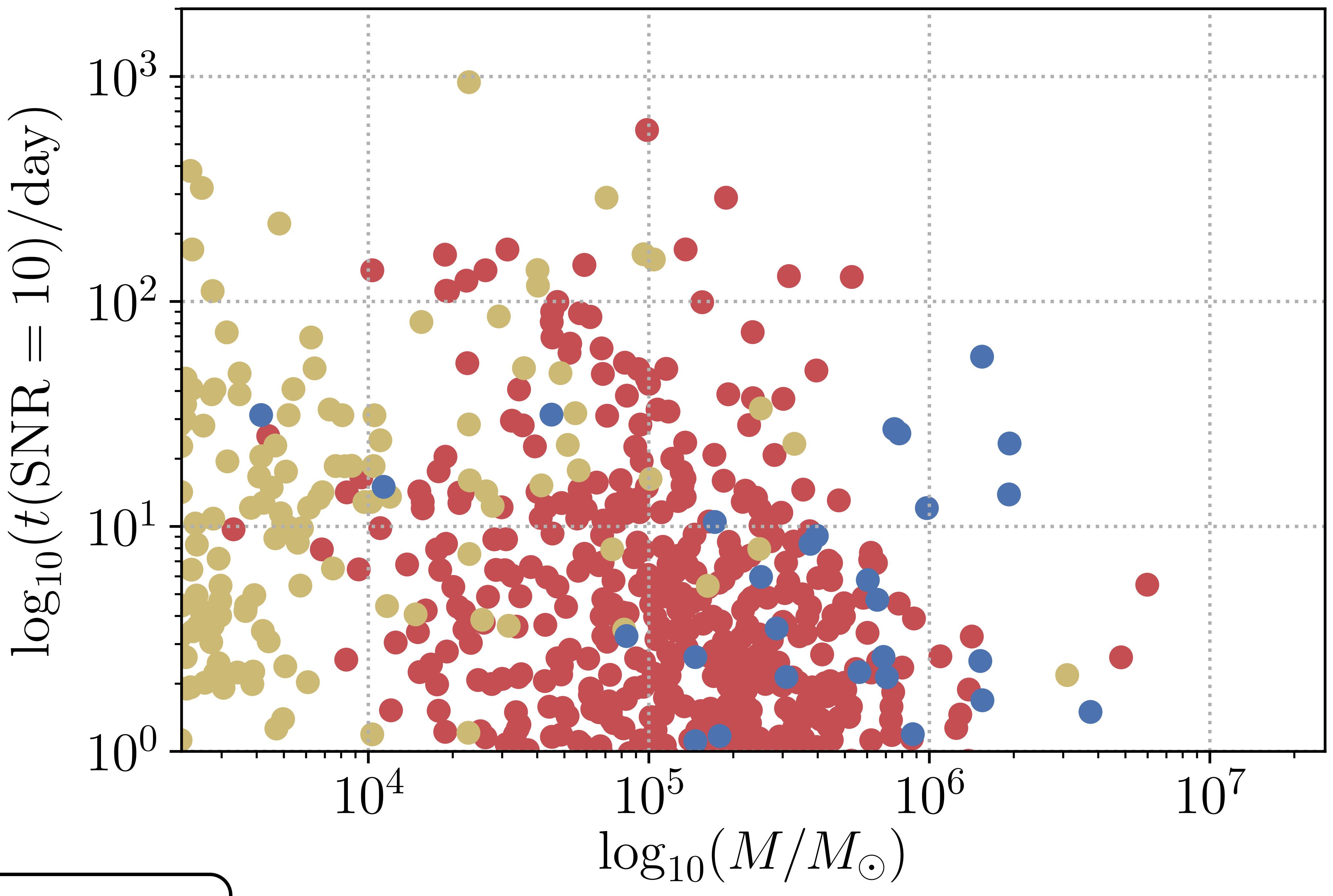
Most of the SNR accumulates in the last hours before coalescence

The length of MBHB signals

- How long before merger can we detect the signal ?
- SNR=10 to claim detection

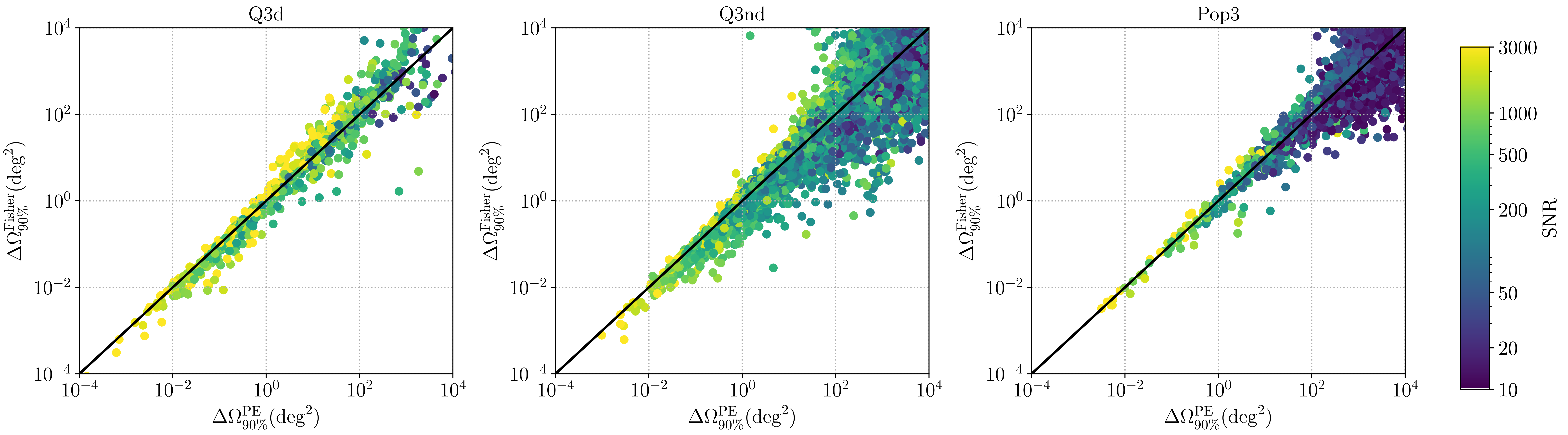


Astrophysical models [Barausse 2012]:
- Heavy seeds - delay
- Heavy seeds - no delay
- PopIII seeds - delay



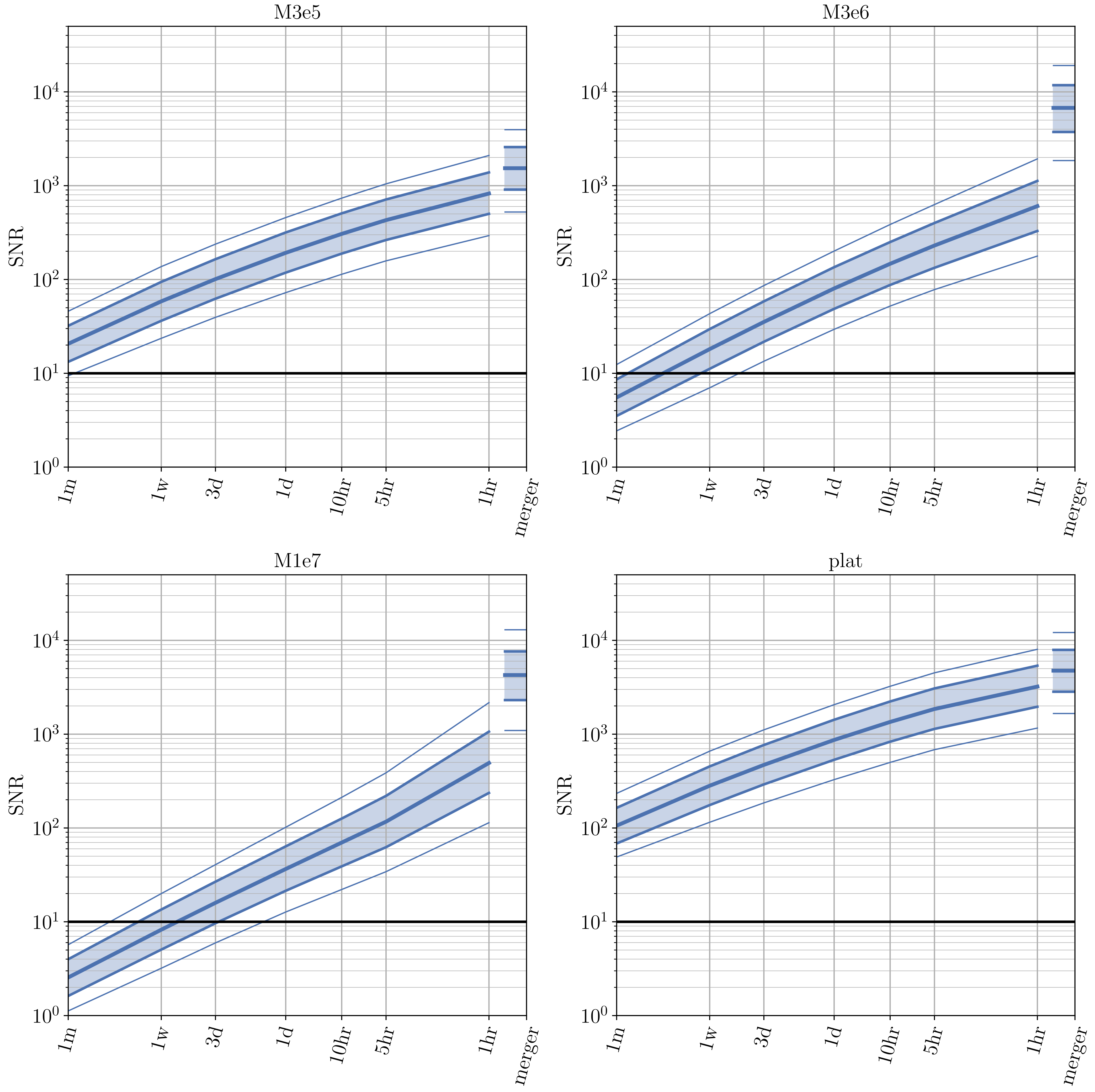
MBHB detected signals:
Bulk shorter than ~10days
Tail extending to ~3months

Fisher vs MCMC localisation



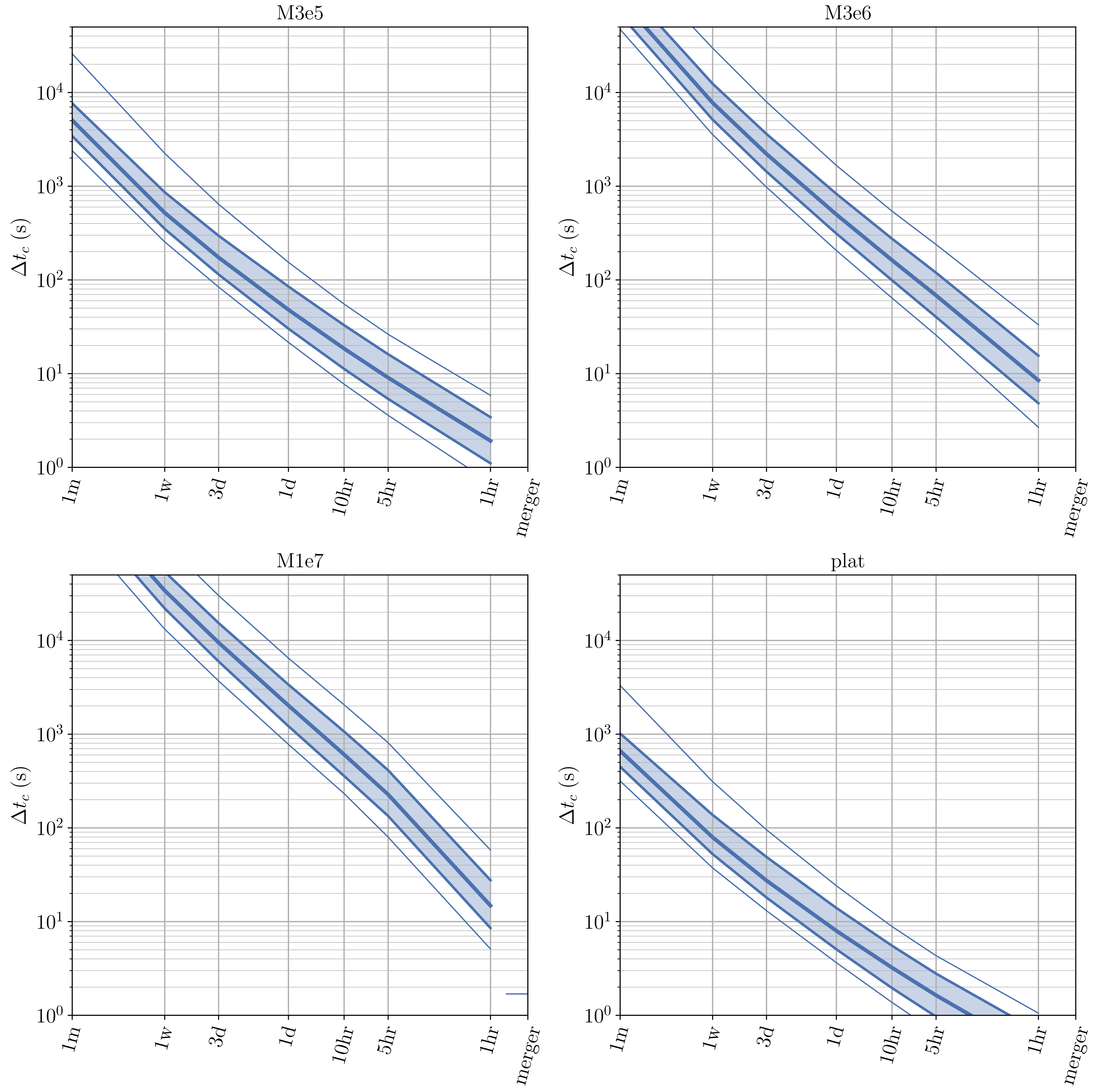
Early detection for ‘golden’ sources

SNR pre-merger and early detection



[See also Mangiagli&al 2020]

Time of coalescence measurement error



Early detection easy for golden sources

Allows protected observation periods

[Piro&al, in prep]

Analysis settings: waveform models

Injections:

NRHybSur3dq8 [Varma&al 2018]
- SXS NR simulations hybridized with long EOB inspirals (covers ~6 months for $M = 10^5 M_\odot$)
- Surrogate interpolant, time-domain

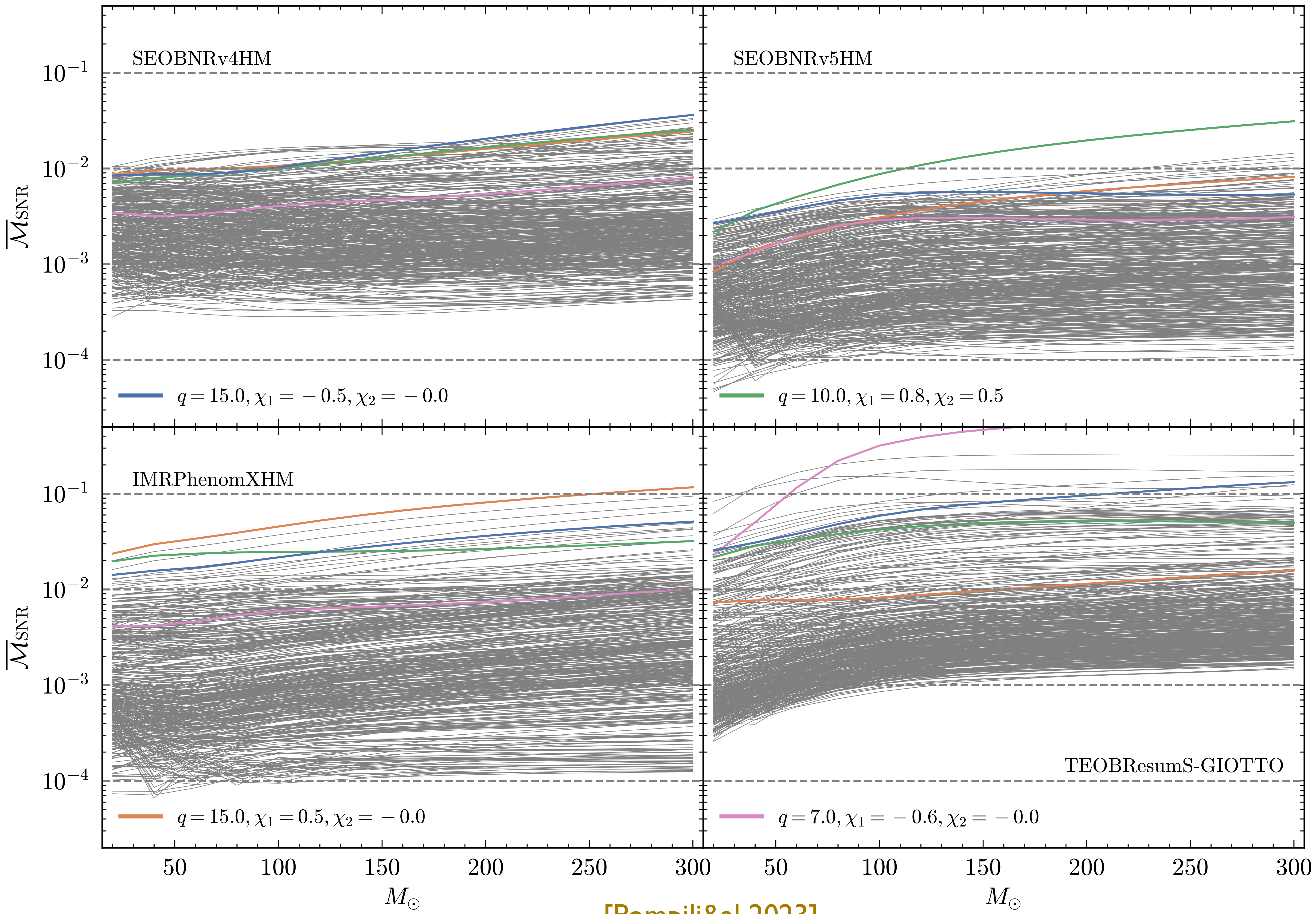
Templates:

Efficient Fourier-domain models from 2 families:
- PhenomHM [London&al 2017]
- PhenomXHM [García-Quirós&al 2020]
- SEOBNRv4HM_ROM [Cotesta&al 2018]
- SEOBNRv5HM_ROM [Pompili&al 2023]

Mode content for all: 22, 21, 33, 44

Limitations:
- **aligned spins only**
- in the inspiral, all based on PN/EOB

Aligned spin case: mismatch with NR ~ $10^{-4} - 10^{-2}$



[Pompili&al 2023]

Pre-merger analysis: accumulation of information with time

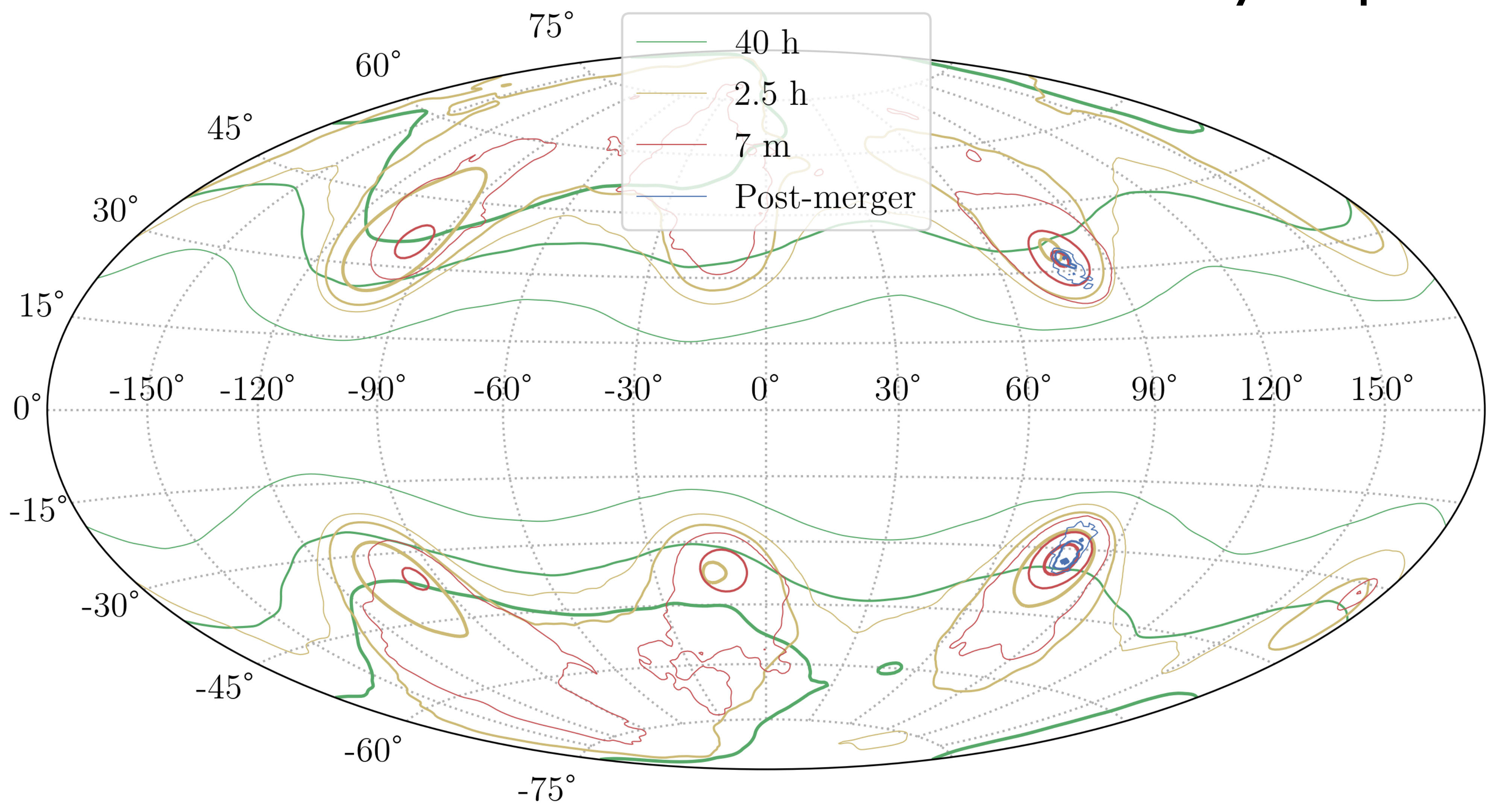
Method

- Represent a cut in time-to-merger by a cut in frequency, becomes inaccurate at merger

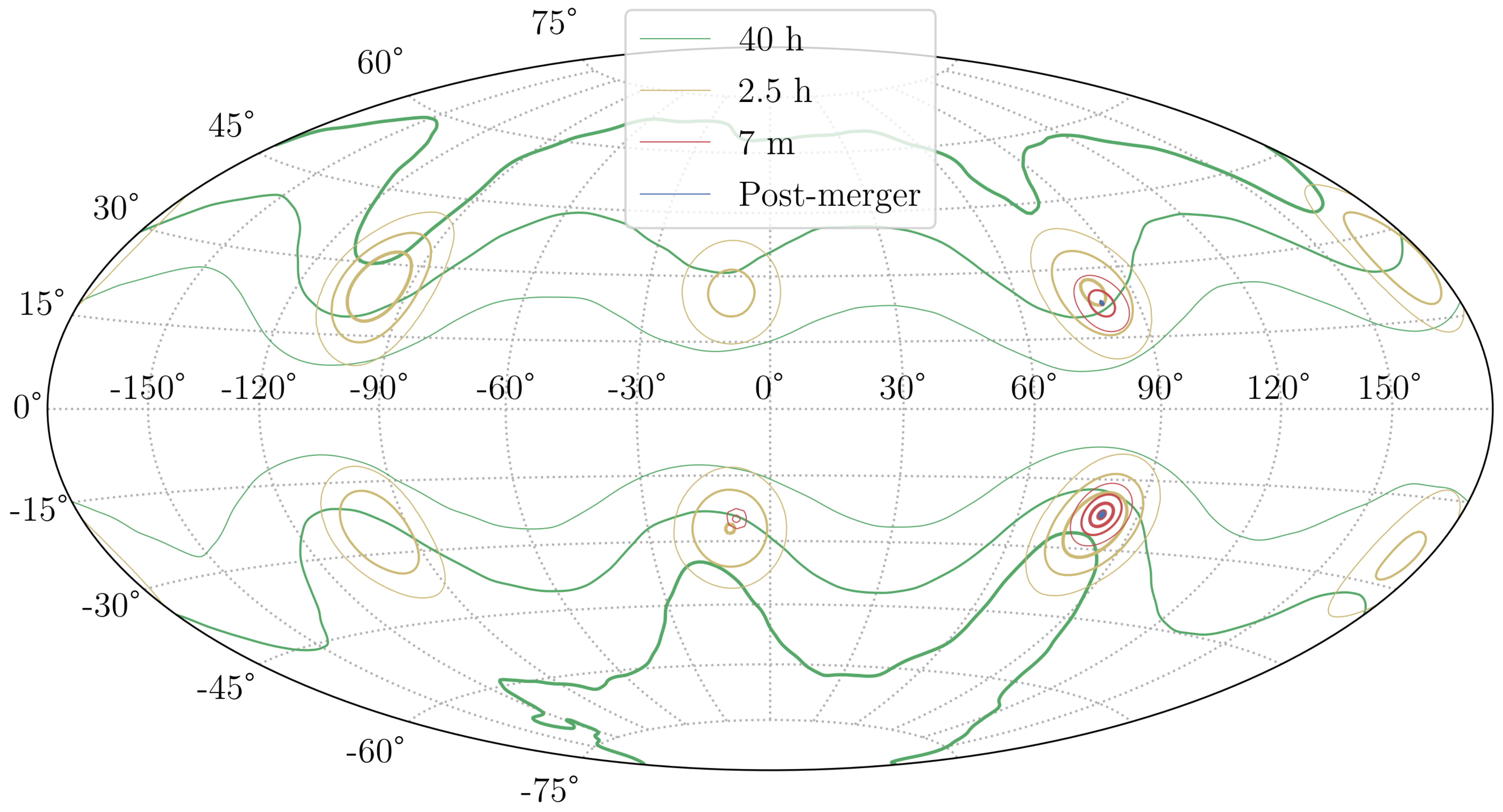
SNR-based time cuts:

SNR	DeltaT
10	40h
42	2.5h
167	7min
666	-

LISA-frame sky map 22



LISA-frame sky map hm



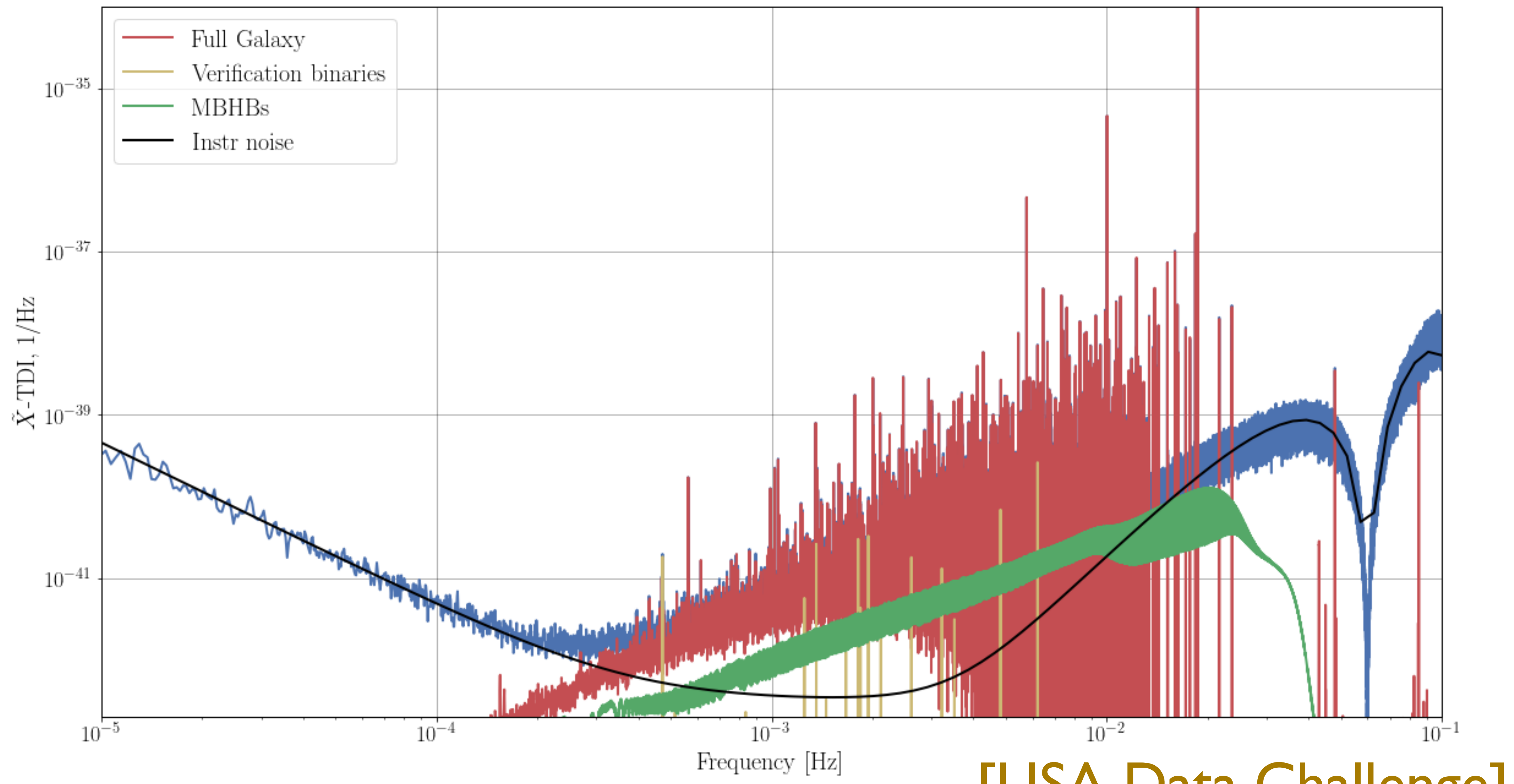
8-maxima sky degeneracy
only broken shortly before merger

2-maxima sky degeneracy
survives after merger (‘Reflected’)

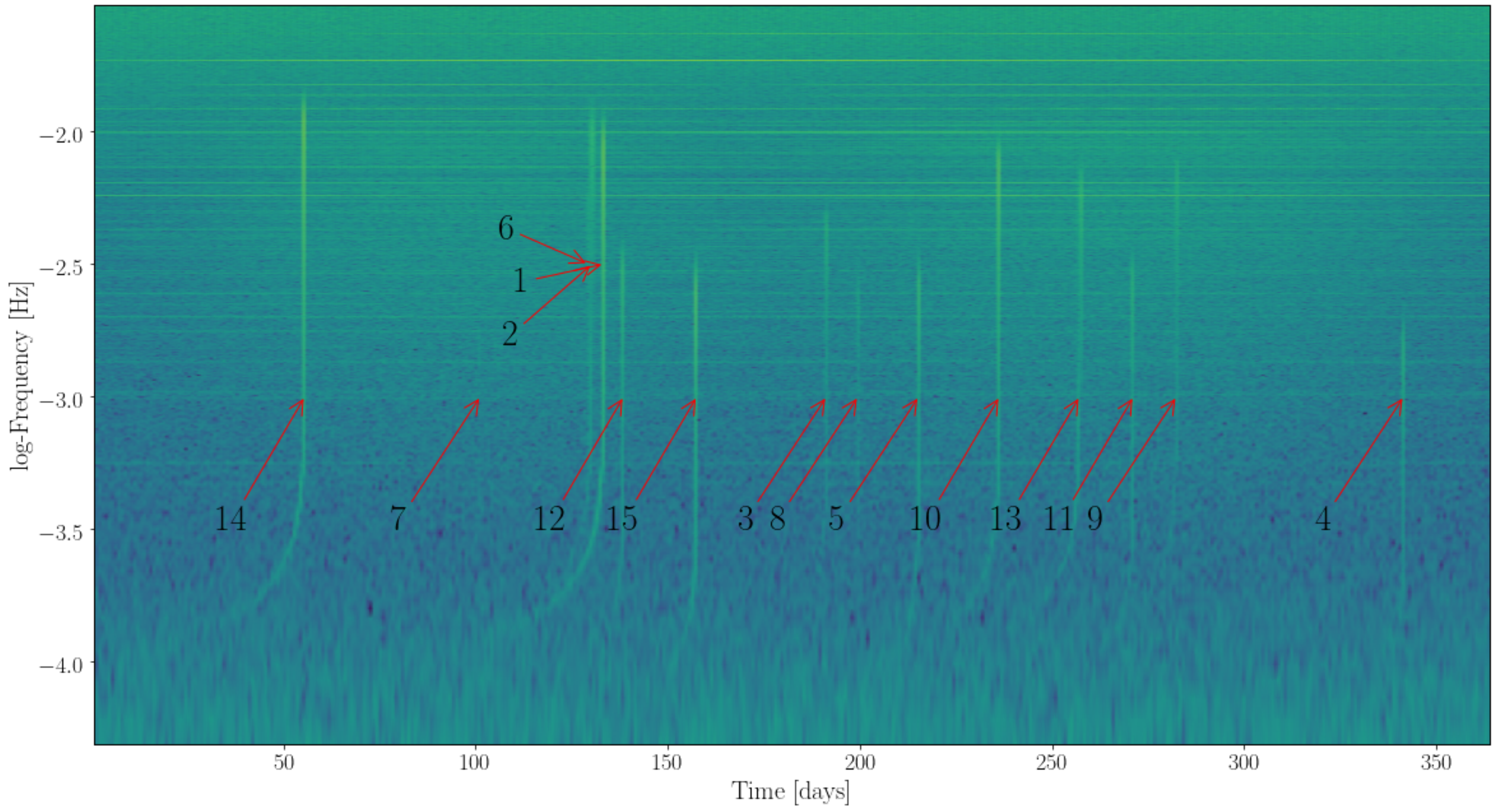
LDC-2: source superposition and global analysis

LISA Data analysis challenges

- Superposition of many sources, with a population of GBs also forming a stochastic background -> Global fit
- High SNR for MBHBs, waveform systematics important
- EMRI waveform models
- Data gaps
- Glitches, instrumental non-stationarity

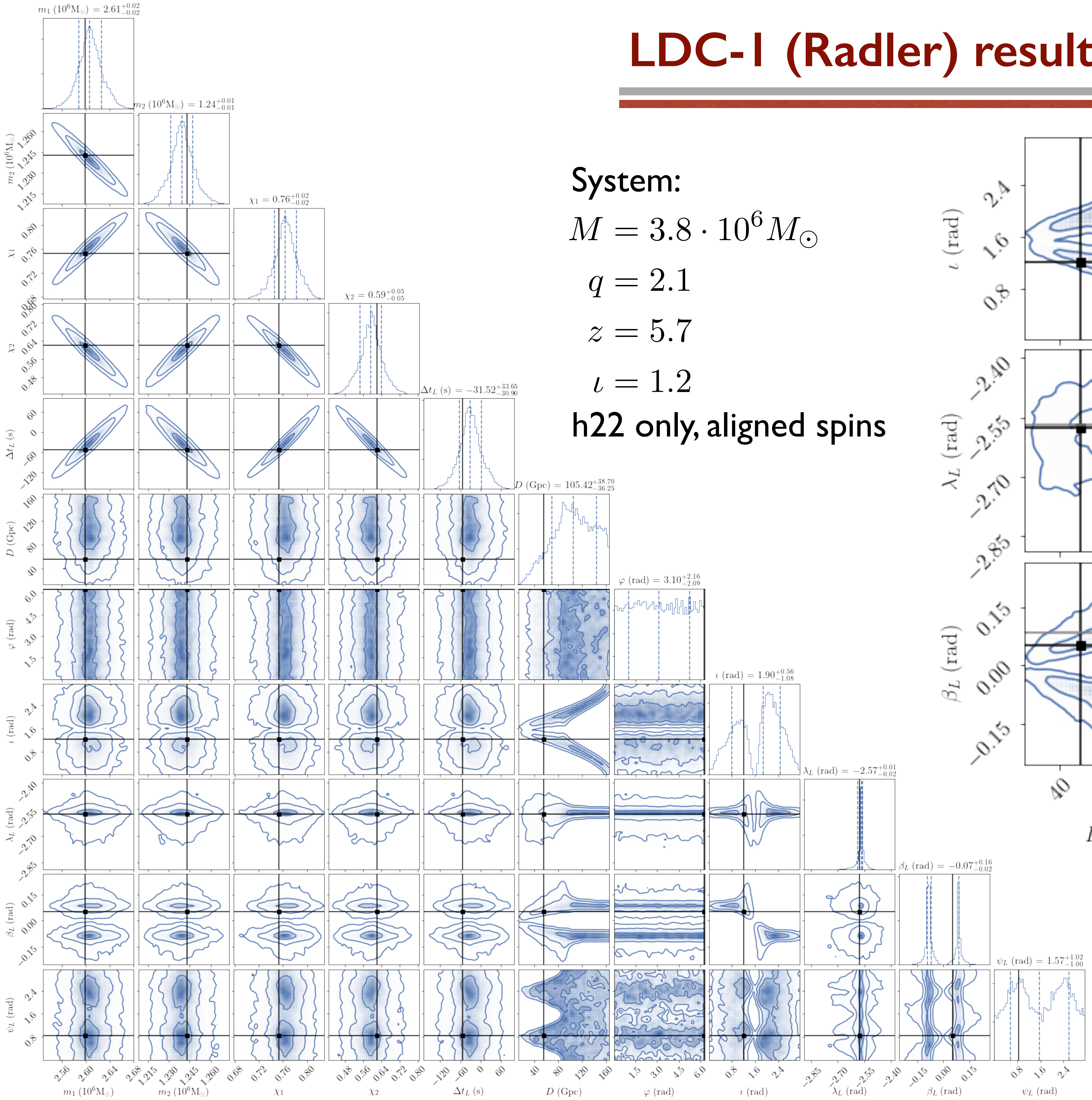


[LISA Data Challenge]



LISA Data Challenge 2 ‘Sangria’

- ~10 massive black holes
- Population of galactic binaries (~10000 resolved)
- Unkown noise level



LDC-1 (Radler) results

System:

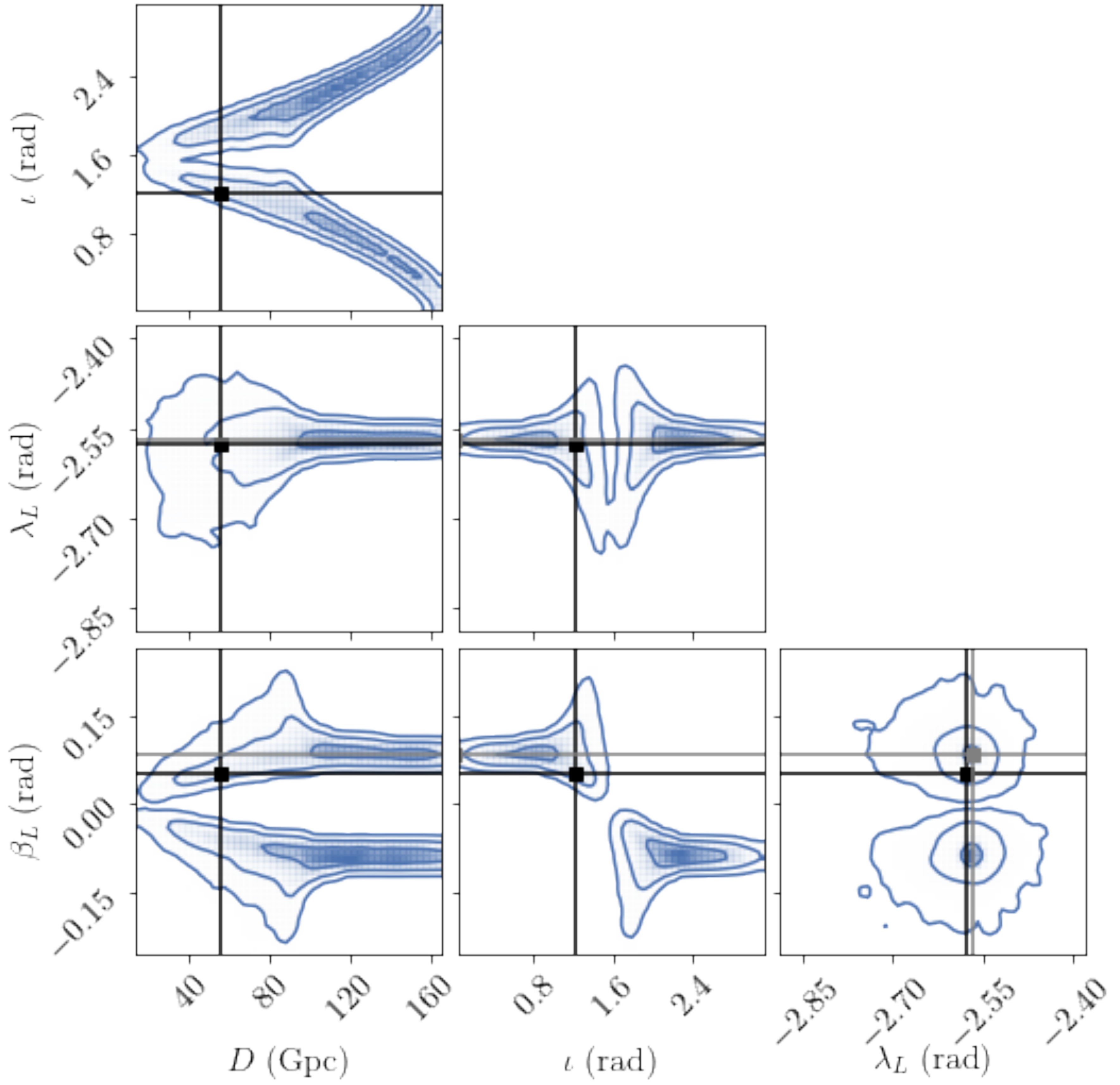
$M = 3.8 \cdot 10^6 M_\odot$

$q = 2.1$

$z = 5.7$

$\iota = 1.2$

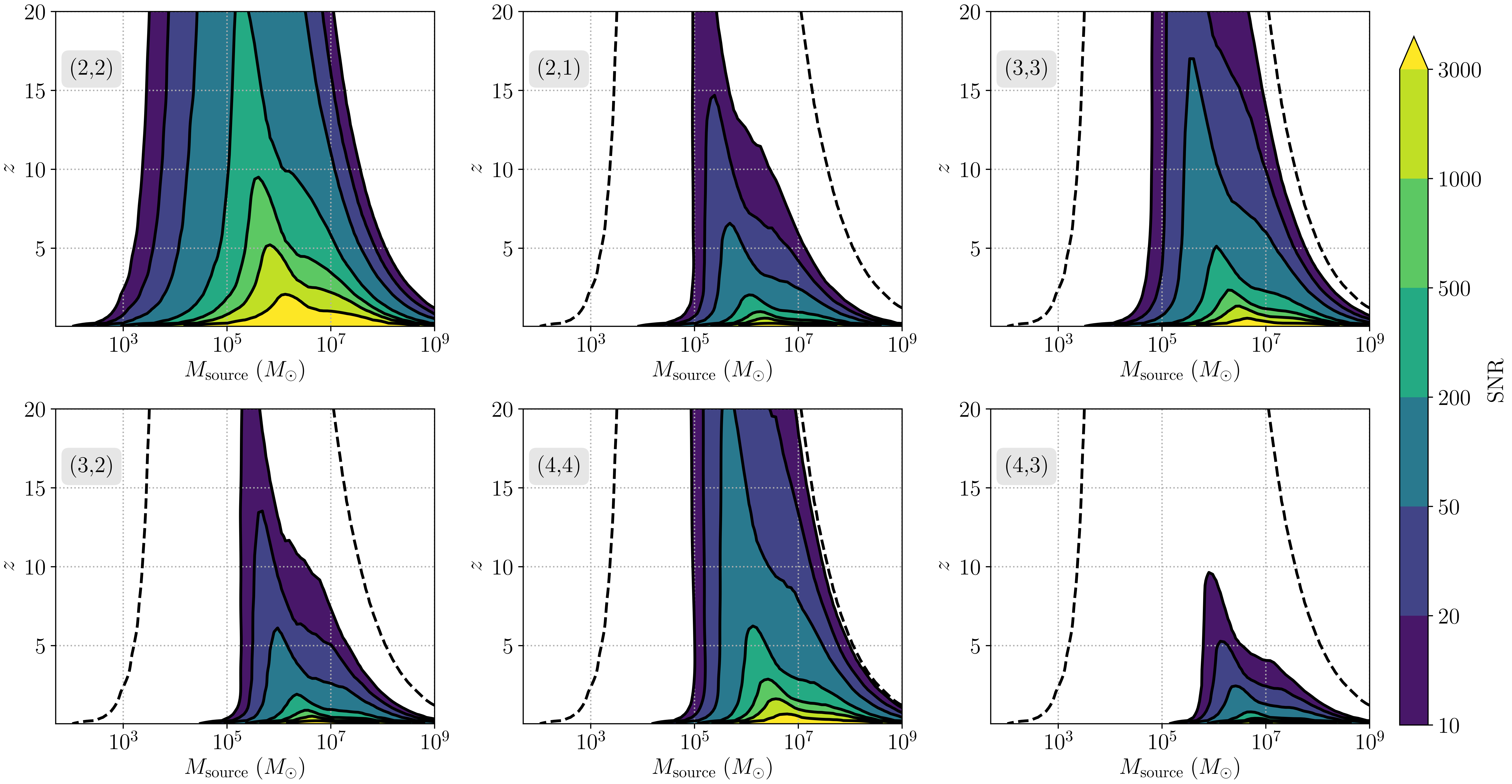
h22 only, aligned spins



Strong 22-mode degeneracies,
multimodal sky (reflected)
Use this knowledge to sample !

The SNR of higher harmonics

PhenomHM, q=5, averaging over spins and orientations



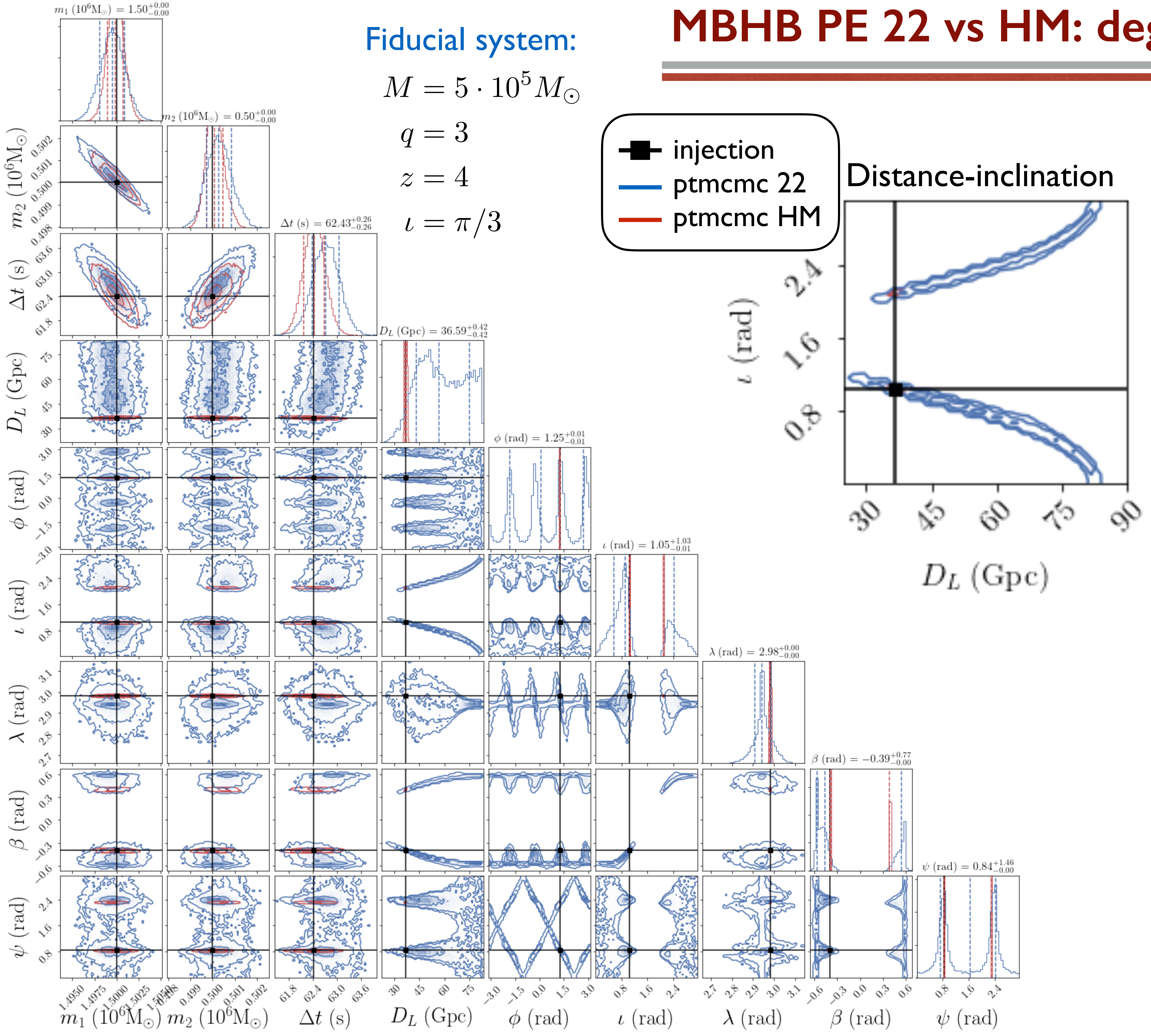
'Mode SNR':

$$\sqrt{(s_{\ell m}|s_{\ell m})}$$

Cross-terms not negligible

$$\sum_{(\ell m)\neq(\ell' m')} (s_{\ell m}|s_{\ell' m'})$$

MBHB PE 22 vs HM: degenerate case



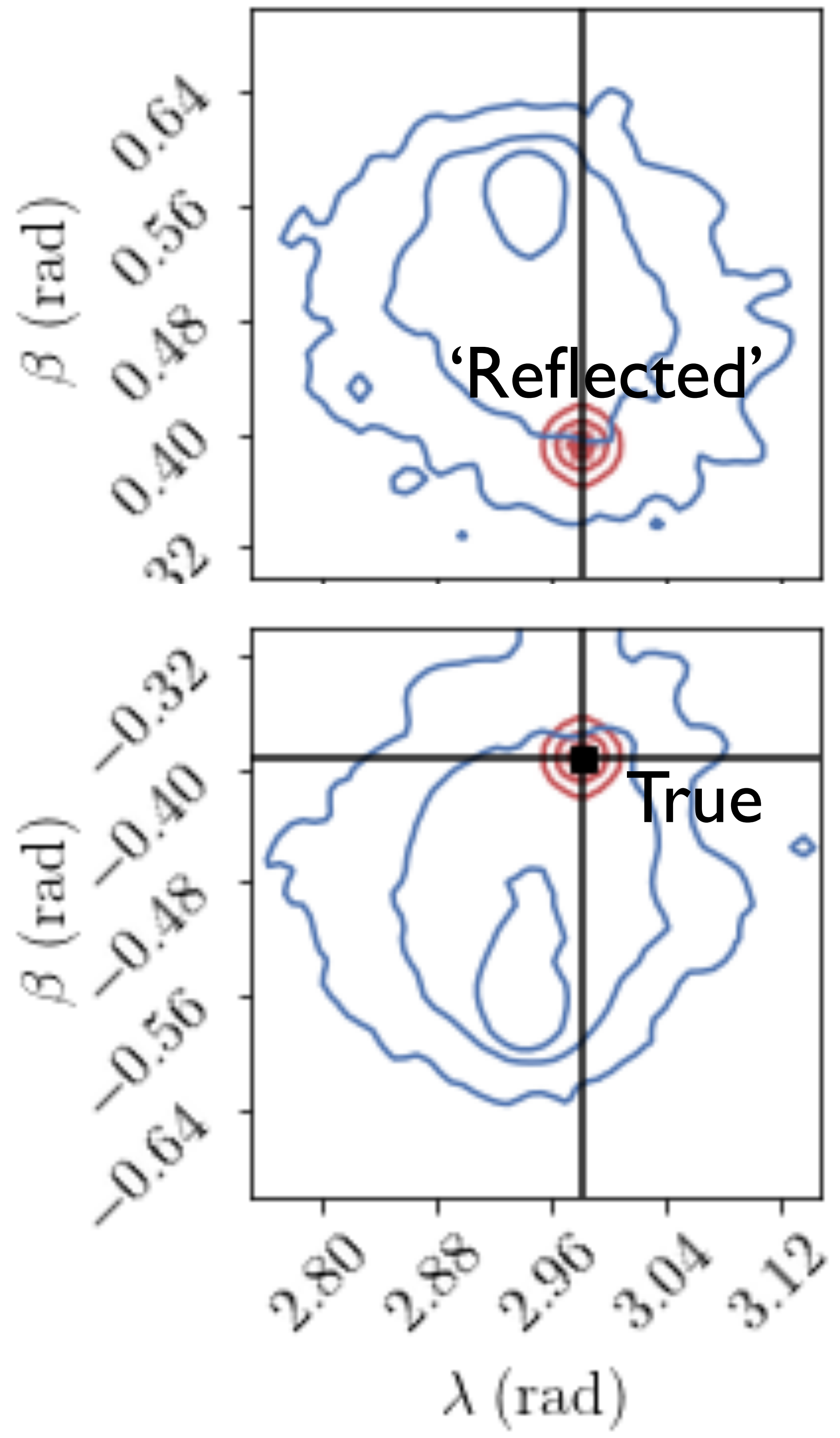
Fiducial system:

$$M = 5 \cdot 10^5 M_\odot$$
$$q = 3$$
$$z = 4$$
$$\iota = \pi/3$$

injection
ptmcmc 22
ptmcmc HM

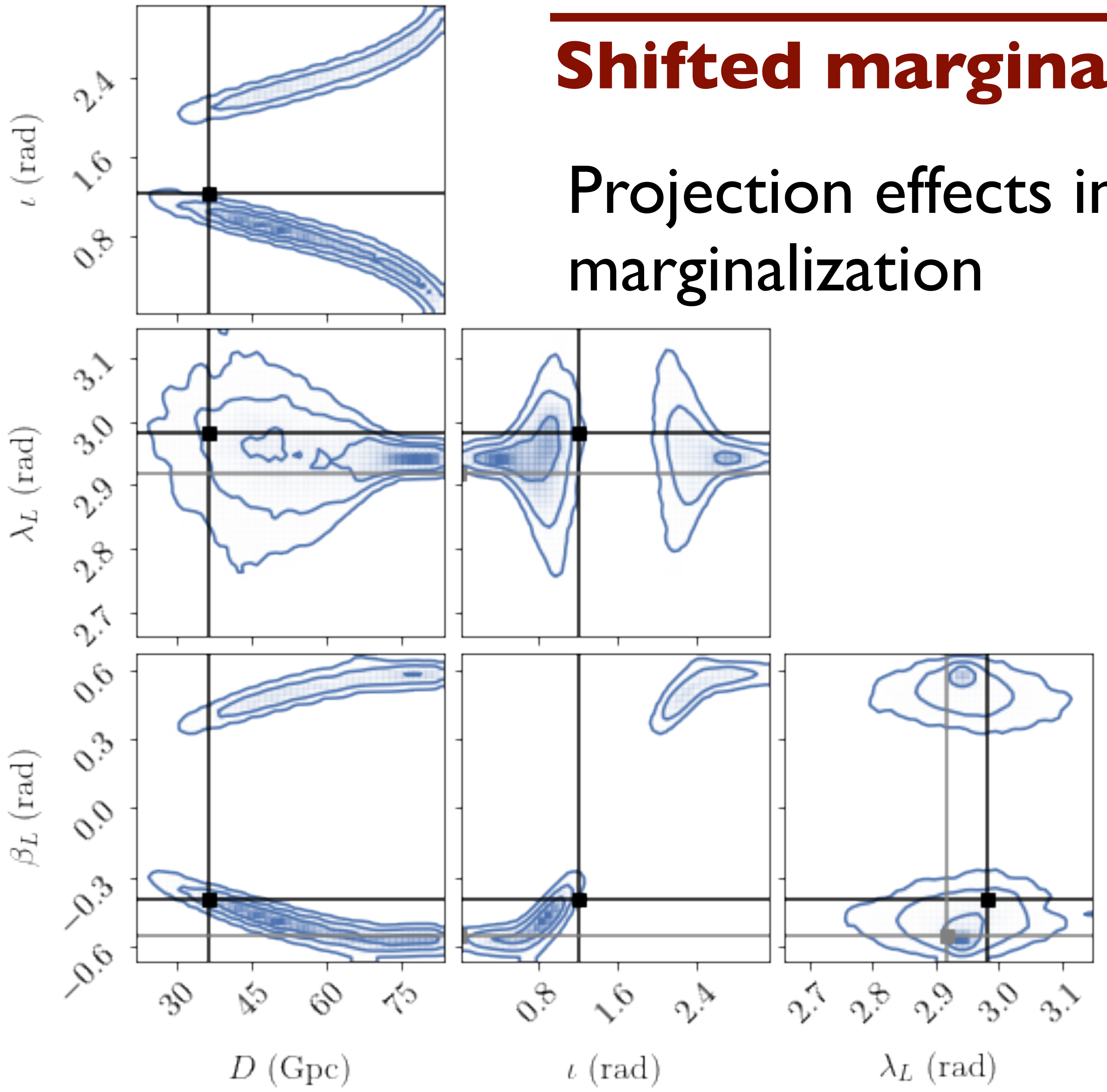
Distance-inclination

Sky position



Higher harmonics crucial in breaking degeneracies

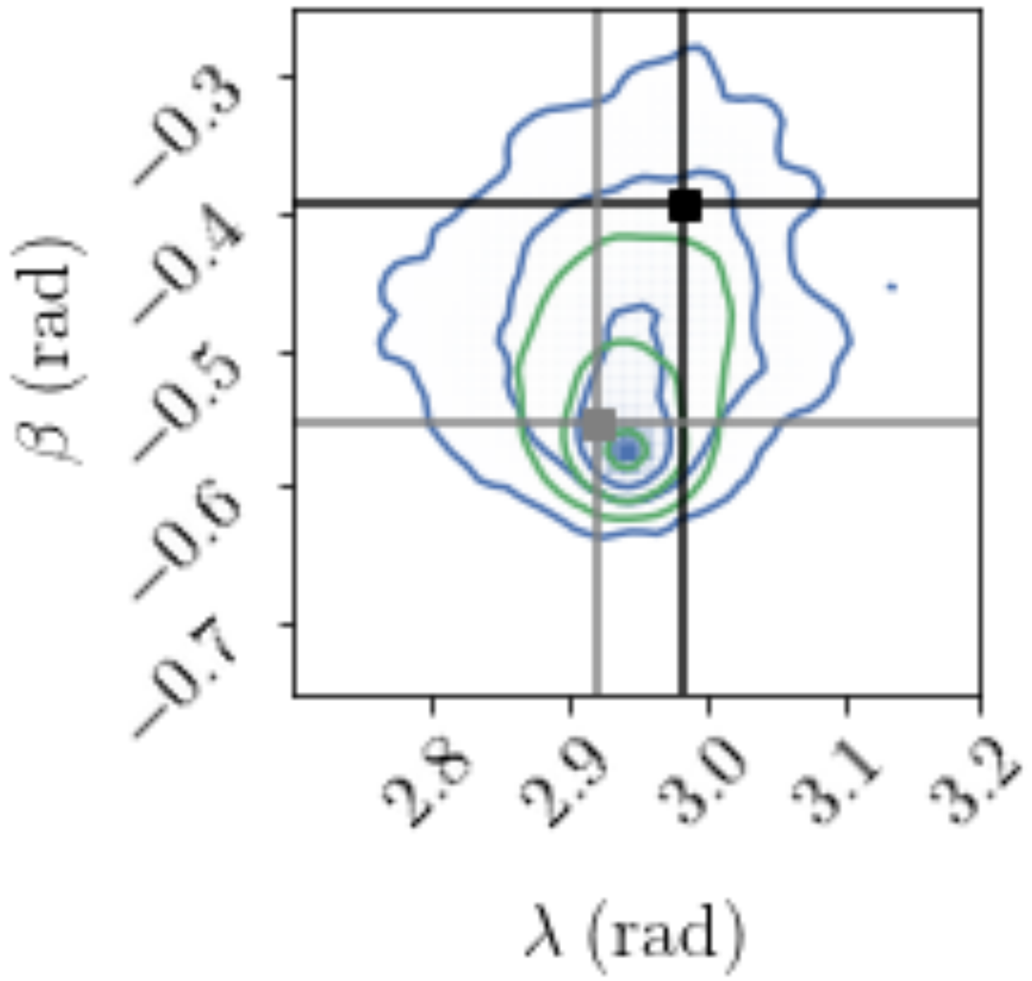
Understanding degeneracy breaking by higher harmonics



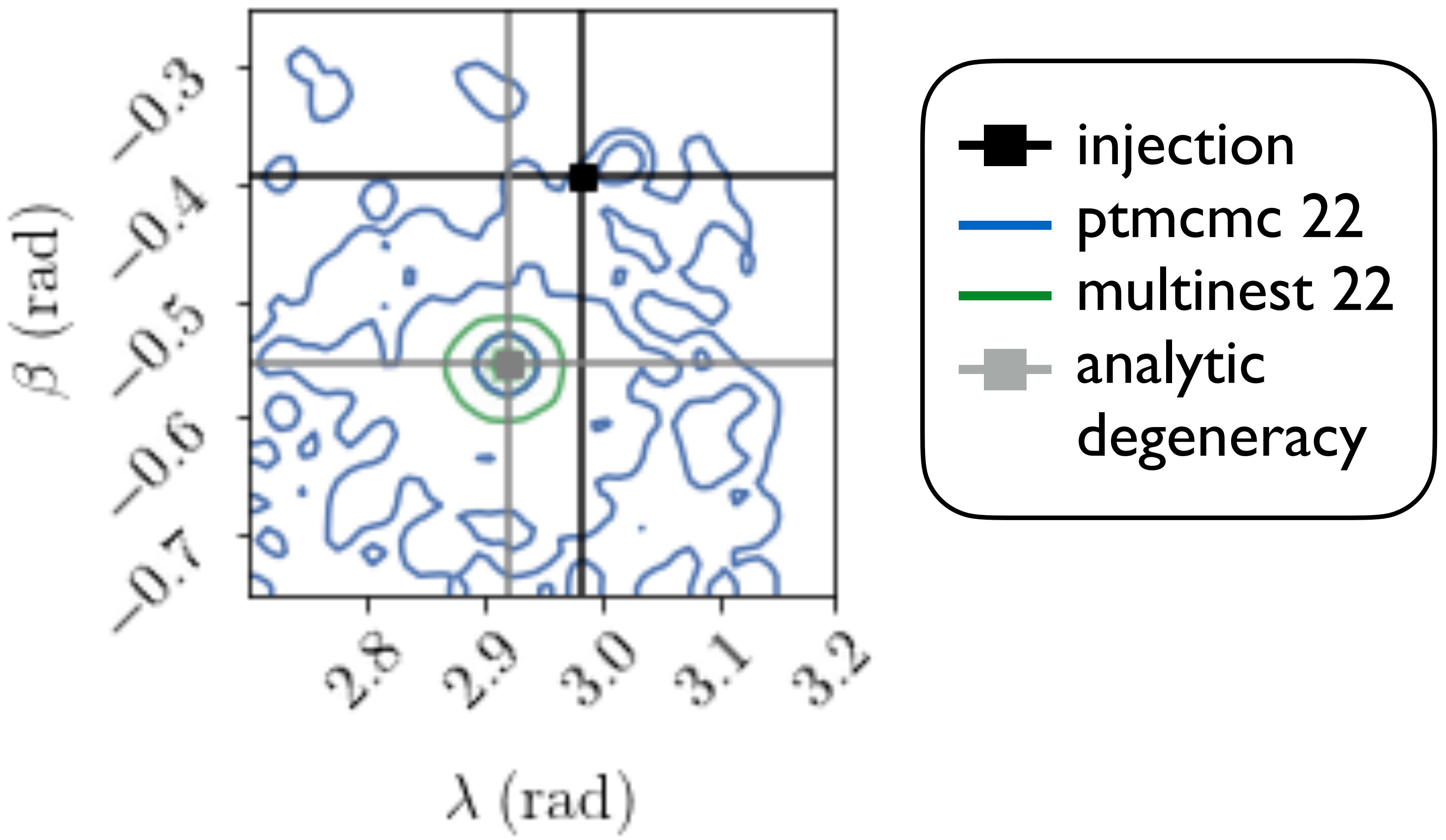
Shifted marginal sky posterior

Projection effects in marginalization

Sky zoom (h22), full response



Sky zoom (h22), no LISA motion and no high-f



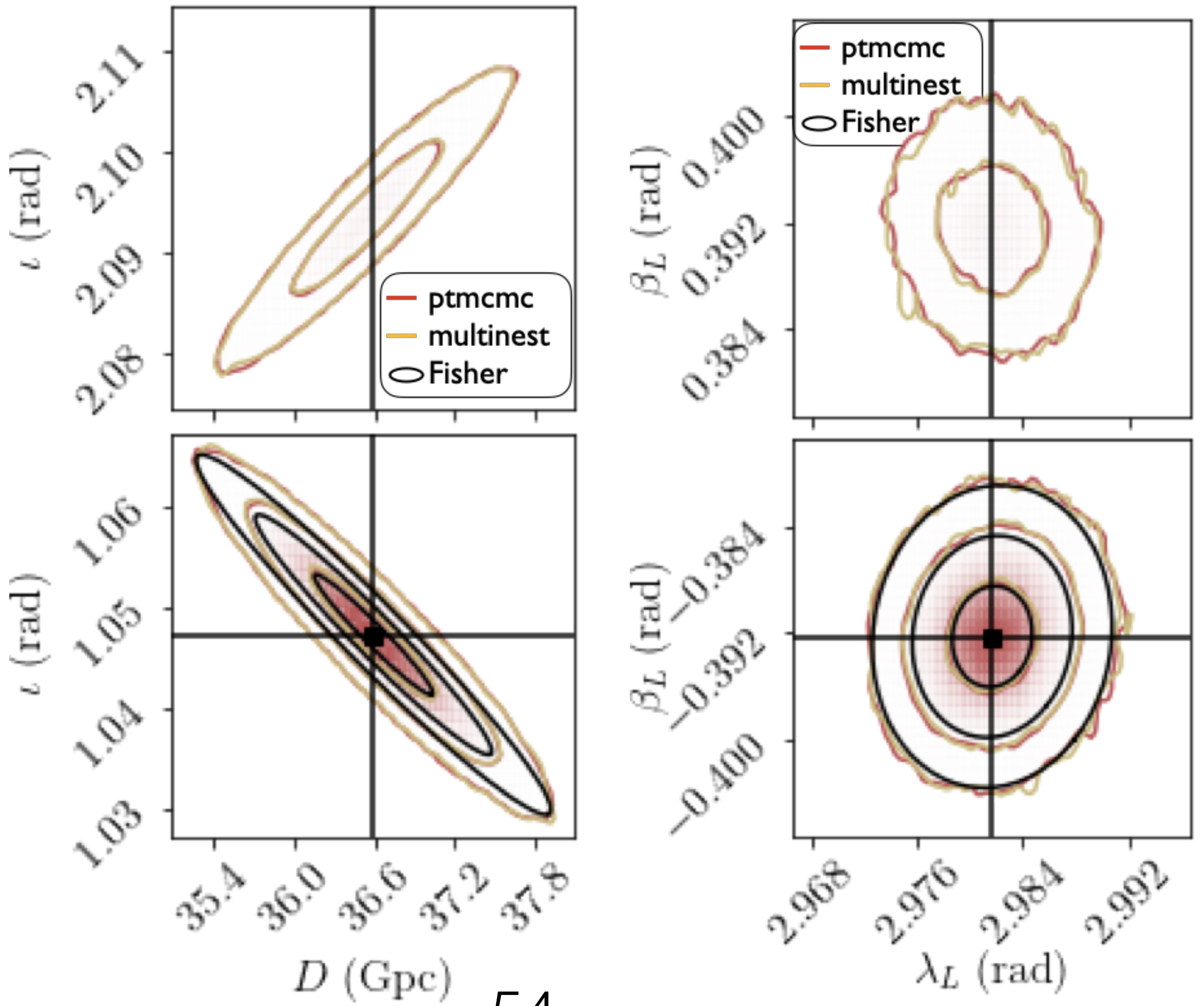
The role of higher harmonics

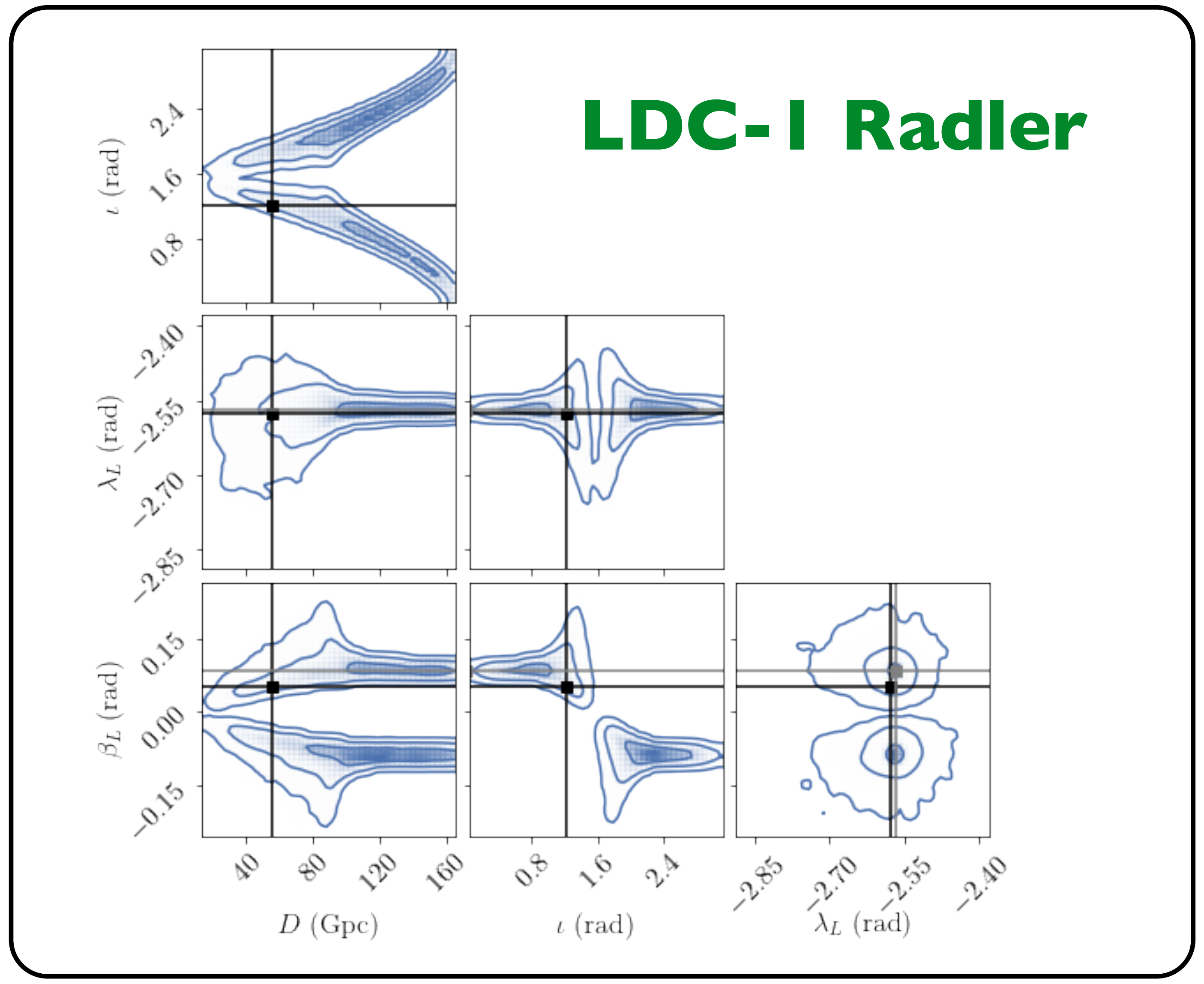
$$h_+ - ih_\times = \sum {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

$${}_{-2}Y_{\ell m}(\iota, \varphi) = {}_{-2}Y_{\ell m}(\iota, 0) e^{im\varphi}$$

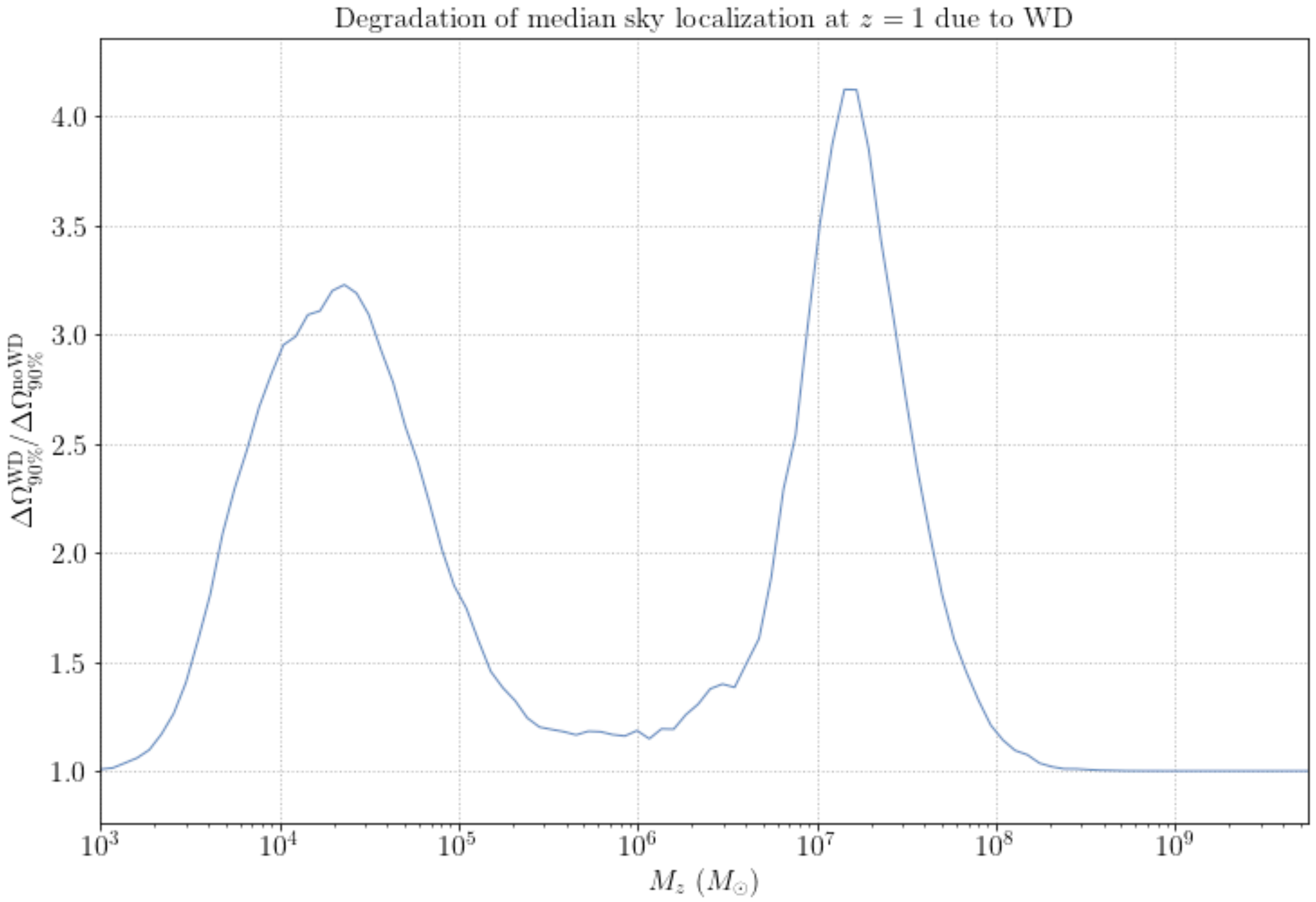
When measuring several modes $h_{\ell m}$:

- Distance/inclination degeneracy broken
- Phase independently measured
- Better sky localization (caveat: edge-on, see [Katz&al 2020])





Sky area: impact of WD background



The WD confusion background matters
Sky localisation will be updated as the GB analysis is refined

Pre-merger localization: role of instrumental response for 'golden' systems

Here: main mode sky area

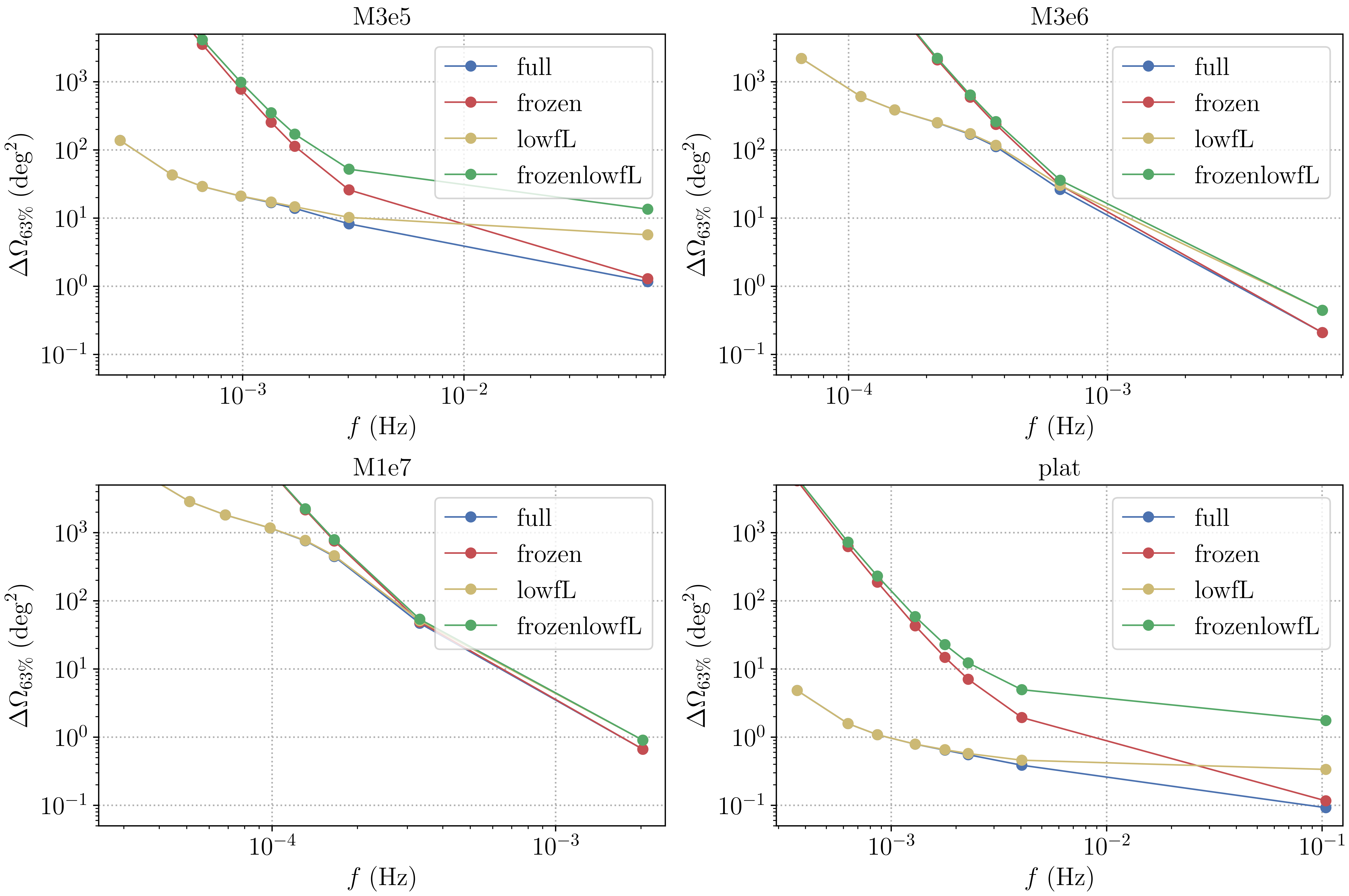
Response (signal with HM here):

- 'Full': keep all terms
- 'Frozen': ignore LISA motion
- 'Low-f': ignore f-dependency
- 'Frozen Low-f': ignore both

For low masses, best candidates for advance localization:

- Localization from the LISA motion saturates reaching merger
- Localization from high-f effects dominates at merger

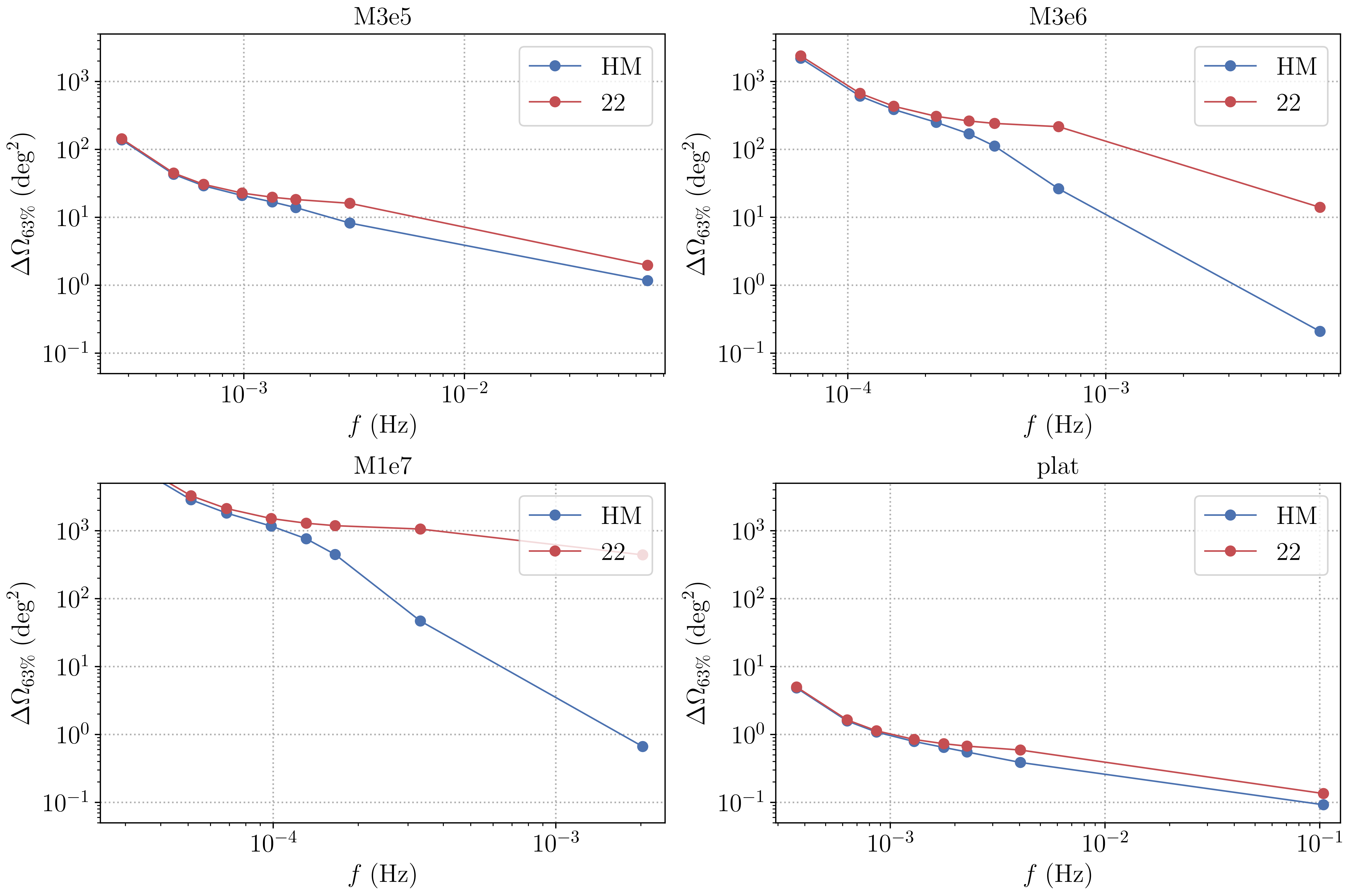
For high masses, HM at merger convey most of the information



Pre-merger localization: role of HM for 'golden' systems

Here: main mode sky area

Higher modes become most important at merger and for high mass



Pre-merger analysis: likelihood with decomposed response

Degeneracy breaking for 8 sky maxima

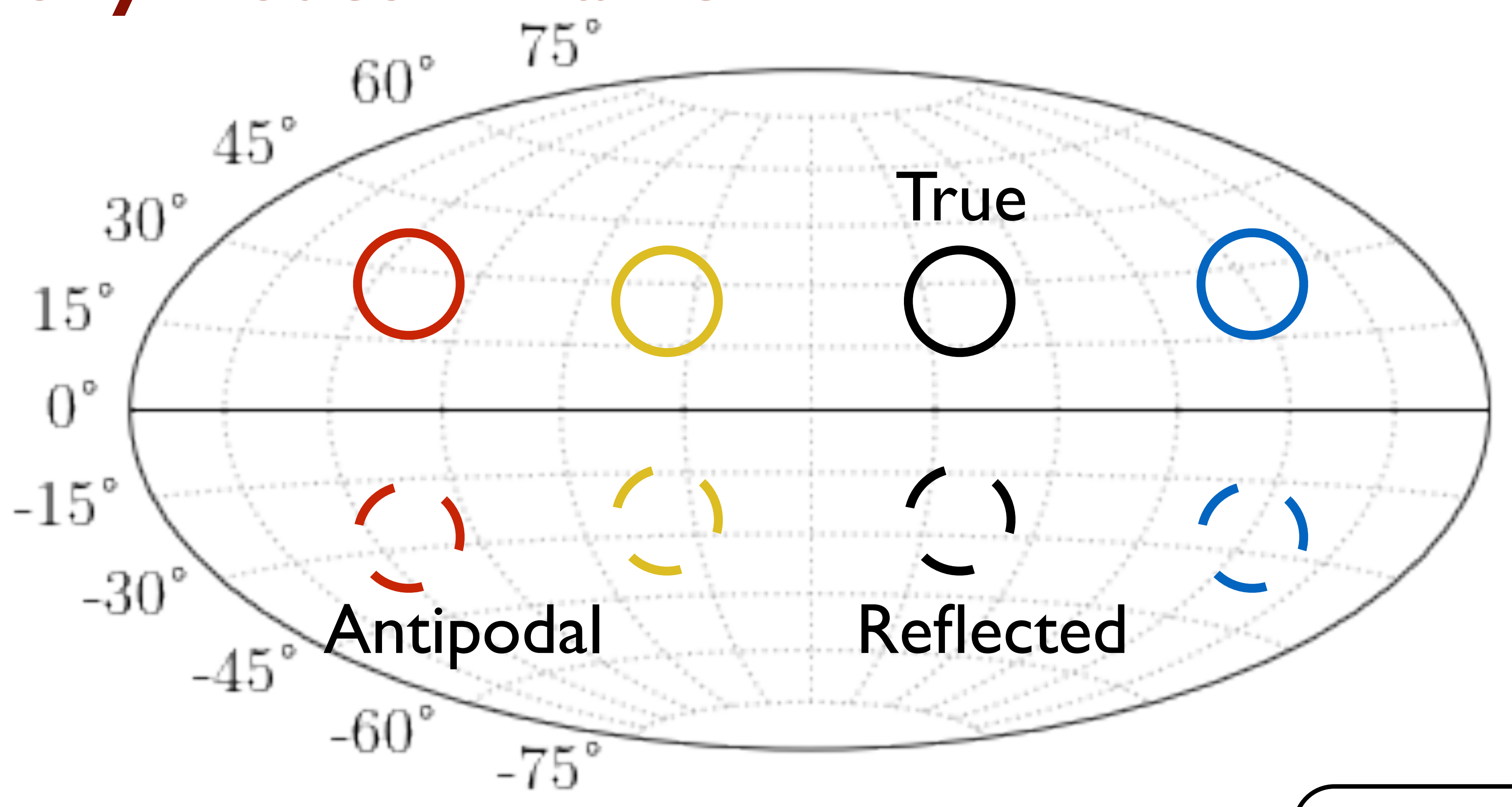
Instrument response:

- 'Full': keep all terms
- 'Frozen': ignore LISA motion
- 'Low-f': ignore f-dependency
- 'Frozen Low-f': ignore both

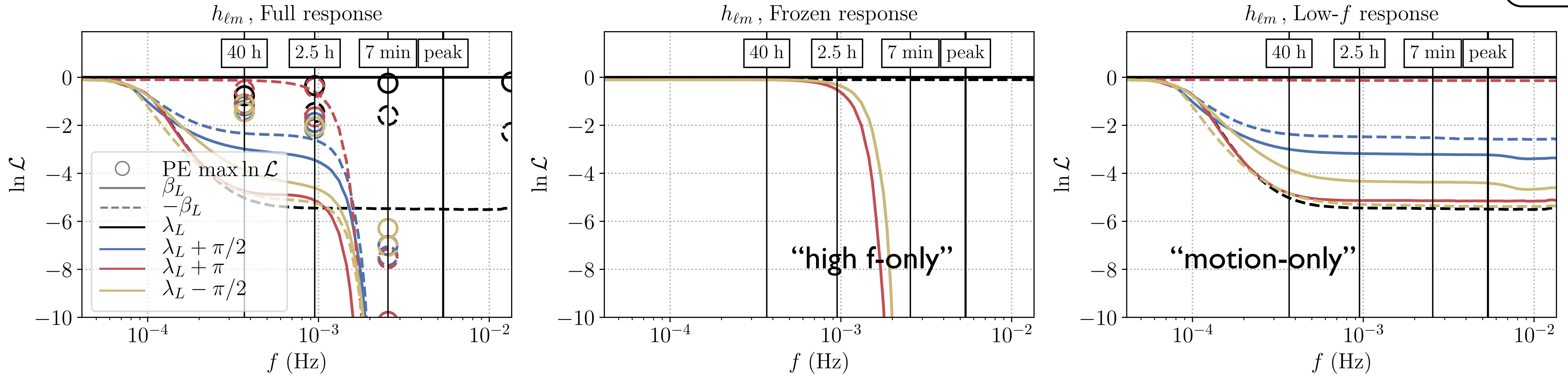
Likelihood: $\ln \mathcal{L}(d|\theta) = -\sum_{\text{channels}} \frac{1}{2}(h(\theta) - d|h(\theta) - d)$

Approximate degeneracy measure: likelihood at the other sky positions

Sky modes L-frame



High-f features crucial



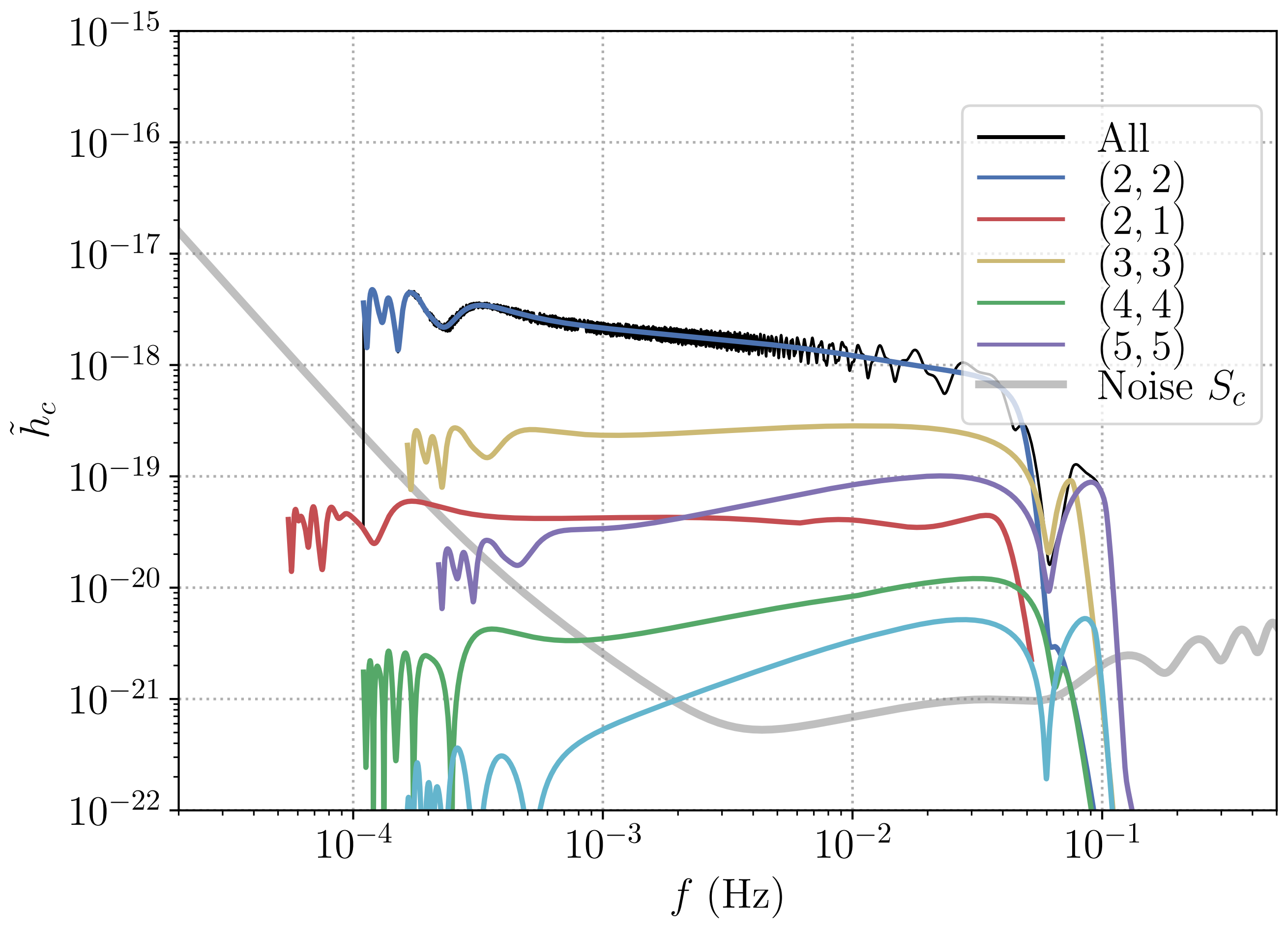
[Marsat&al 2020]

LISA/Athena candidates

‘Platinum’

$$M_{\rm source} = 3 \times 10^5 M_\odot \,, z = 0.3$$

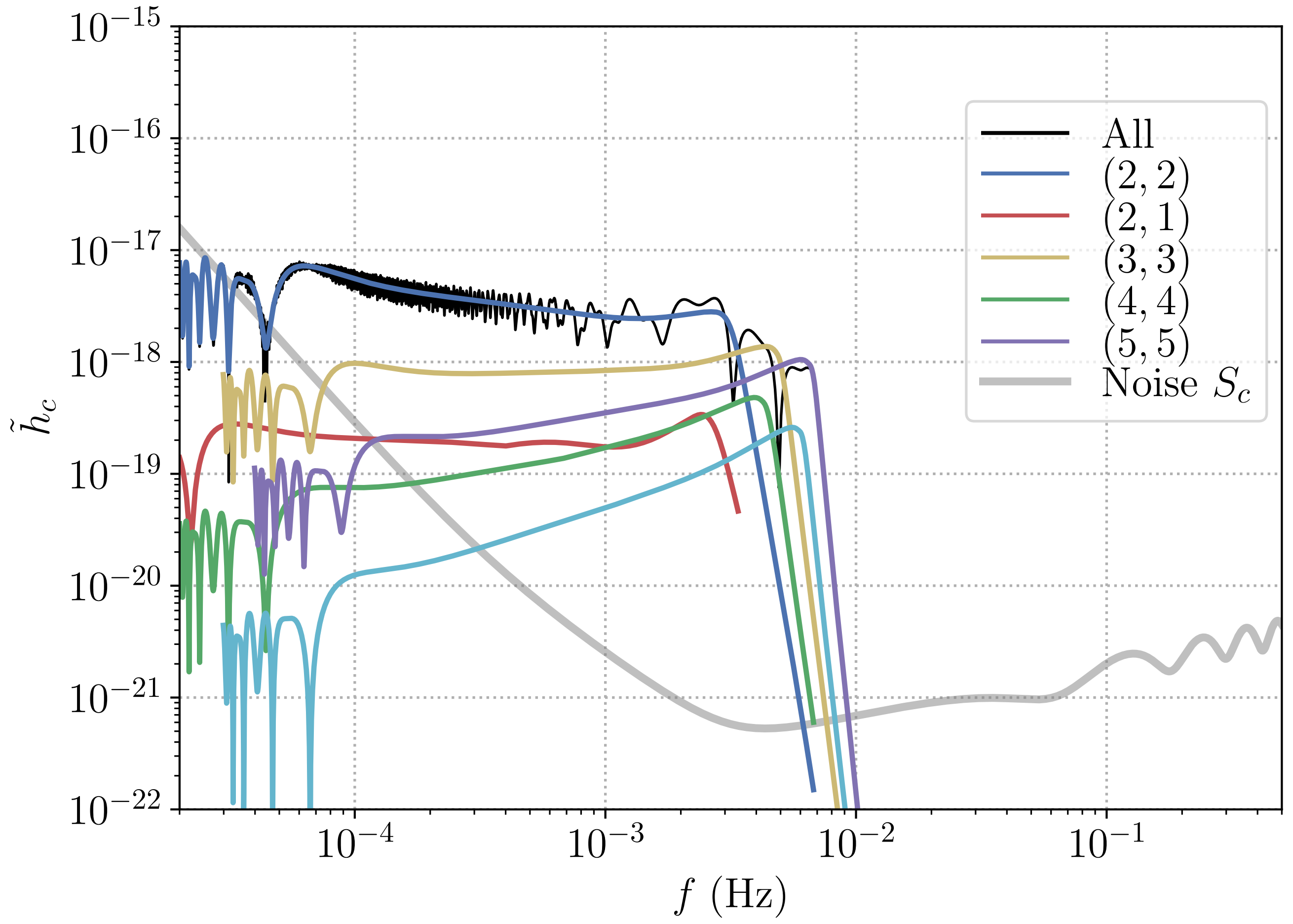
- Very long: >1yr
- Localization unimodal early on, no sky degeneracies

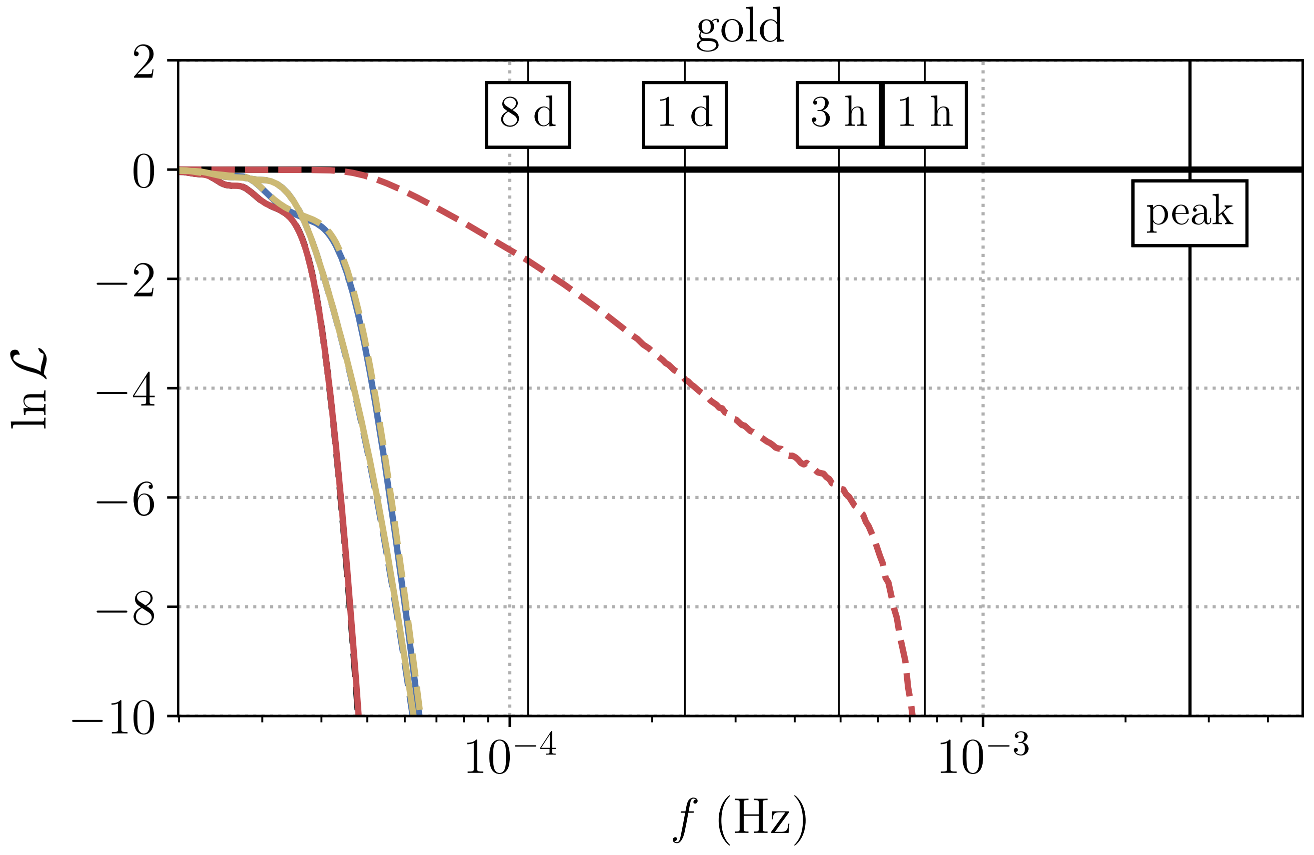


‘M3e6’

$$M_{\rm source} = 3 \times 10^6 M_\odot \,, z = 1$$

- Observable for ~2w

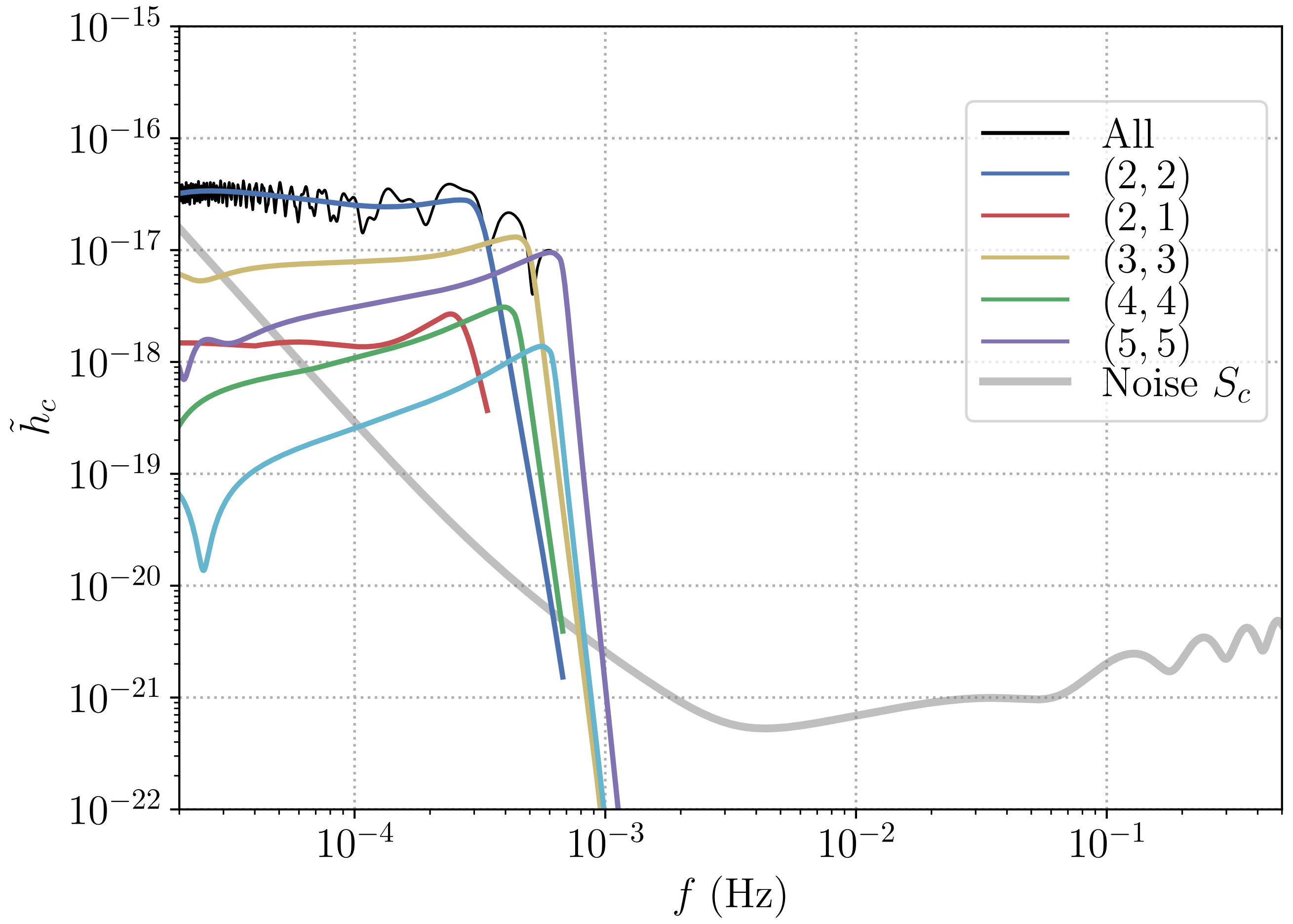


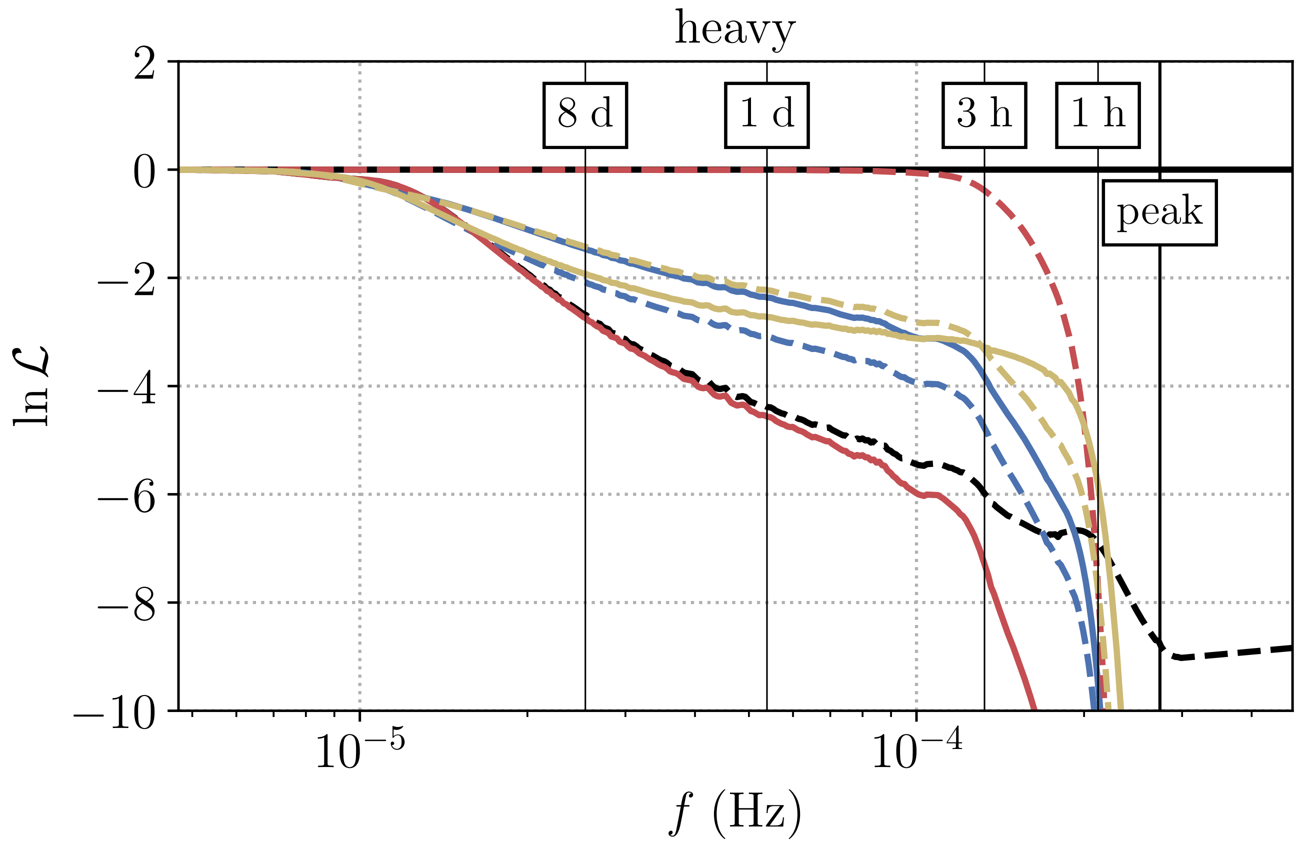


‘M1e7’

$$M_{\rm source} = 3 \times 10^7 M_\odot \,, z = 1$$

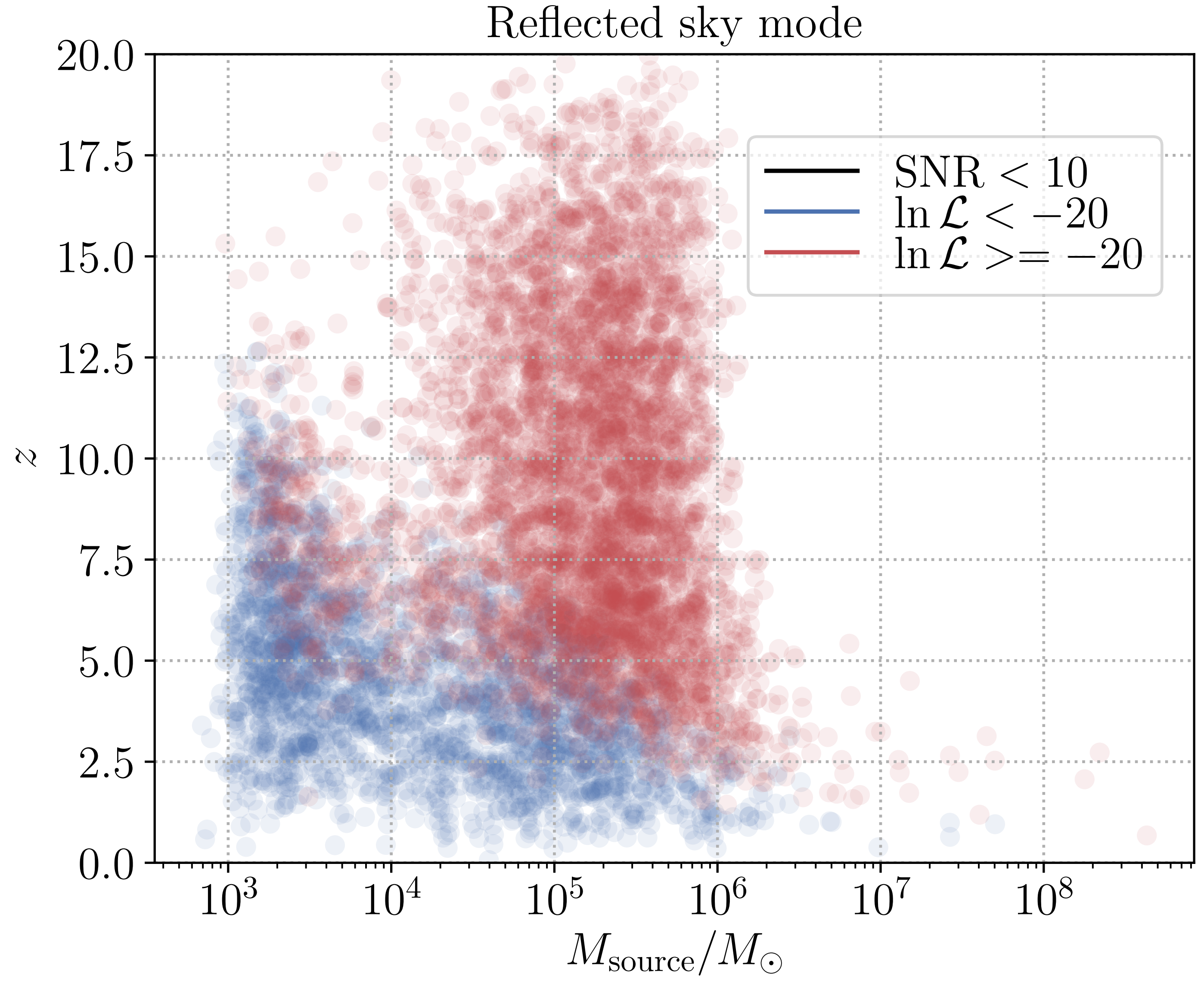
- Observable for ~2d

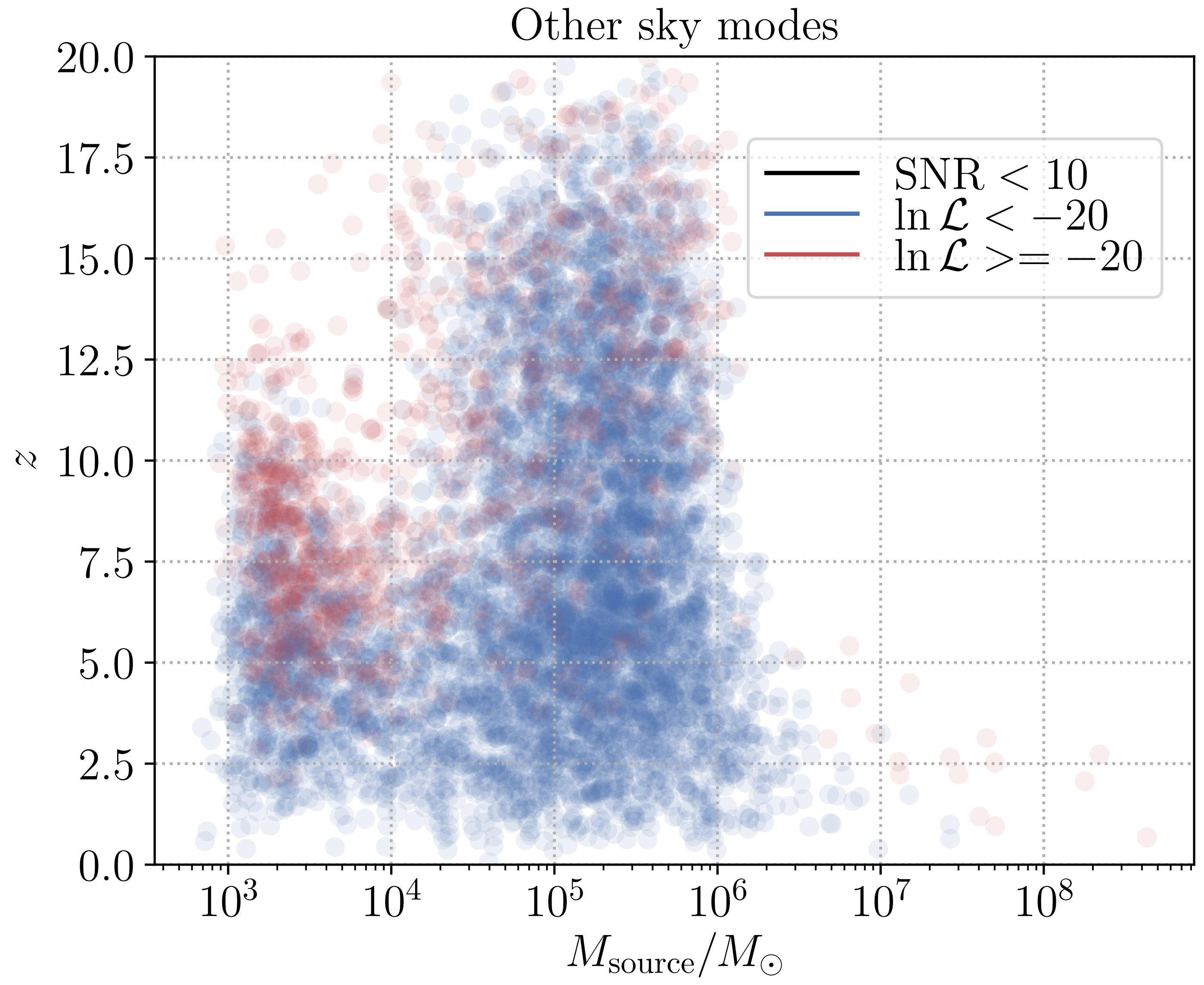




Multimodality of the sky localization: astrophysical catalogs

Post-merger



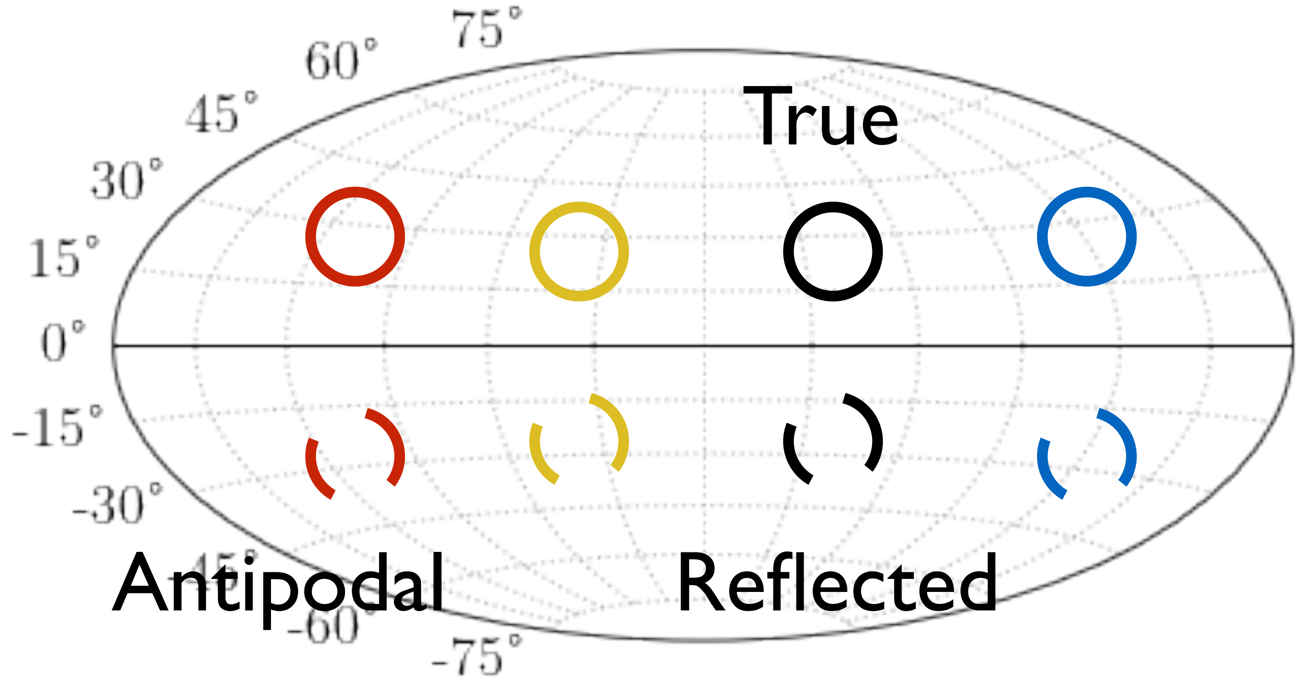


Approximate degeneracy measure: likelihood at the other sky positions

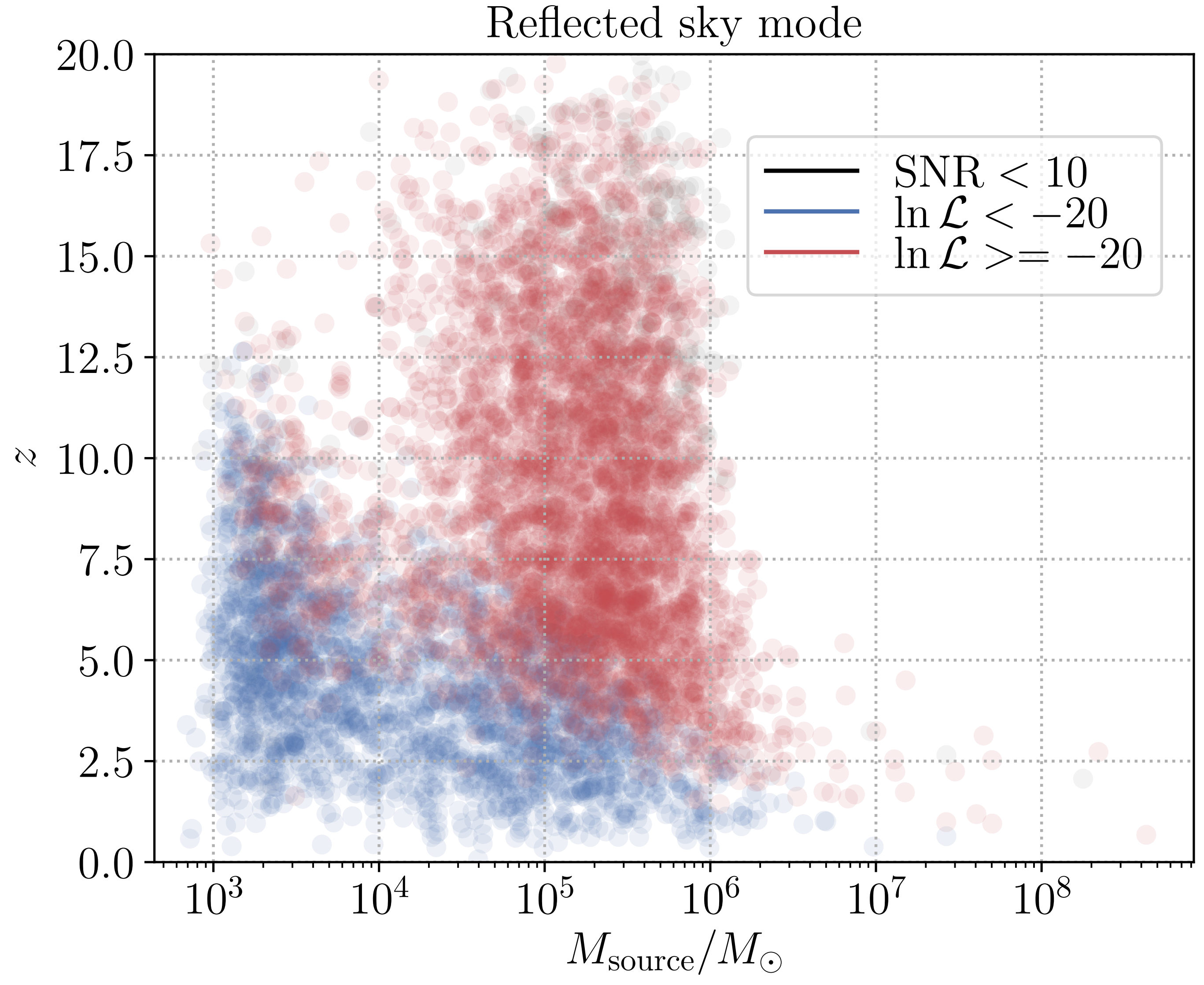
Threshold used here (a bit arbitrary):

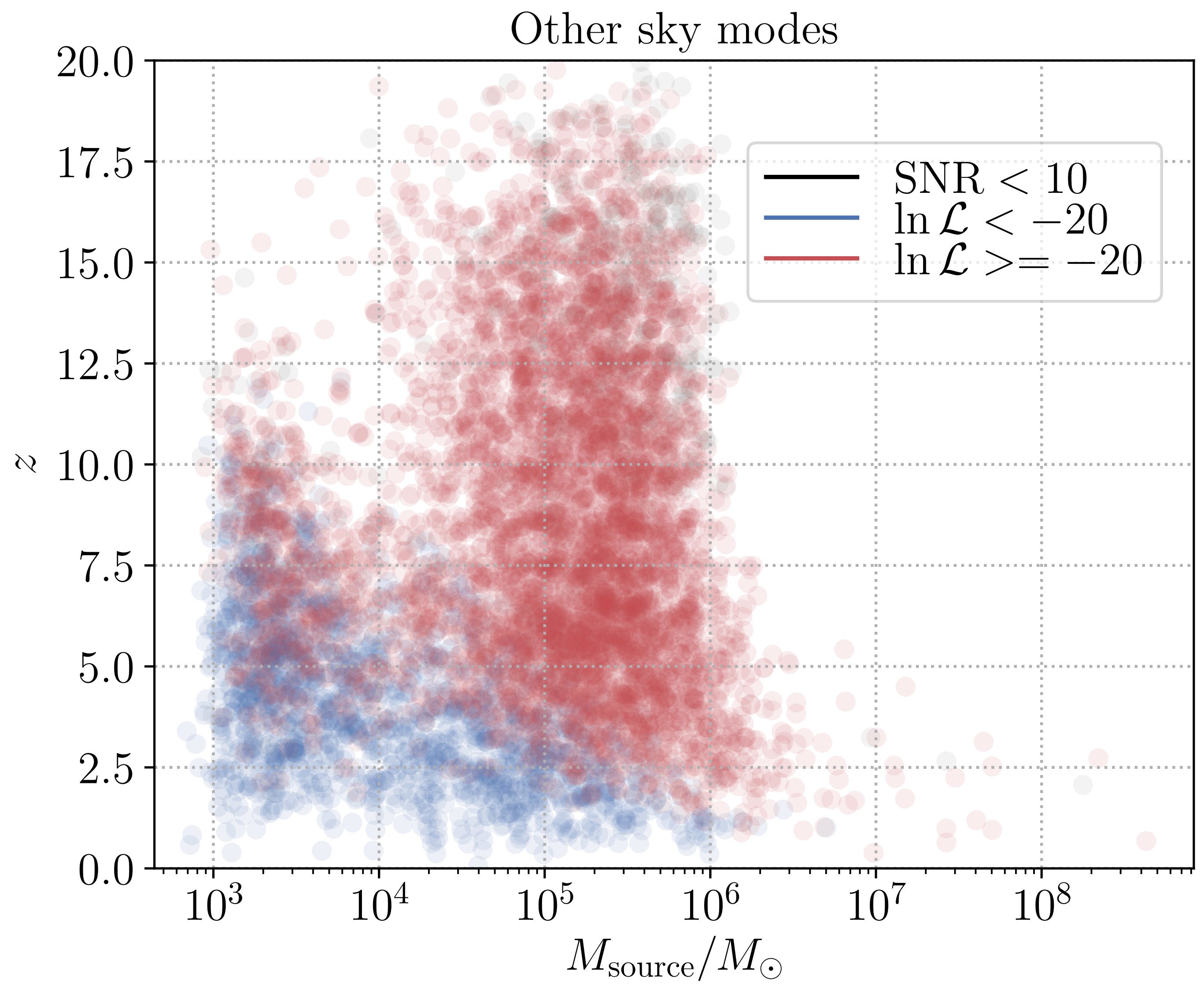
$$\ln\mathcal{L} > -20$$

Blue: presumably not degenerate
Red: presumably degenerate



Pre-merger 5hr

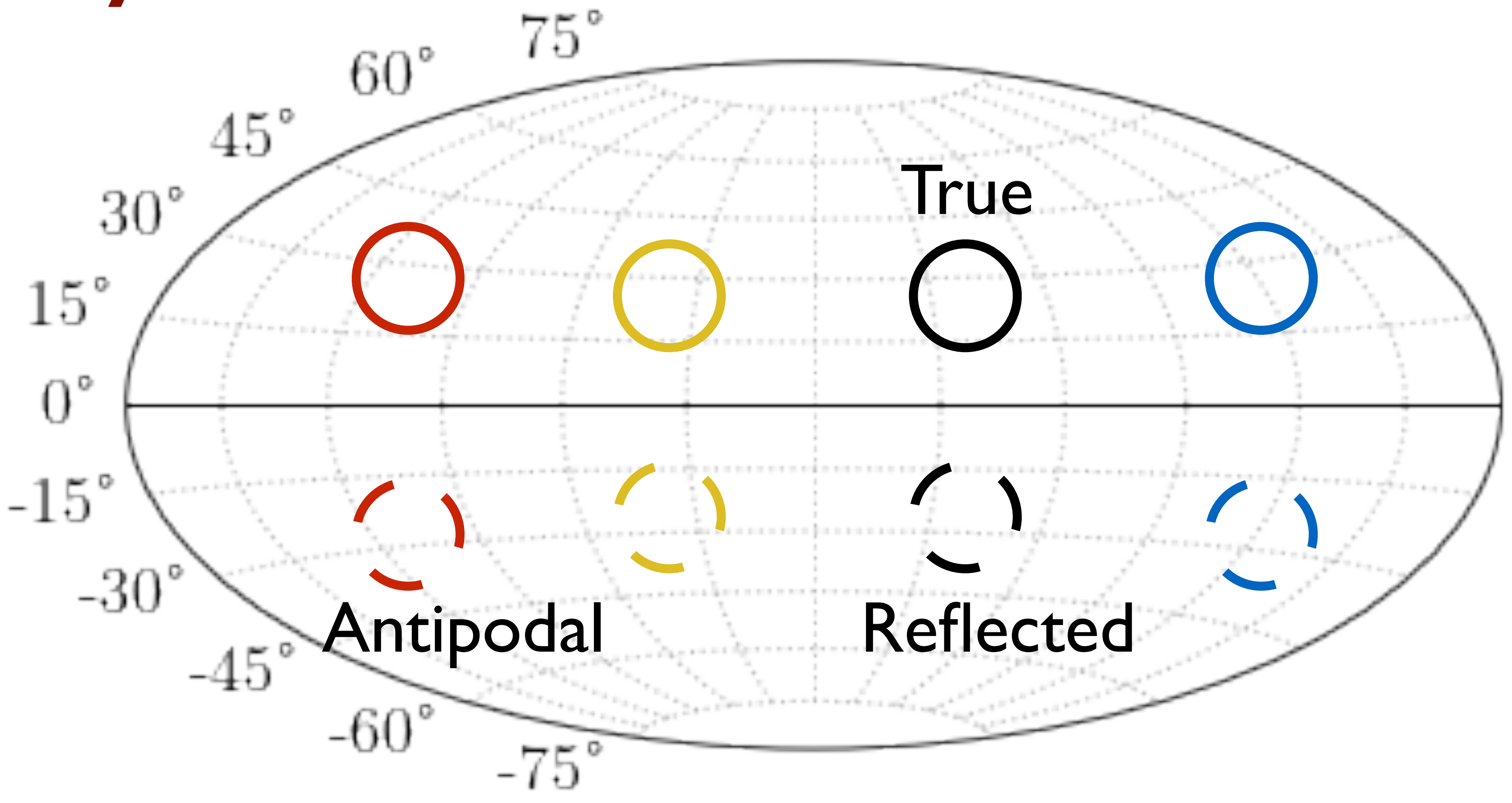


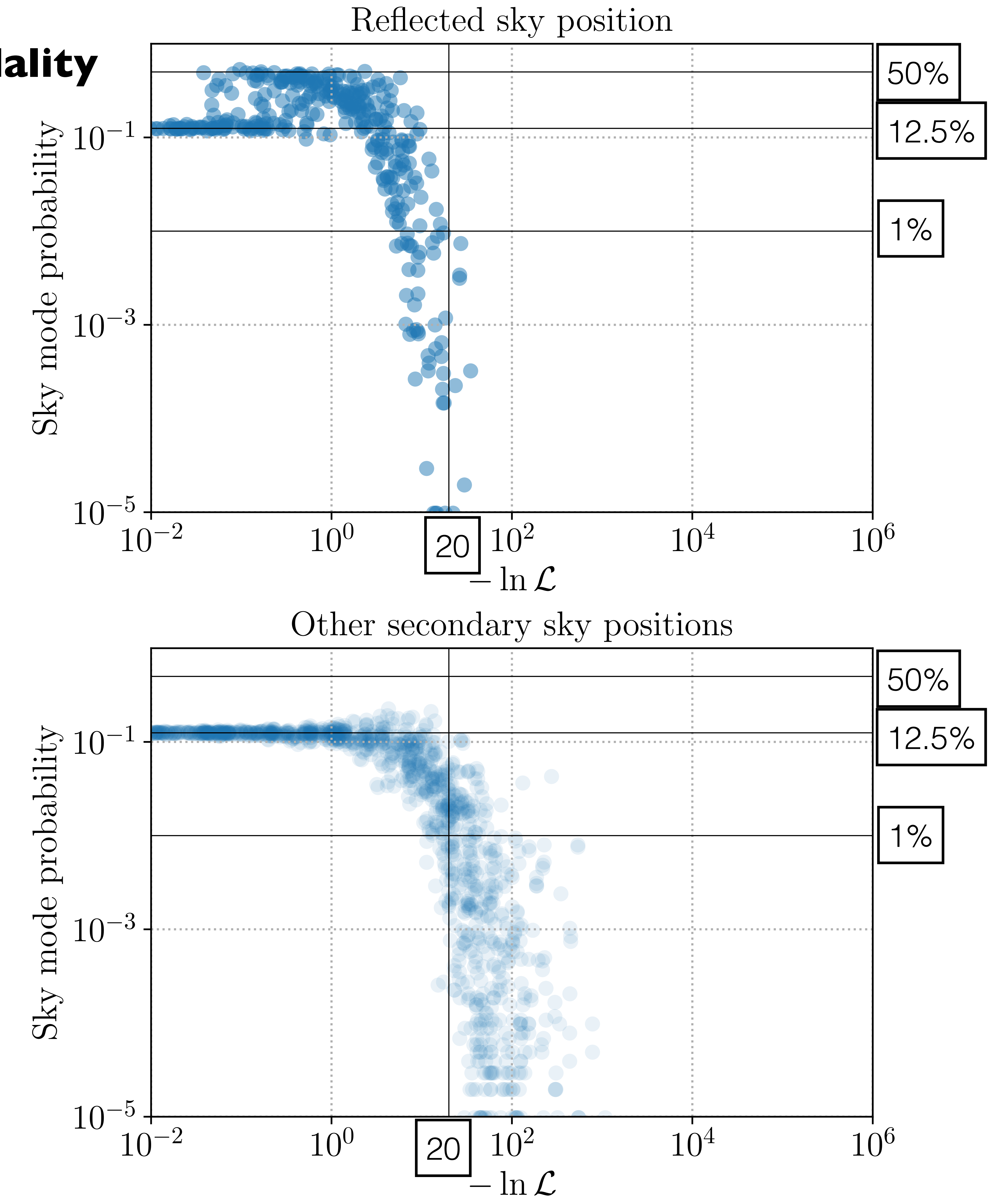


Multimodality of the sky localization: a likelihood estimator ?

400 post-merger PE runs
at 'boundary' of multimodality

Sky modes L-frame





Point-like estimate of multimodality: likelihood at the sky modes.

Relation to posterior probability weights from actual (expensive) sampling ?

Dispersion in this relation; thresholding approximate