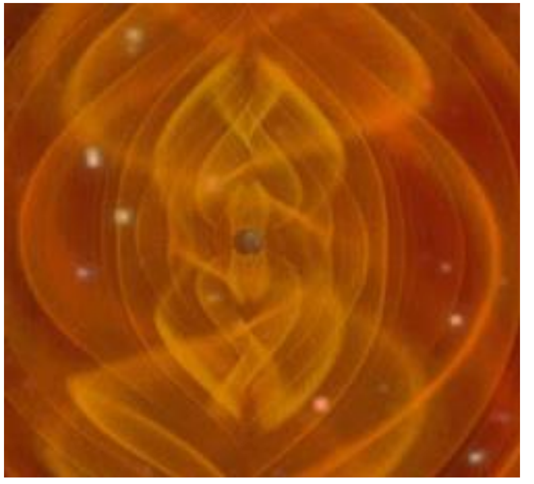


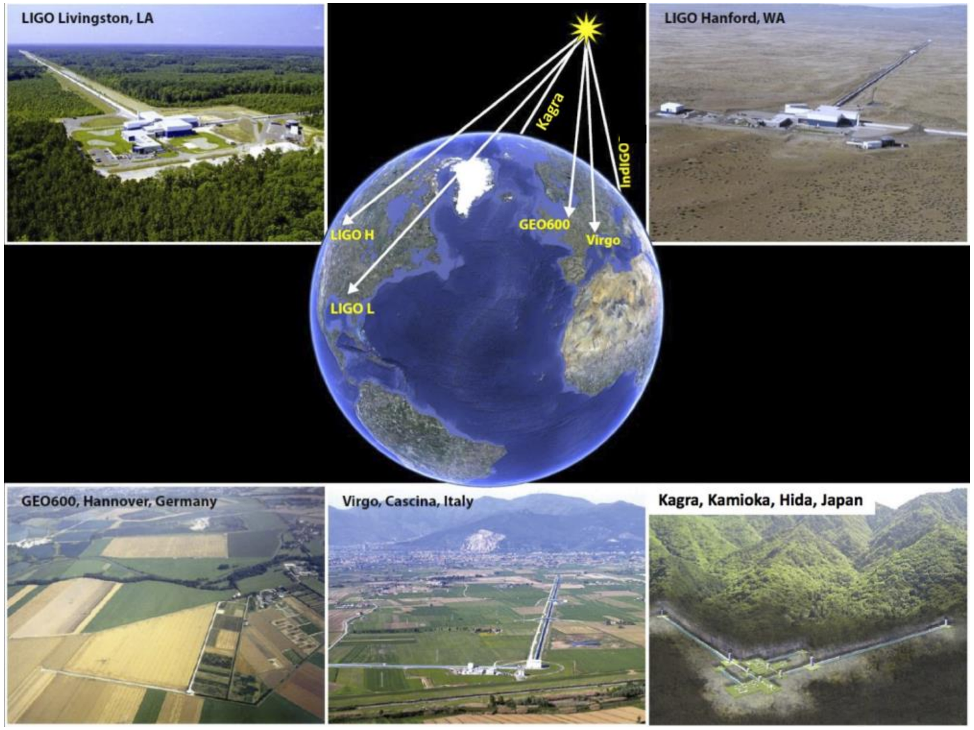
Testing general relativity with gravitational waves

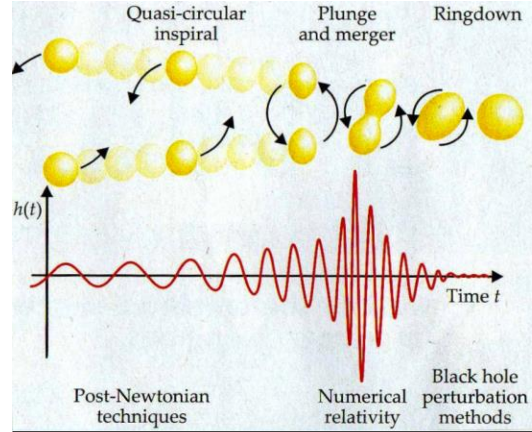
Chris Van Den Broeck

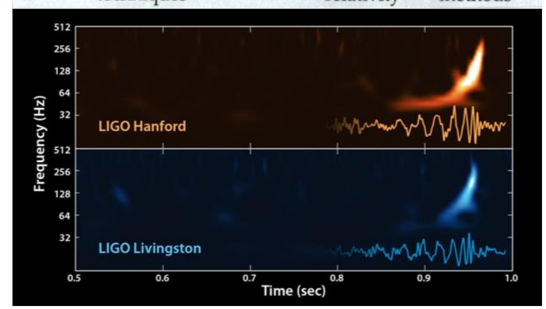
Universiteit Utrecht

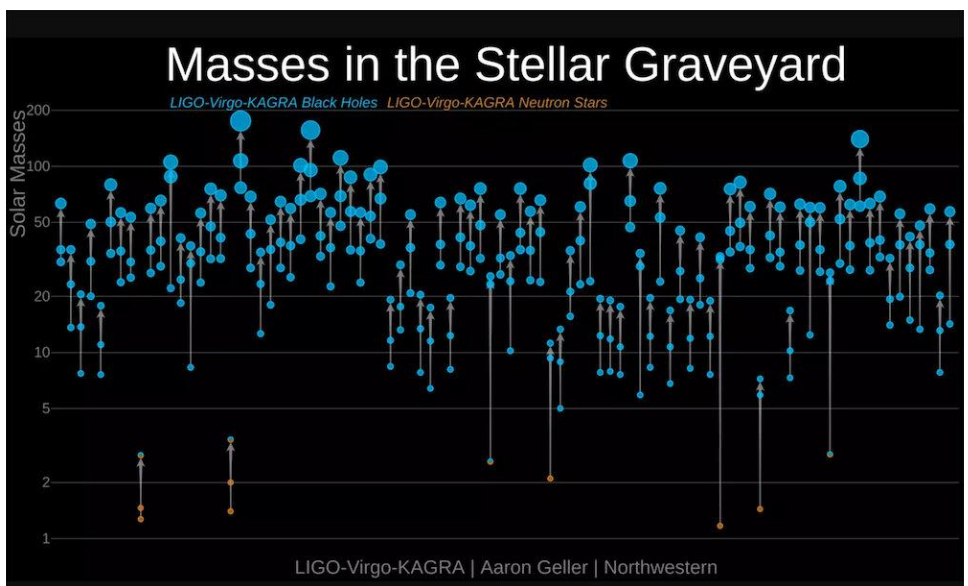
Nikhef

First ACME Workshop: The gravitational wave sky and complementary observations
Toulouse, 7-11 April 2025


LIGO Livingston, LA
LIGO Hanford, WA
Kagra
IndIGO
GEO600
Virgo
LIGO H
LIGO L
GEO600, Hannover, Germany
Virgo, Cascina, Italy
Kagra, Kamioka, Hida, Japan

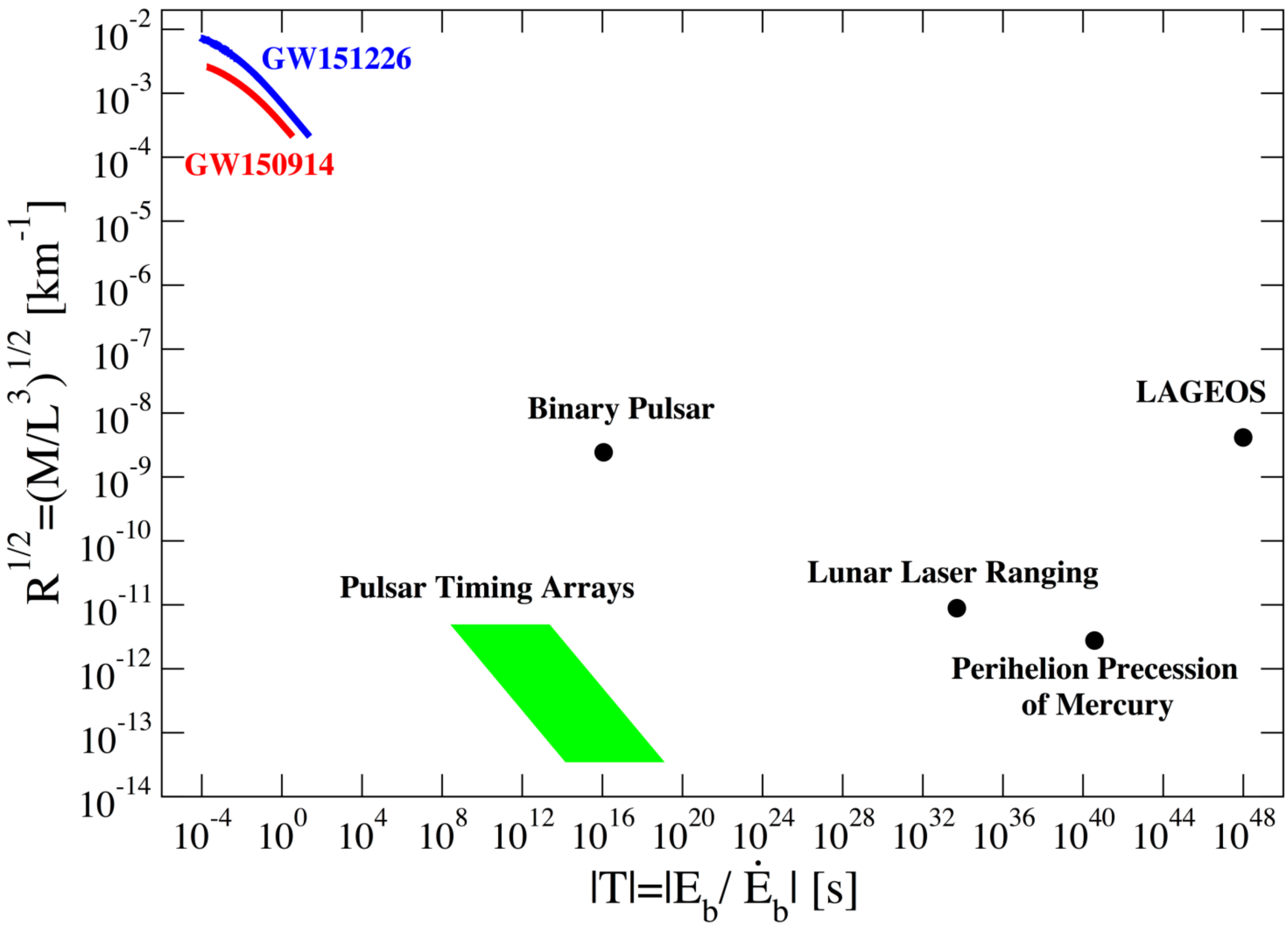

Quasi-circular inspiral
Plunge and merger
Ringdown
h(t)
Time t
Post-Newtonian techniques
Numerical relativity
Black hole perturbation methods


Frequency (Hz)
LIGO Hanford
LIGO Livingston
Time (sec)


Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
Solar Masses
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Access to strongly curved, dynamical spacetime

Curvature of spacetime



Characteristic timescale

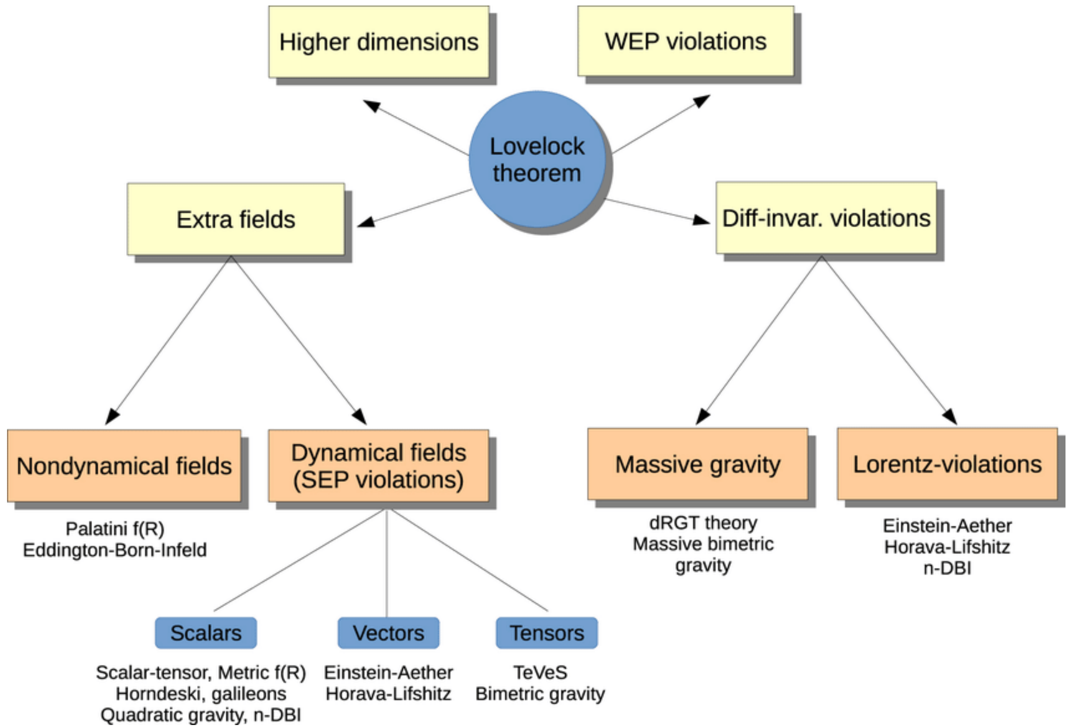
Yunes et al., PRD **94**, 084002 (2016)

The nature of gravity

- Lovelock's theorem:

  "In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term."

- Relaxing one or more of the assumptions allows for a plethora of alternative theories:



Berti et al., CQG **32**, 243001 (2015)

- Most alternative theories: no full inspiral-merger-ringdown waveforms known
  - Most current tests are **model-independent**

Fundamental physics with gravitational waves

1. The strong-field dynamics of spacetime
    - Is the inspiral-merger-ringdown process consistent with the predictions of GR?

2. The propagation of gravitational waves
    - Evidence for dispersion?

3. What is the nature of compact objects?

    Are the observed massive objects the "standard" black holes of classical general relativity?

    - Are there unexpected effects during inspiral?
    - Is the remnant object consistent with the no-hair conjecture? Is it consistent with Hawking's area increase theorem?
    - Searching for gravitational wave echoes

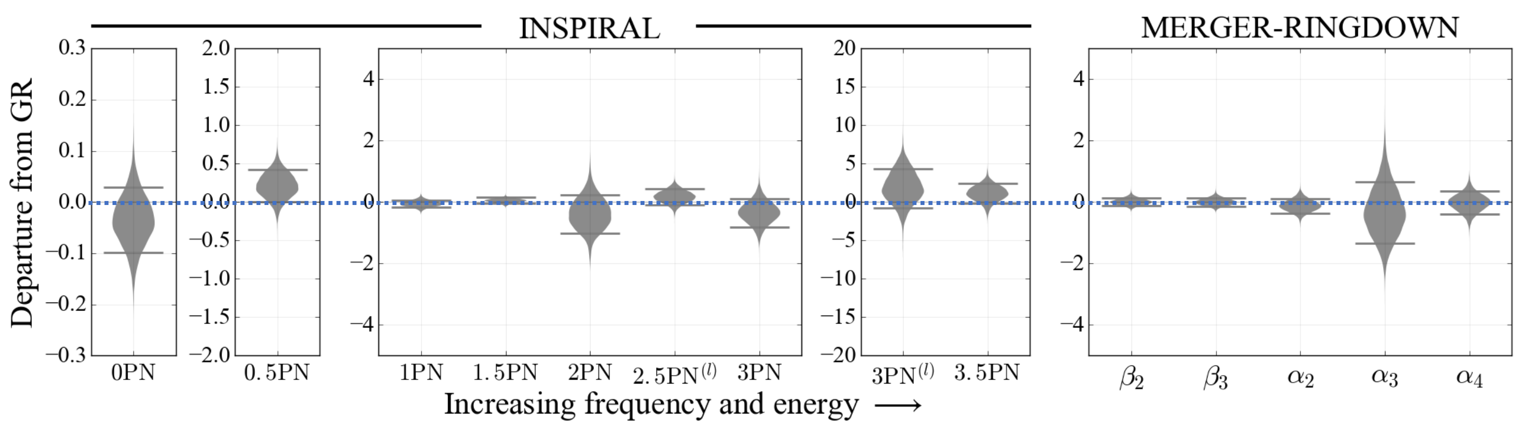
1. The strong-field dynamics of spacetime

- Inspiral-merger-ringdown process
  - Post-Newtonian description of inspiral phase

$$\Phi(v) = \left(\frac{v}{c}\right)^{-5}\left[\varphi_{0\mathrm{PN}} + \varphi_{0.5\mathrm{PN}}\left(\frac{v}{c}\right) + \varphi_{1\mathrm{PN}}\left(\frac{v}{c}\right)^{2} + \ldots + \varphi_{2.5\mathrm{PN}^{(l)}}\log\left(\frac{v}{c}\right)\left(\frac{v}{c}\right)^{5} + \ldots + \varphi_{3.5\mathrm{PN}}\left(\frac{v}{c}\right)^{7}\right]$$

  - Merger-ringdown governed by additional parameters β_n, α_n

- Place bounds on deviations in these parameters:



LIGO + Virgo, arXiv:2112.06861

- Rich physics:
  Dynamical self-interaction of spacetime, spin-orbit and spin-spin interactions

- Can combine information from multiple detections
  - Bounds will get tighter roughly as $1/\sqrt{N_{\mathrm{det}}}$

A theory-specific test with GW230529

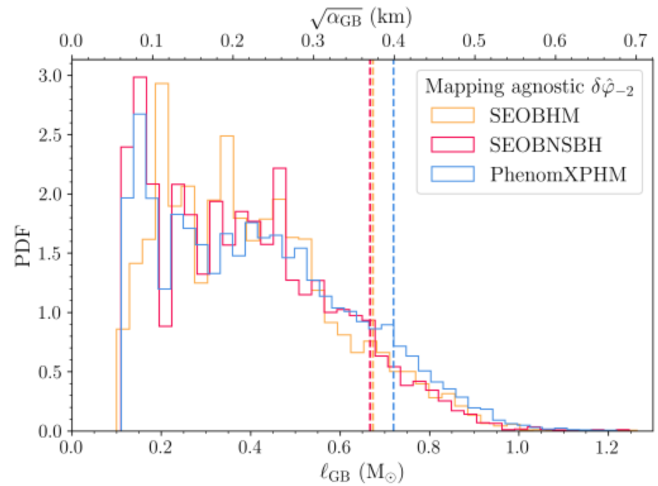
- Neutron star merging with lower mass-gap event
- Strongest constraints to date on the -1PN coefficient
- Einstein-scalar-Gauss-Bonnet theory:

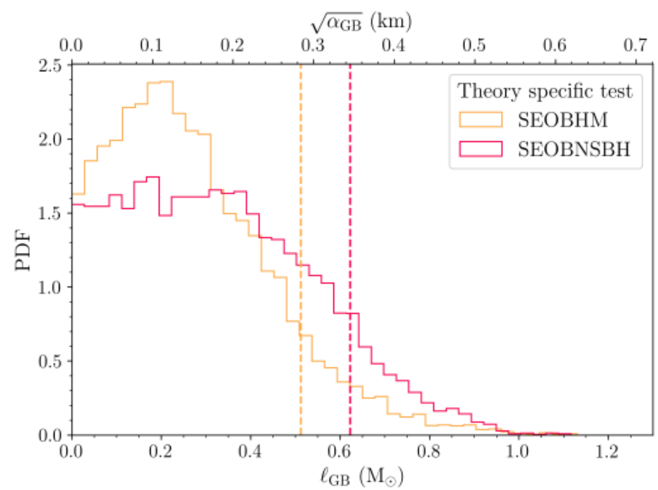
$$S = \frac{1}{16\pi}\int \mathrm{d}x^4 \sqrt{-g}\left(R - 2(\partial\phi)^2 + \ell_{\mathrm{GB}}^2 f(\phi)\mathcal{G}\right)$$

with Gauss-Bonnet invariant

$$\mathcal{G} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$$

and $f(\phi) = 2\phi + \mathcal{O}(\phi^2)$





- "Agnostic" test:

$$\delta\hat{\varphi}_{-2} = \frac{-5\ell_{\mathrm{GB}}^4}{168 m_1^4}$$

- Theory-specific test (adding corrections up to 1.5PN):

$$\ell_{\mathrm{GB}} \lesssim 0.51\ M_\odot$$

- Best bound so far!

Saenger et al., arXiv:2406.03568

2. The propagation of gravitational waves

- Dispersion of gravitational waves?
  E.g. as a result of **non-zero graviton mass**:
  - Dispersion relation:

    $$E^2 = p^2c^2 + m_g^2c^4$$

  - Graviton speed:

    $$v_g/c = 1 - m_g^2c^4/2E^2$$

  - Modification to gravitational wave phase:

    $$\delta\Psi = -\pi Dc/[\lambda_g^2(1+z)\,f] \qquad\qquad \lambda_g = h/(m_g c)$$

- Bound on graviton mass:

  $$\boxed{m_g \leq 1.76 \times 10^{-23}\,\mathrm{eV}/c^2}$$

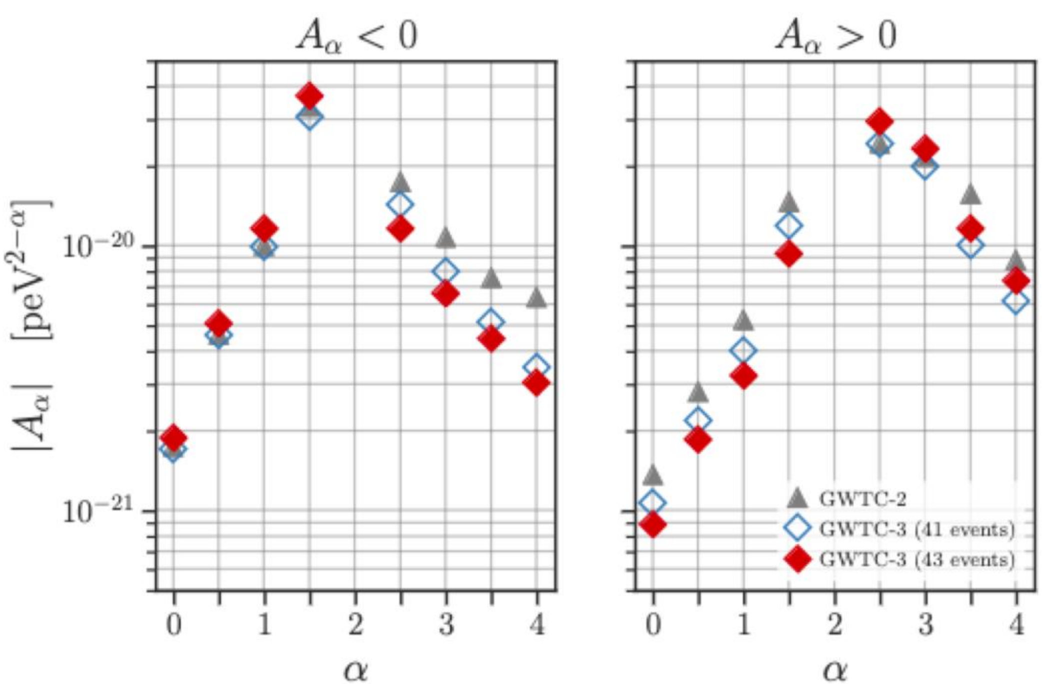
LIGO + Virgo, arXiv:2112.06861

2. The propagation of gravitational waves

- More general forms of dispersion:

$$E^2 = p^2c^2 + Ap^\alpha c^\alpha$$

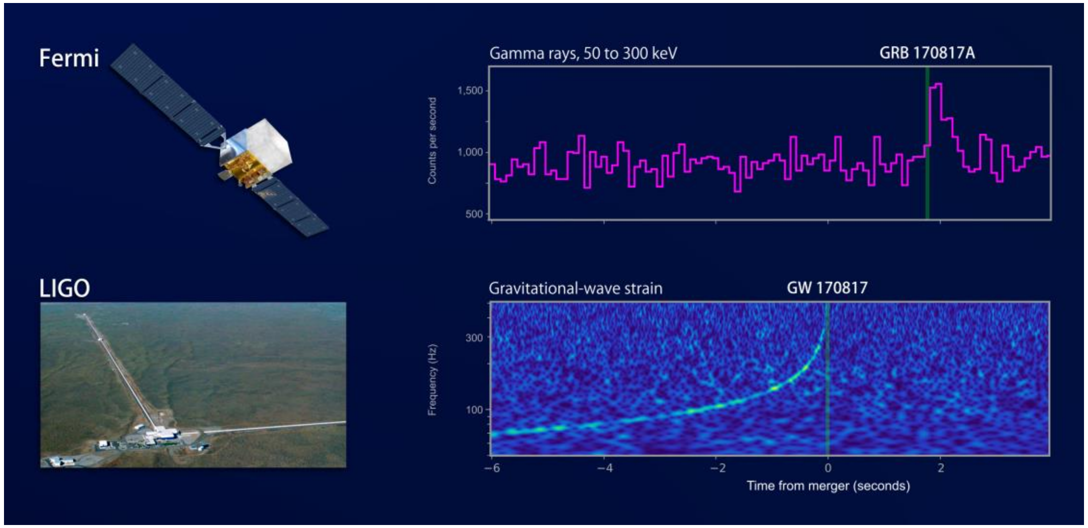
  - $\alpha \neq 0$ corresponds to violation of local Lorentz invariance
  - $\alpha = 2.5$ multi-fractal spacetime
  - $\alpha = 3$ doubly special relativity
  - $\alpha = 4$ higher-dimensional theories



LIGO + Virgo, arXiv:2112.06861

The propagation of gravitational waves

- **Does the speed of gravity equal the speed of light?**
- The binary neutron star coalescence GW170817 came with gamma ray burst, **1.74 seconds afterwards**



- With a conservative lower bound on the distance to the source:

$$\boxed{-3 \times 10^{-15} < (v_{GW} - v_{EM})/v_{EM} < +7 \times 10^{-16}}$$

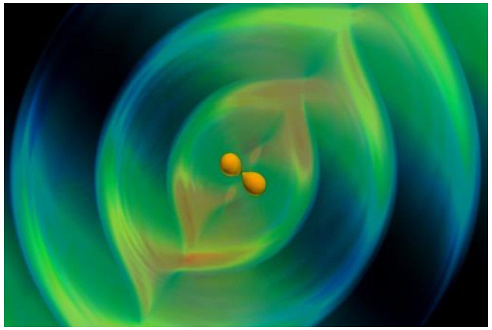
- Excluded certain alternative theories of gravity designed to explain dark matter or dark energy in a dynamical way

LIGO + Virgo + Fermi-GBM + INTEGRAL, ApJ. **848**, L13 (2017)
LIGO + Virgo, PRL **123**, 011102 (2019)

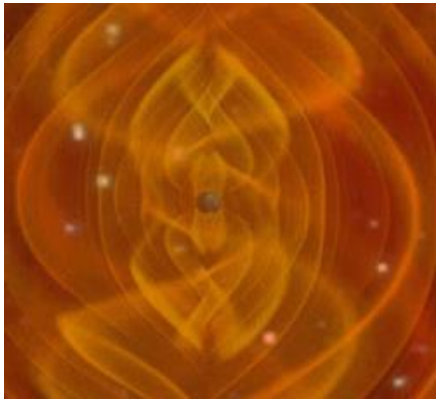
3. What is the nature of compact objects?

- Black holes, or still more exotic objects?
  - Boson stars
  - Dark matter stars
  - Clouds of ultralight bosons surrounding black holes
  - Gravastars
  - Wormholes
  - Firewalls, fuzzballs
  - *The unknown*

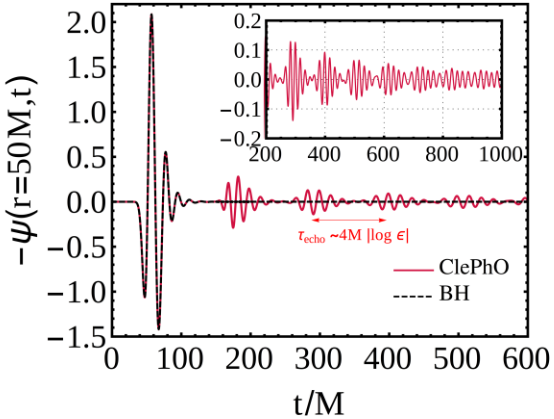
3. What is the nature of compact objects?



Anomalous effects during inspiral

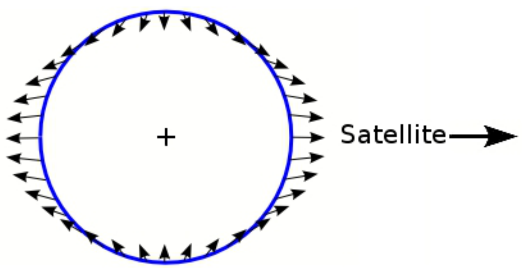


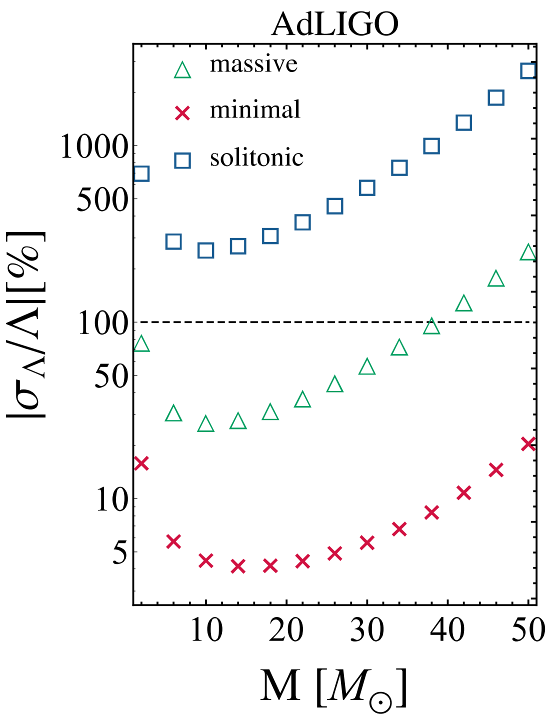
Ringdown of newly formed object



Gravitational wave echoes

Anomalous effects during inspiral





- Tidal field of one body causes quadrupole deformation in the other:

  $$Q_{ij} = -\lambda(\mathrm{EOS}; m)\,\mathcal{E}_{ij}$$

  where $\lambda(\mathrm{EOS}; m)$ depends on internal structure (equation of state)
  - Black holes: $\lambda \equiv 0$
  - Boson stars, dark matter stars: $\lambda > 0$
  - Gravastars: $\lambda < 0$

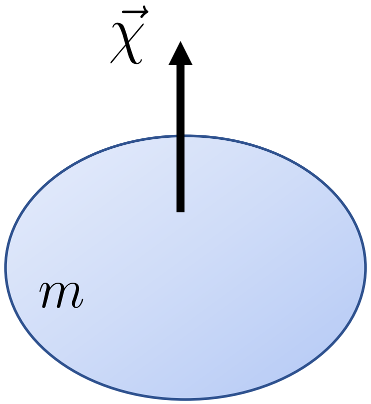
- Enters inspiral phase at order $(v/c)^{10}$, through

  $$\lambda(m)/m^5 \propto (R/m)^5$$

  - *O*(10^2 - 10^3) for neutron stars
  - Can also be measurable for black hole mimickers, e.g. boson stars

Cardoso et al., PRD **95**, 084014 (2017)
Johnson-McDaniel et al., arXiv:1804.0826

Anomalous effects during inspiral



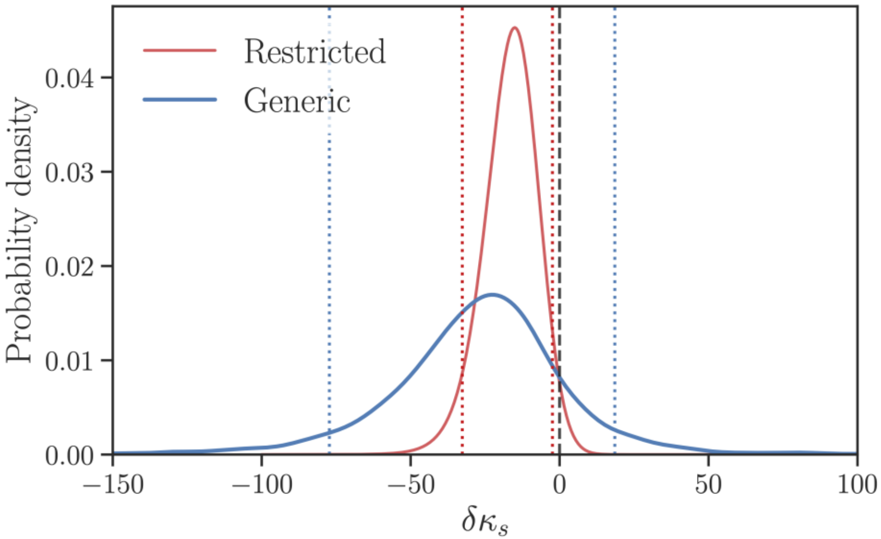
- Spin of an individual compact object also induces a quadrupole moment:

$$Q = -\kappa\,\chi^2 m^3$$

  - Black holes: $\kappa = 1$
  - Boson stars, dark matter stars: $\kappa > 0$
  - Gravastars: $\kappa < 0$

- Allow for deviations from black hole value:

$$Q = -(1+\delta\kappa)\,\chi^2 m^3$$



Possible theoretical values for boson stars:
$\kappa \sim 10 - 150$

… hence constraints are already of interest!

Krishnendu et al., PRD **100**, 104019 (2019)
LIGO + Virgo, arXiv:2112.06861

Ringdown of newly formed black hole

- Ringdown regime: Kerr metric + linear perturbations
  - Ringdown signal is a superposition of damped sinusoids

$$h(t) = \sum_{lmn} \mathcal{A}_{lmn}\, e^{-t/\tau_{lmn}} \cos(2\pi f_{lmn} t + \phi_{lmn})$$

  - Characteristic frequencies f_{lmn} and damping times τ_{lmn}

- No-hair conjecture: stationary, electrically neutral black hole completely characterized by mass M , spin χ
  - Linearized Einstein equations around Kerr background enforce specific dependences:

$$f_{lmn} = f_{lmn}(M,\chi)$$
$$\tau_{lmn} = \tau_{lmn}(M,\chi)$$

Berti et al., PRD **73**, 064030 (2006)

  - Look for deviations from the expressions for frequencies, damping times:

$$f_{lmn}(M,\chi) \quad \to \quad (1+\delta\hat{f}_{lmn})\, f_{lmn}(M,\chi)$$
$$\tau_{lmn}(M,\chi) \quad \to \quad (1+\delta\hat{\tau}_{lmn})\, \tau_{lmn}(M,\chi)$$

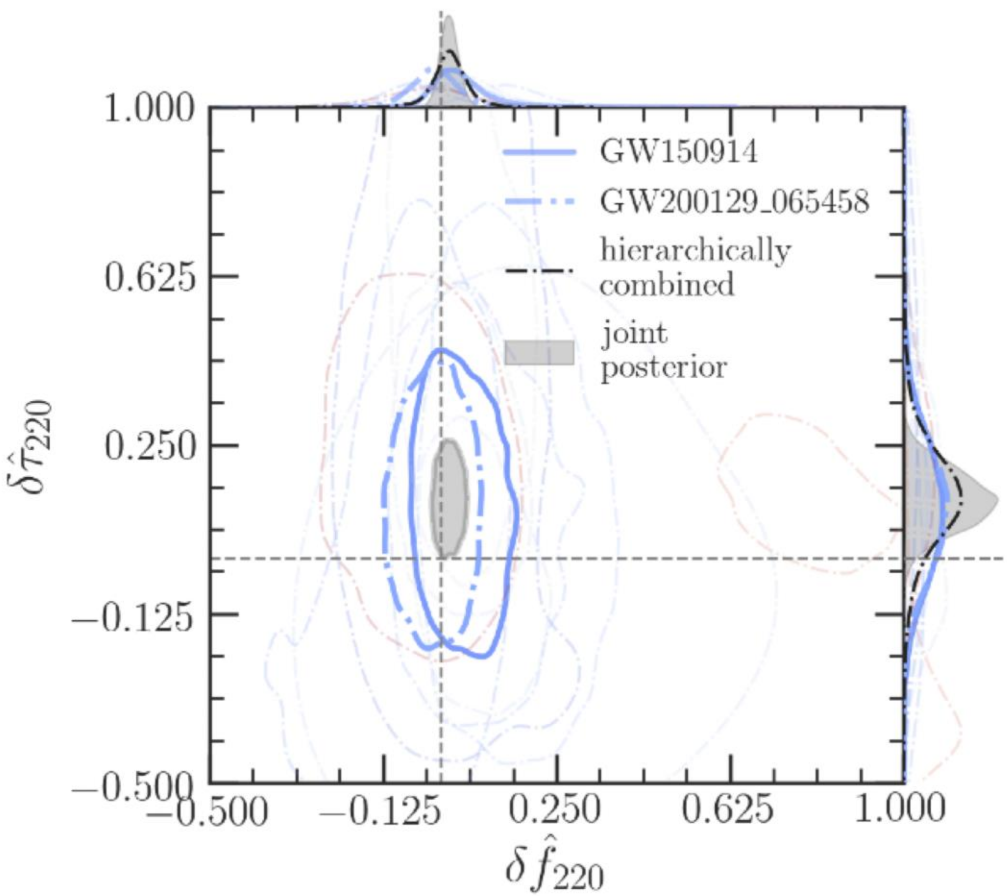
Carulllo et al., PRD **98**, 104020 (2018)
Brito et al., PRD **98**, 084038 (2018)

Ringdown of newly formed black hole

- Look for deviations from the expressions for frequencies, damping times:

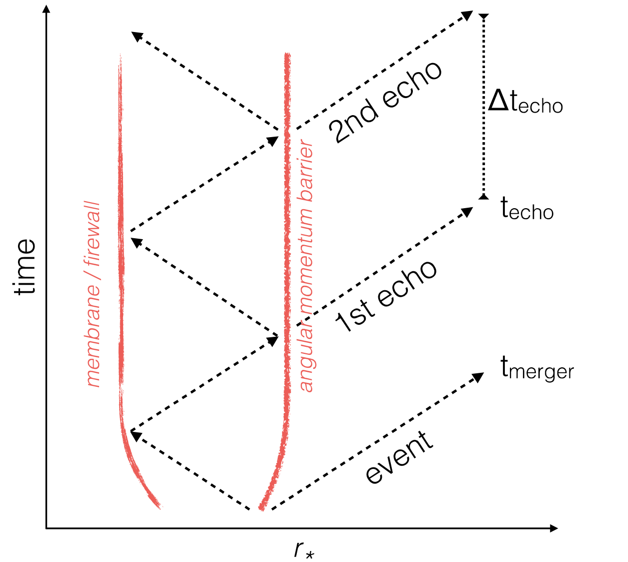
$$f_{lmn}(M,\chi) \rightarrow (1+\delta\hat{f}_{lmn})\, f_{lmn}(M,\chi)$$
$$\tau_{lmn}(M,\chi) \rightarrow (1+\delta\hat{\tau}_{lmn})\, \tau_{lmn}(M,\chi)$$

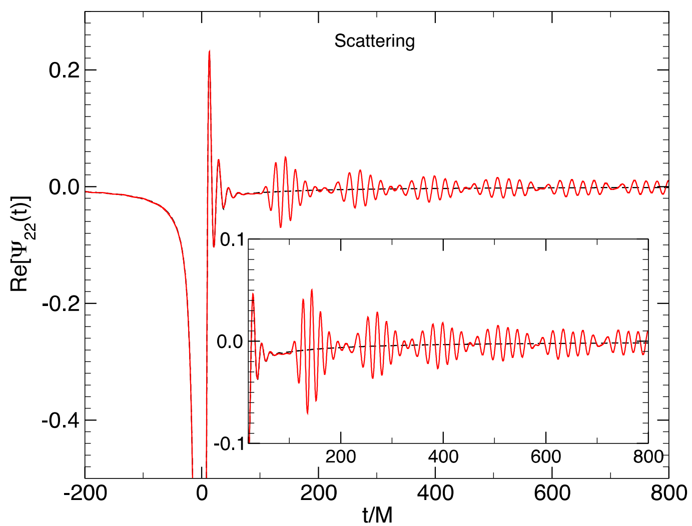
- Recent measurements:



LIGO + Virgo, arXiv:2112.06861

Gravitational wave echoes





- Exotic objects with corrections near horizon: continuing bursts of radiation called *echoes*
- If microscopic horizon modification $\ell \ll M$ then time between successive echoes

$M = 65\, M_\odot$

$$\Delta t \sim -nM \log\left(\frac{\ell}{M}\right)$$

where n is set by nature of object:

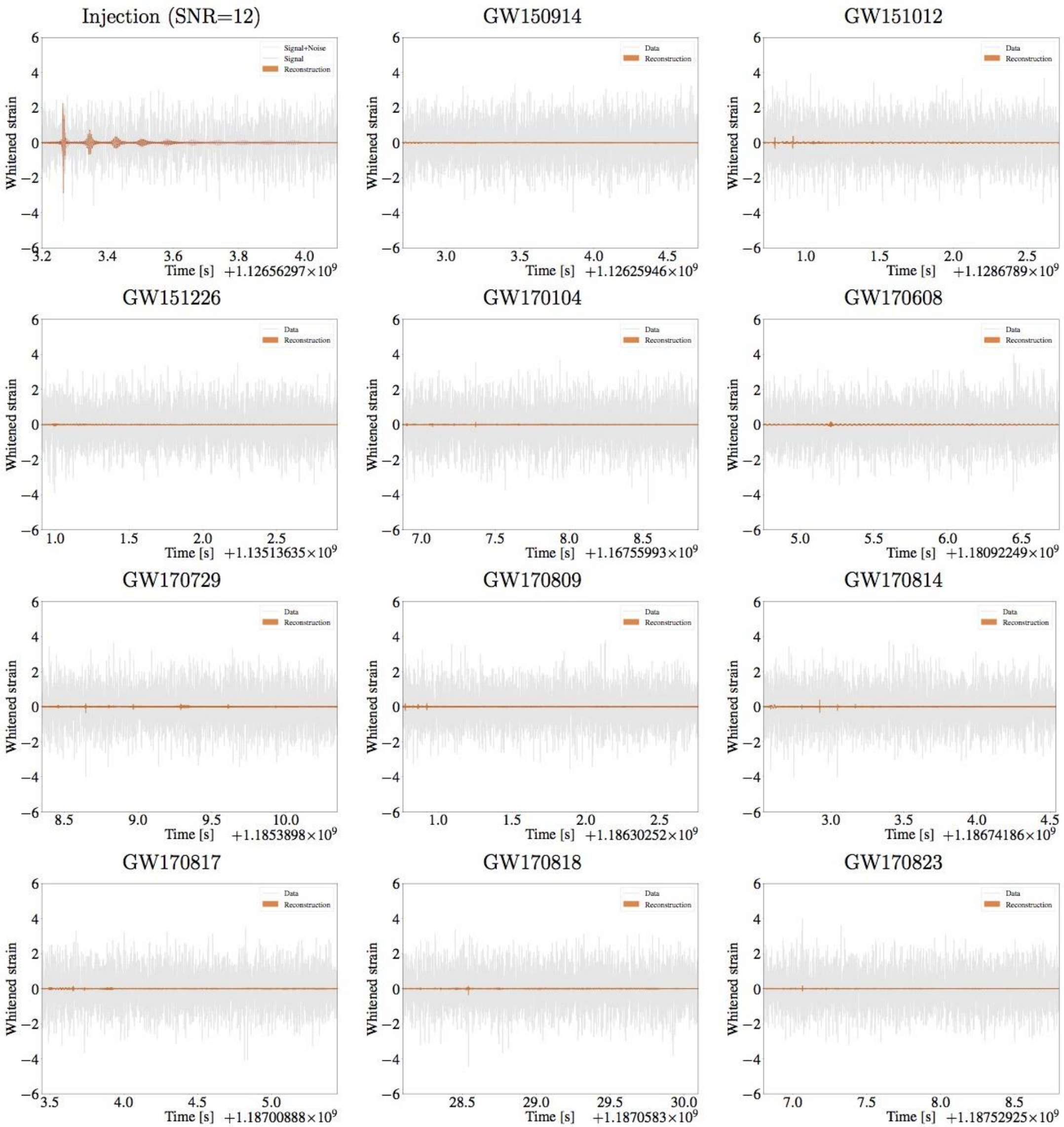
  - $n = 8$ for wormholes
  - $n = 6$ for thin-shell gravastars
  - $n = 4$ for empty shell
- For GW150914 ($M = 65\, M_\odot$), taking $\ell = \ell_{\mathrm{Planck}}$, and $n = 4$: $\Delta t = 117\,\mathrm{ms}$

Cardoso et al., PRL **116**, 171101 (2016)
Cardoso et al., PRD **94**, 084031 (2016)
Abedi et al., PRD **96**, 082004 (2017)
Westerweck et al., PRD **97**, 124037 (2018)

Tsang et al., PRD **98**, 024023 (2018)
Tsang et al., PRD **101**, 064012 (2020)
LIGO + Virgo + KAGRA, arXiv:2112.06861

Gravitational wave echoes

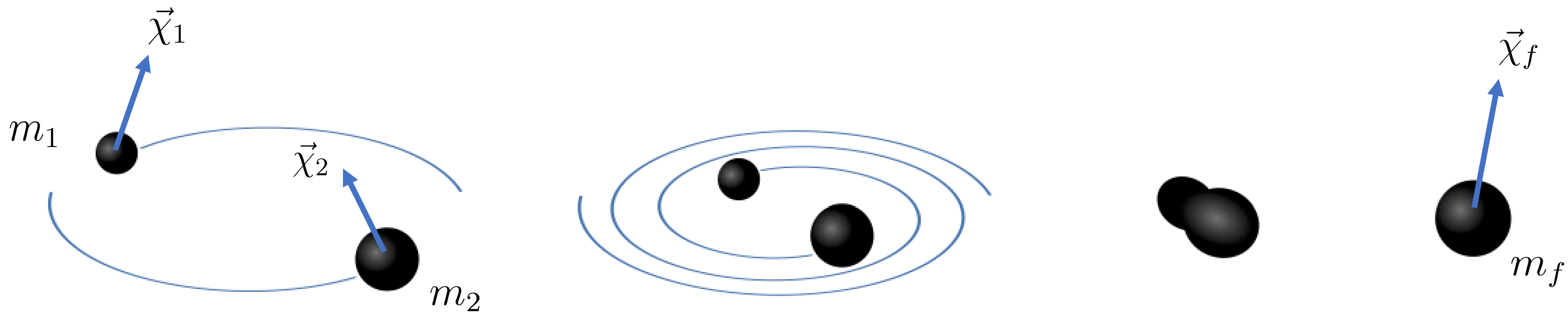
- *Morphology-independent* search for echoes: wavelet decomposition



Tsang et al., PRD **101**, 064012 (2020)
LIGO + Virgo + KAGRA, arXiv:2112.06861

First tests of Hawking's area increase theorem

- During binary black hole merger, horizon area should not decrease



- "Ingoing" black holes considered Kerr
  - Measure masses m_1 , m_2 and initial spins χ_1, χ_2 from inspiral signal
  - Total initial horizon area:

    $\mathcal{A}_0 = \mathcal{A}(m_1, \chi_1) + \mathcal{A}(m_2, \chi_2)$ where $\mathcal{A}(m, \chi) = 8\pi m^2(1 + \sqrt{1 - \chi^2})$

- Final black hole also Kerr
  - Obtain mass m_f and spin χ_f from ringdown frequencies and damping times
  - Final horizon area:

    $\mathcal{A}_f = \mathcal{A}(m_f, \chi_f)$

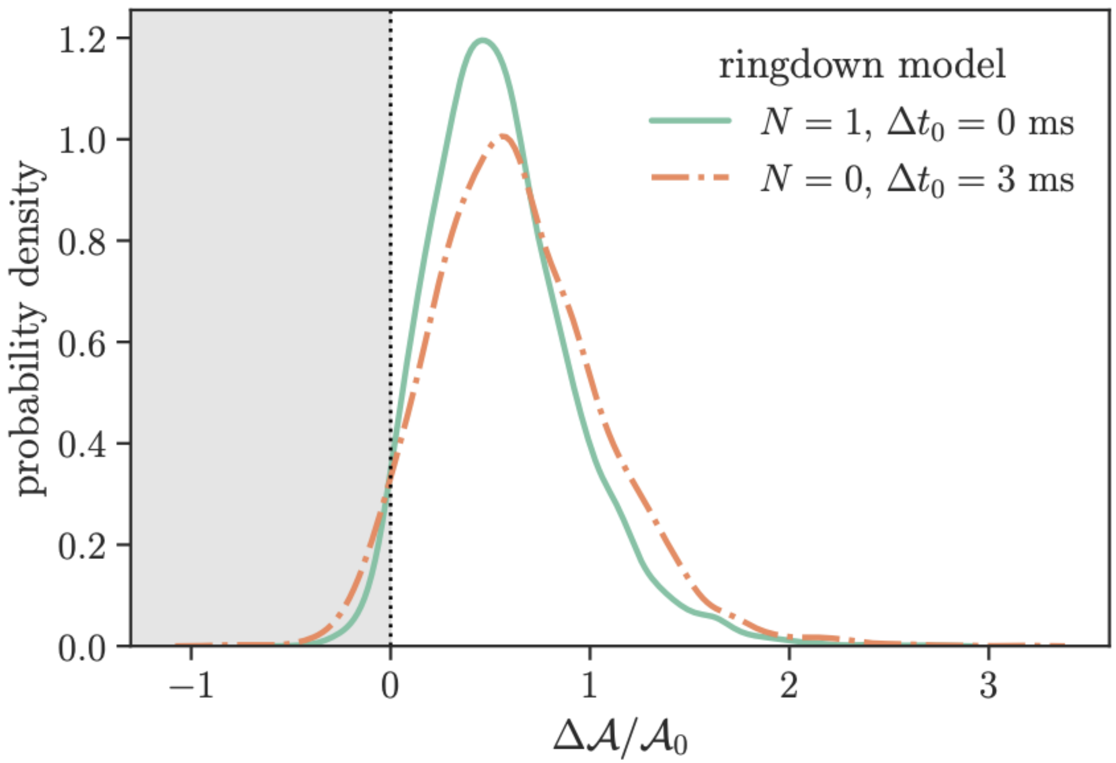
- According to the theorem: $\Delta\mathcal{A}/\mathcal{A}_0 = (\mathcal{A}_f - \mathcal{A}_0)/\mathcal{A}_0 \geq 0$

First tests of Hawking's area increase theorem

- According to the theorem:

$$\Delta\mathcal{A}/\mathcal{A}_0 = (\mathcal{A}_f - \mathcal{A}_0)/\mathcal{A}_0 \geq 0$$

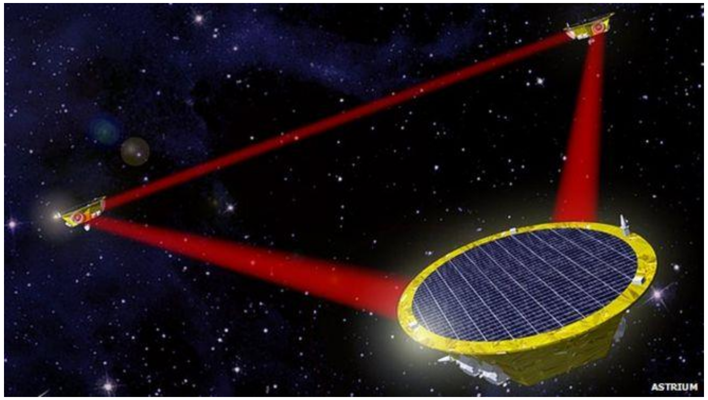
- Measurement on GW150914:

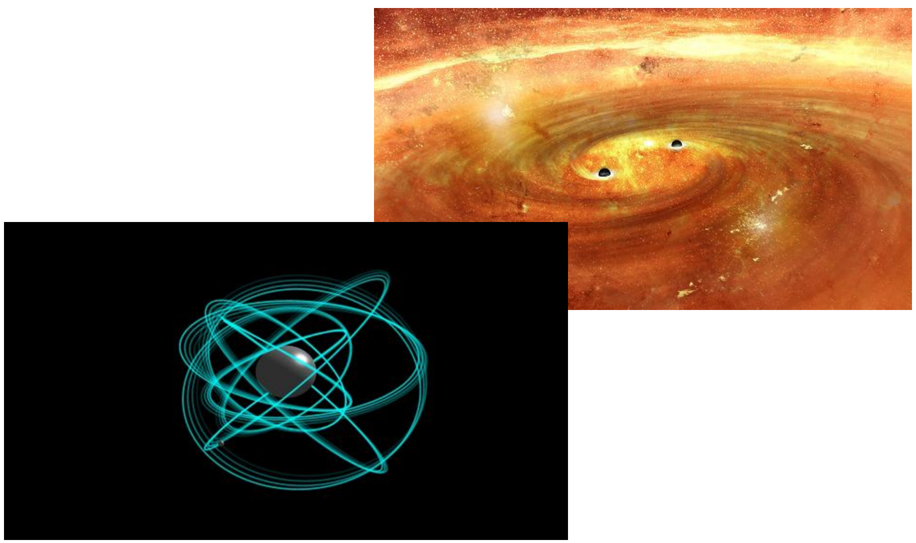


Cabero et al., arXiv:1711.09073
Isi et al., arXiv:2012.04486

- Agreement at > 95% probability

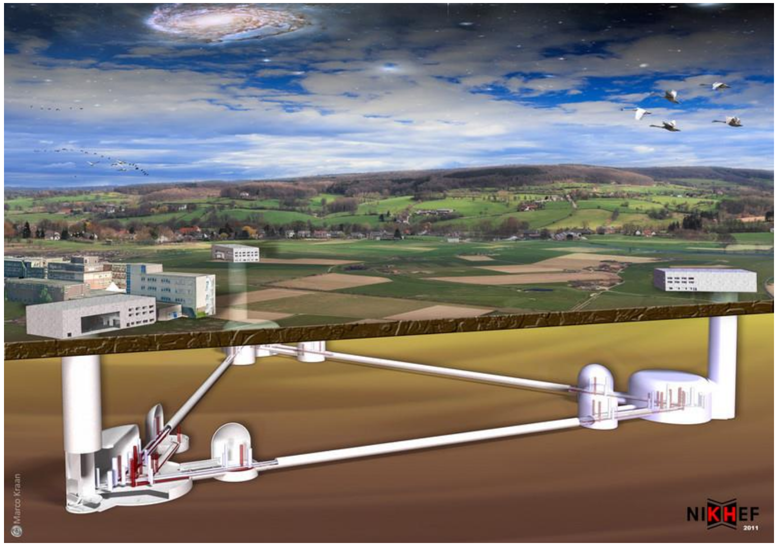
LISA: A gravitational wave detector in space (2034)

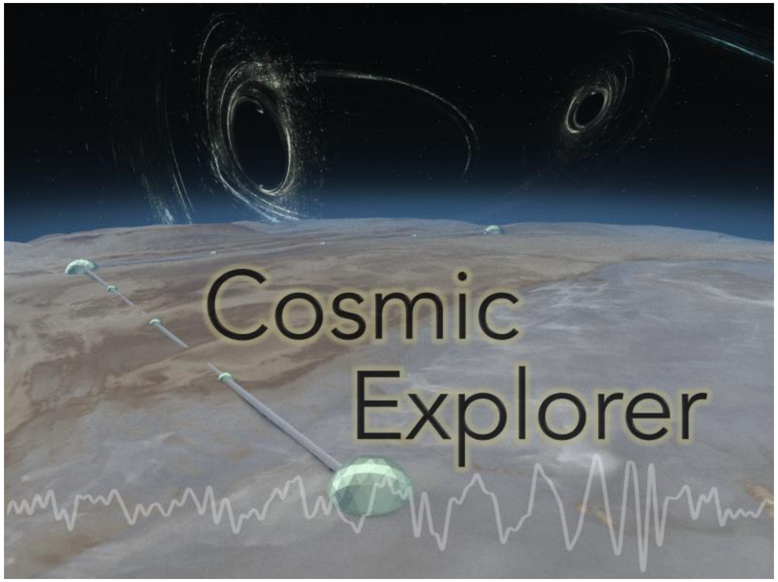




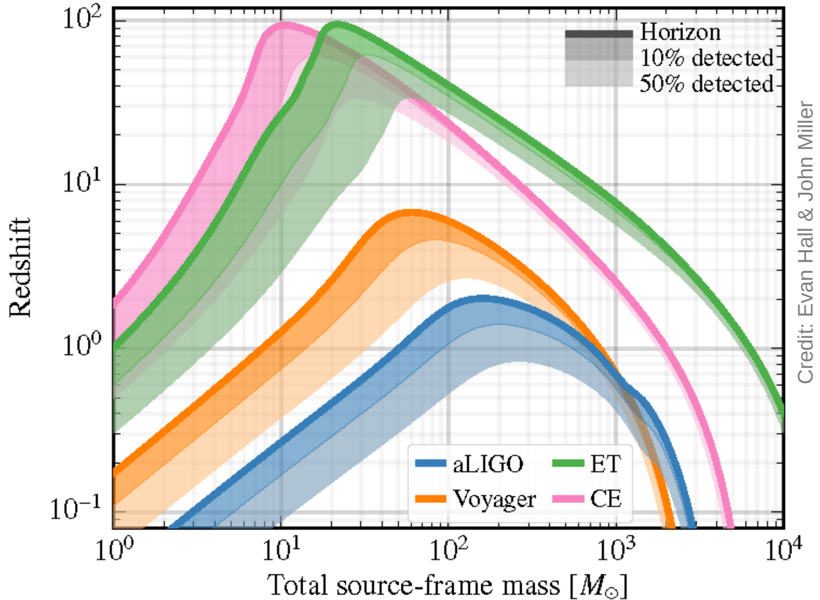
- Laser Interferometer Space Antenna
- Three probes in orbit around the Sun, exchanging laser beams
  - Triangle with sides of a few million kilometers
  - Sensitive to low frequencies (10^{-4} Hz - 0.1 Hz)
  - **January 2024: definitive approval by ESA!**
- Different kinds of sources:
  - Merging *supermassive* binary black holes ($10^5 – 10^{10}$ M_{sun})
  - Smaller objects in complicated orbits around supermassive black hole

Einstein Telescope and Cosmic Explorer (2035?)





- Next-generation ground-based facilities
  - Factor 10 improvement in sensitivity over LIGO/Virgo design sensitivity
  - Merging binary black holes ($3 - 10^4\ M_{sun}$) and neutron stars throughout the visible Universe
  - 10^5 detections per year!



Summary

- The first direct detection of gravitational waves has enabled unprecedented tests of general relativity:
  - First access to genuinely strong-field dynamics of vacuum spacetime
  - Propagation of gravitational waves over large distances
  - Probing the nature of compact objects

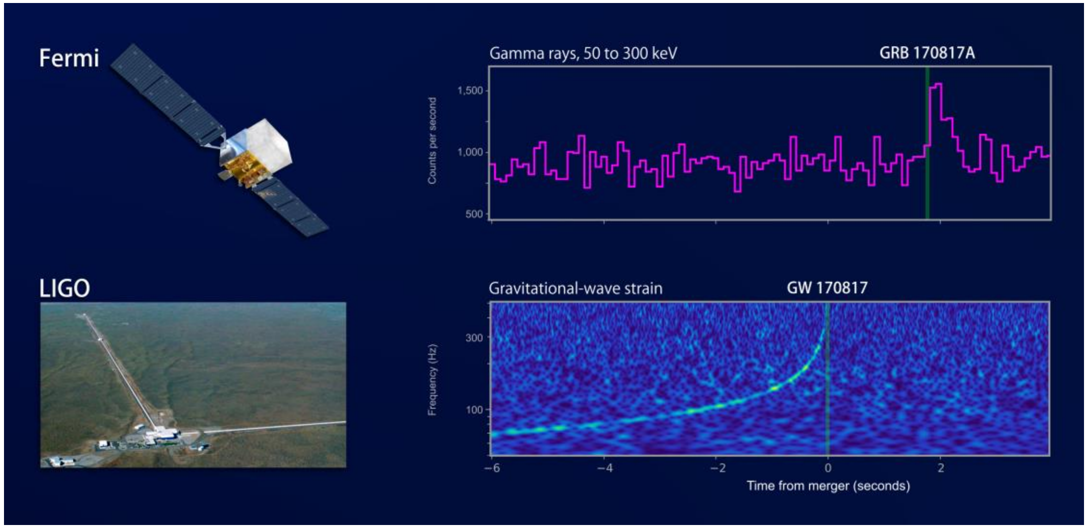
- Some highlights:
  - Higher-order phase coefficients constrained at ~10% level
  - Graviton mass $m_g < 1.76 \times 10^{-23}$ eV/c^2
  - Spin-induced quadrupole moment during inspiral:
    Access to expected values for boson stars
  - No-hair test consistent with no deviations at ~25% level
  - Area increase theorem passes at > 95% probability

- High-precision tests with next-generation observatories:
  LISA, Einstein Telescope, Cosmic Explorer
  - Higher accuracy
  - Larger number of sources
  - Propagation of gravitational waves over cosmological distances

Backup slides

2. The propagation of gravitational waves

- Does the speed of gravity equal the speed of light?
- The binary neutron star coalescence GW170817 came with gamma ray burst, **1.74 seconds afterwards**



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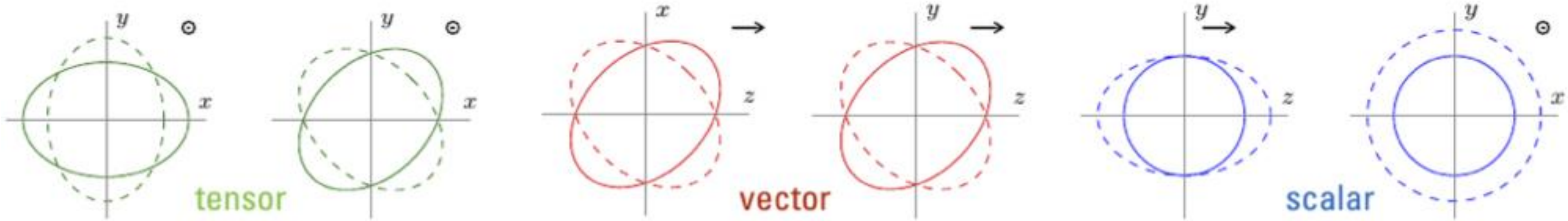
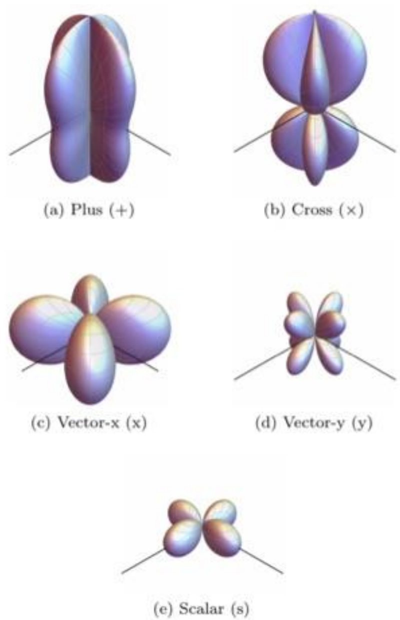
- Excluded certain alternative theories of gravity designed to explain dark matter or dark energy in a dynamical way

LIGO + Virgo + Fermi-GBM + INTEGRAL, ApJ. **848**, L13 (2017)
LIGO + Virgo, PRL **123**, 011102 (2019)

2. The propagation of gravitational waves



- Metric theories of gravity allow up to 6 polarizations
- Distinct antenna patterns:



$$|F_{\rm t}^I(\alpha,\delta)| \equiv \sqrt{F_+^I(\alpha,\delta)^2 + F_\times^I(\alpha,\delta)^2}\,,$$

$$|F_{\rm v}^I(\alpha,\delta)| \equiv \sqrt{F_{\rm x}^I(\alpha,\delta)^2 + F_{\rm y}^I(\alpha,\delta)^2}\,,$$

$$|F_{\rm s}^I(\alpha,\delta)| \equiv \sqrt{F_{\rm b}^I(\alpha,\delta)^2 + F_{\rm l}^I(\alpha,\delta)^2}$$

Isi & Weinstein, PRD **96**, 042001 (2017)

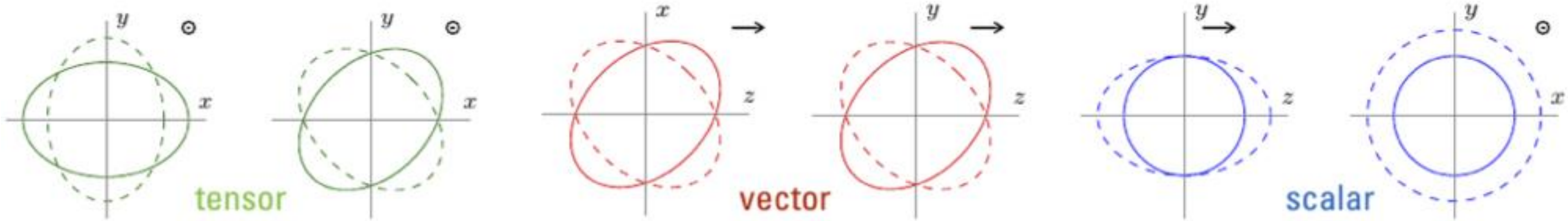
- In the case of GW170817, sky position was known from EM counterpart
  - Pure tensor / pure vector = 10^{21} / 1
  - Pure tensor / pure scalar = 10^{23} / 1

LIGO + Virgo, PRL **123**, 011102 (2019)

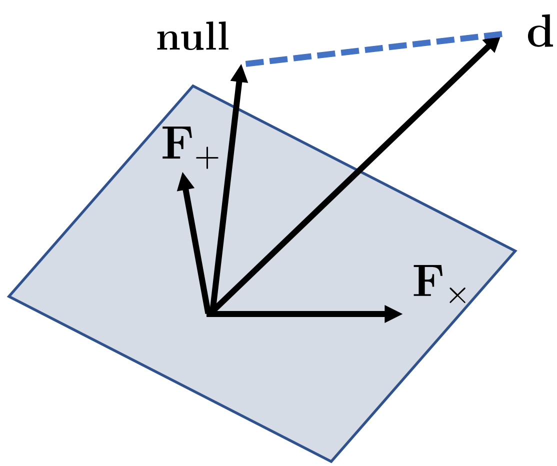
- Using a “null stream”: also look for a mixture

Pang et al., PRD **101**, 104055 (2020)

Alternative polarizations: null stream



- Using a **null stream**: can look for non-tensorial polarizations (without necessarily being able to tell which ones are present)



- Data from D detectors:

$$\mathbf{d} = \begin{pmatrix} d_0 \\ \vdots \\ d_{D-1} \end{pmatrix}$$

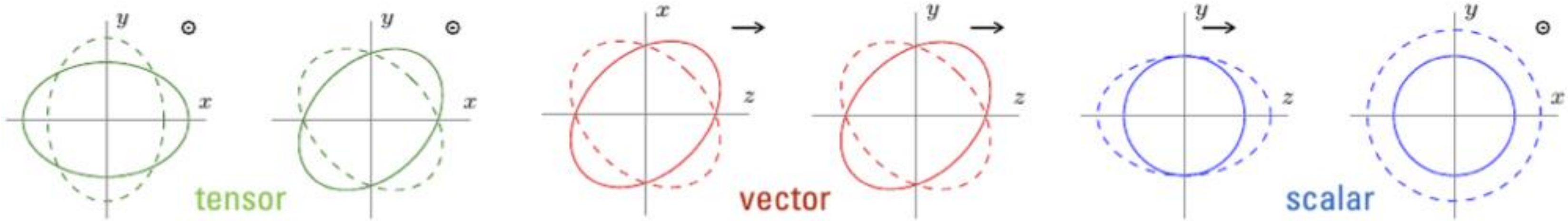
- Antenna pattern functions, known sky location:

$$\mathbf{F} = (\mathbf{F}_+ \ \ \mathbf{F}_\times) = \begin{pmatrix} F_{+,0} & F_{\times,0} \\ \vdots & \vdots \\ F_{+,D-1} & F_{\times,D-1} \end{pmatrix}.$$

- Null stream projects out tensorial content
  - What remains can only contain (mixture of) vector and scalar modes
- No evidence for alternative polarizations in GW170817

Pang et al., PRD **101**, 104055 (2020)

Alternative polarizations in pulsar signals



- **Continuous waves from known pulsars**: sky position (α, δ) also known
- Consider hypotheses $\mathcal{H}_m$ that detector output is

$$h_m(t) = \sum_{p \in m} F_p(\alpha, \delta; t)\, h_p(t)$$

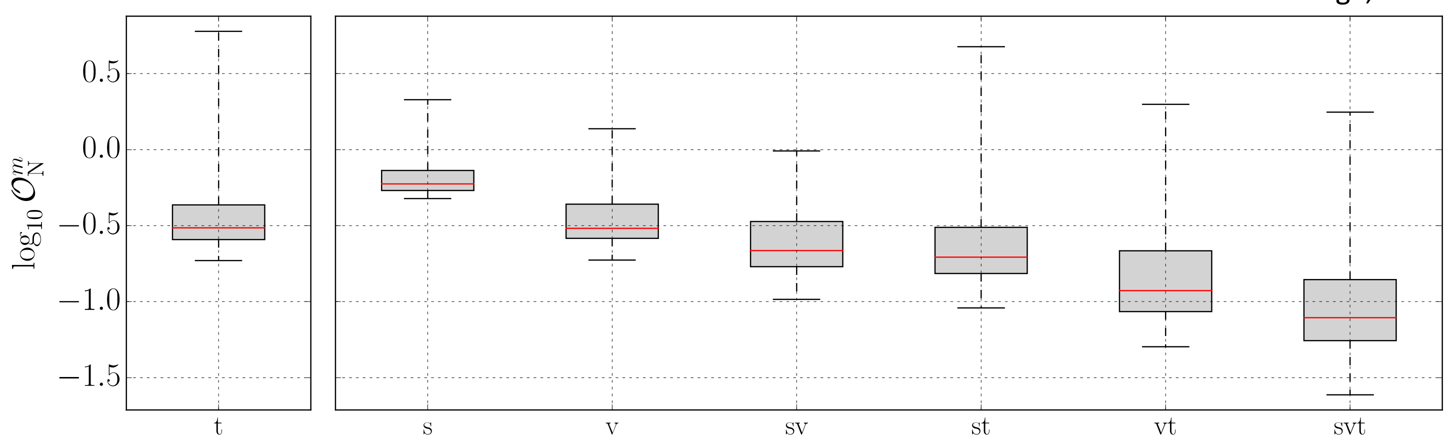
where m is any subset of $\{+, \times, v_X, v_Y, s\}$

- Calculate odds ratios

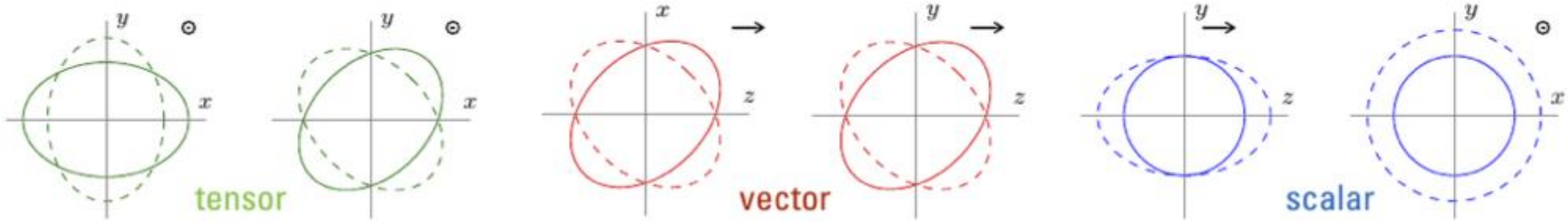
$$\mathcal{O}_N^m = \frac{\mathrm{Prob}(\mathcal{H}_m|d)}{\mathrm{Prob}(\mathcal{H}_N|d)}$$ where $\mathcal{H}_N$ is the noise-only hypothesis

- Results for 200 pulsars analyzed:

LIGO + Virgo, PRL **120**, 031104 (2018)



Alternative polarizations in stochastic backgrounds



- **Search for stochastic backgrounds** through cross-correlations of detector outputs:

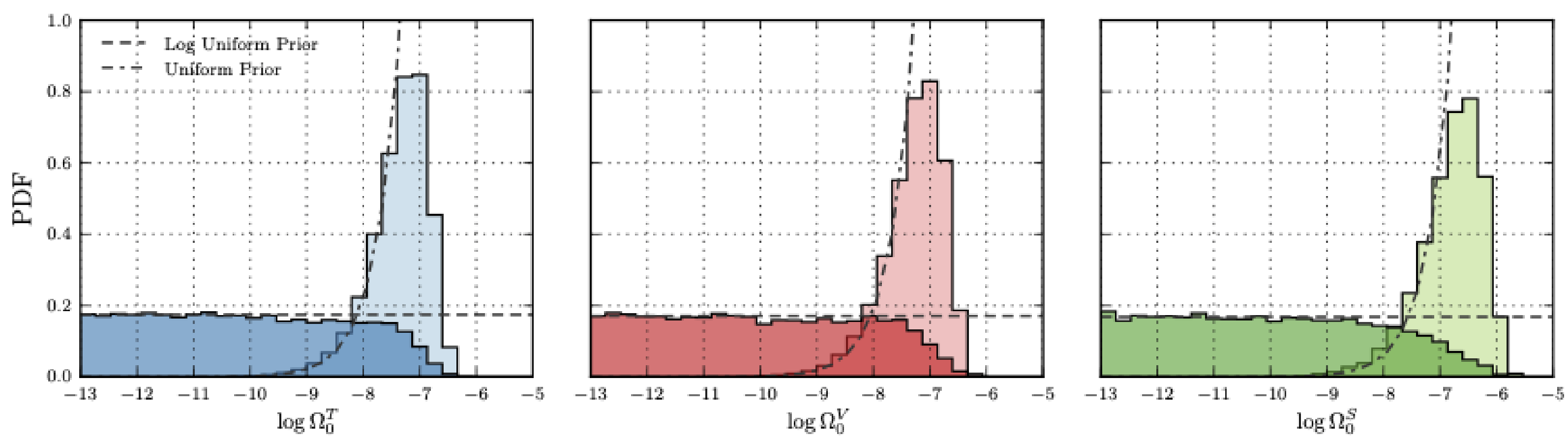
$$Y = \sum_p \int \tilde{s}^*(f)\,\tilde{Q}_p(f)\,\tilde{s}_2(f)\,df \quad \text{with optimal filter} \quad \tilde{Q}_p(f) \propto \frac{\gamma_p(f)\,\Omega_p(f)}{f^3 S_1(f)\,S_2(f)}$$

where $\gamma_p(f)$ the overlap reduction function for polarization p and the energy densities $\Omega_p(f)$ are contributions to

$$\Omega(f) = \Omega_0^T\left(\frac{f}{f_0}\right)^{\alpha_T} + \Omega_0^V\left(\frac{f}{f_0}\right)^{\alpha_V} + \Omega_0^S\left(\frac{f}{f_0}\right)^{\alpha_S}$$

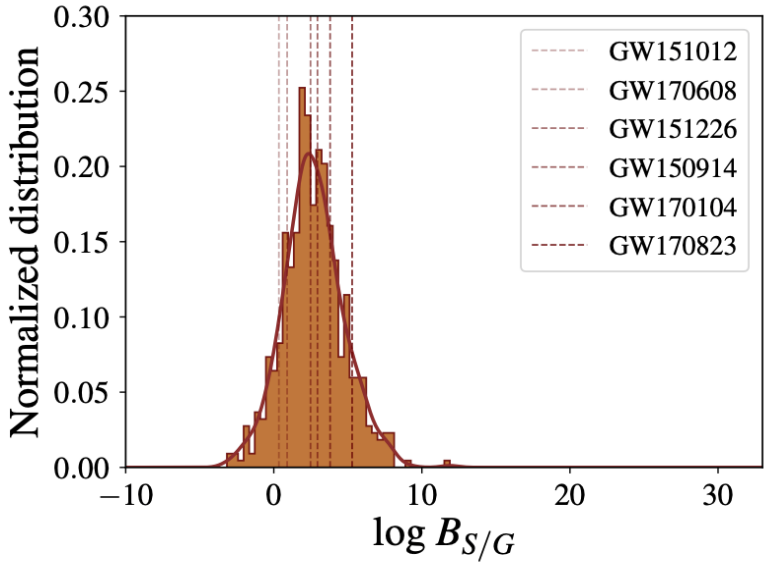
- Parameter estimation on $\Omega_0^T, \Omega_0^V, \Omega_0^S$:

LIGO + Virgo, PRL **120**, 201102 (2018)

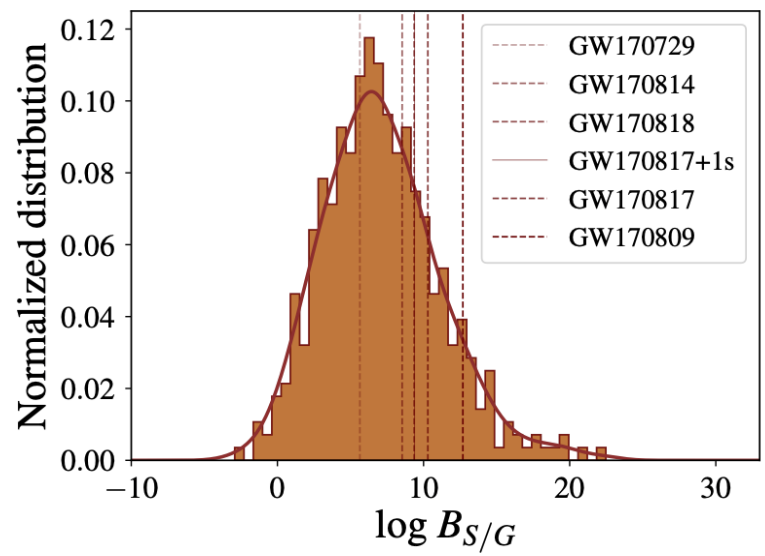


Gravitational wave echoes

- Ratio of evidences for signal versus glitch: Bayes factor $B_{S/G} = \frac{\mathrm{Prob}(\mathbf{d}|\mathcal{H}_{\mathrm{signal}})}{\mathrm{Prob}(\mathbf{d}|\mathcal{H}_{\mathrm{glitch}})}$
- Analysis of data following the detections of binary coalescences in the 1st and 2nd observing runs of Advanced LIGO/Virgo:



2-detector events



3-detector events

- Similarly for Bayes factor signal versus noise, $B_{S/N} = \frac{\mathrm{Prob}(\mathbf{d}|\mathcal{H}_{\mathrm{signal}})}{\mathrm{Prob}(\mathbf{d}|\mathcal{H}_{\mathrm{noise}})}$
- No statistically significant evidence for echoes following these events

Tsang et al., PRD **98**, 024023 (2018)
Tsang et al., PRD **101**, 064012 (2020)

Primordial stochastic backgrounds