additional structure in the observed black hole population and implications for cosmology

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Apr 10, 2025

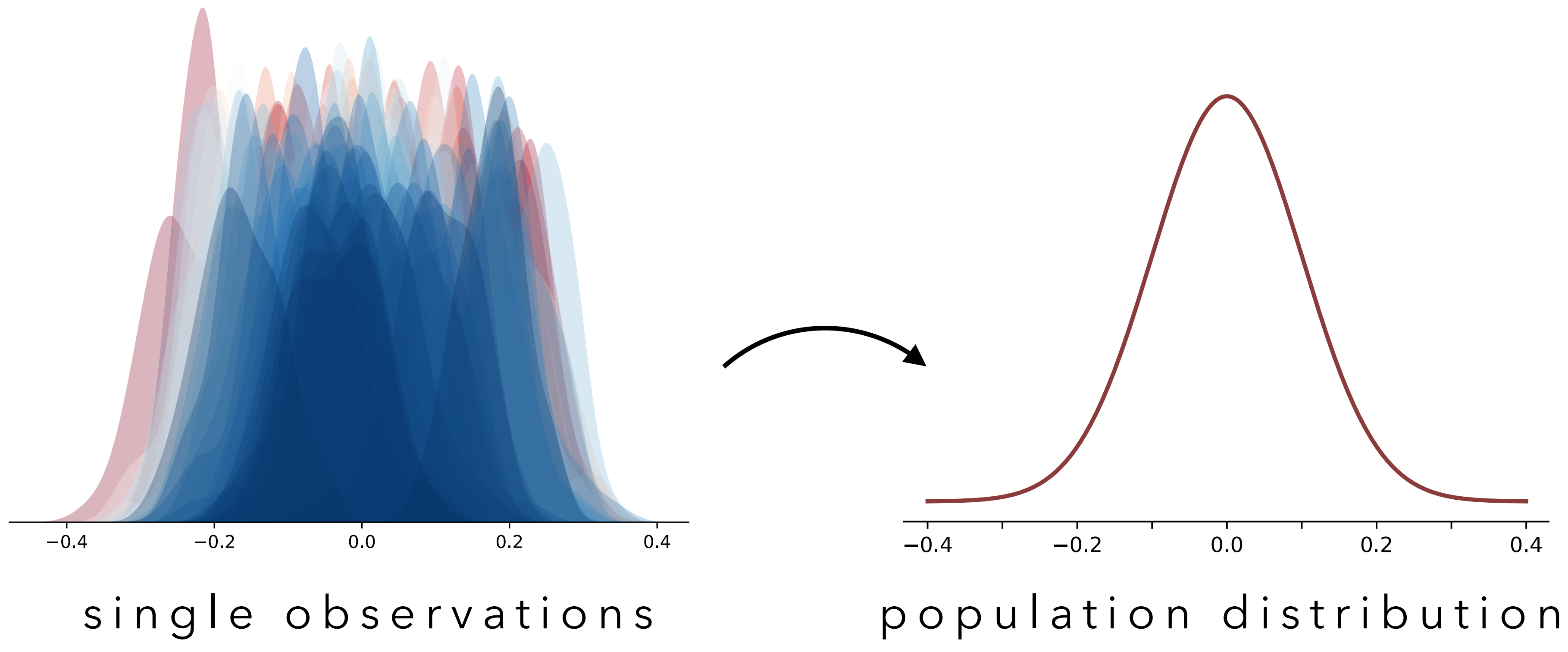
ACME Workshop, Toulouse

1. population studies

STATISTICAL FRAMEWORK

[1809.02063, 2007.05579, 1809.03815]

combine multiple measurements to find the most likely intrinsic probability distribution from which the observed events are drawn



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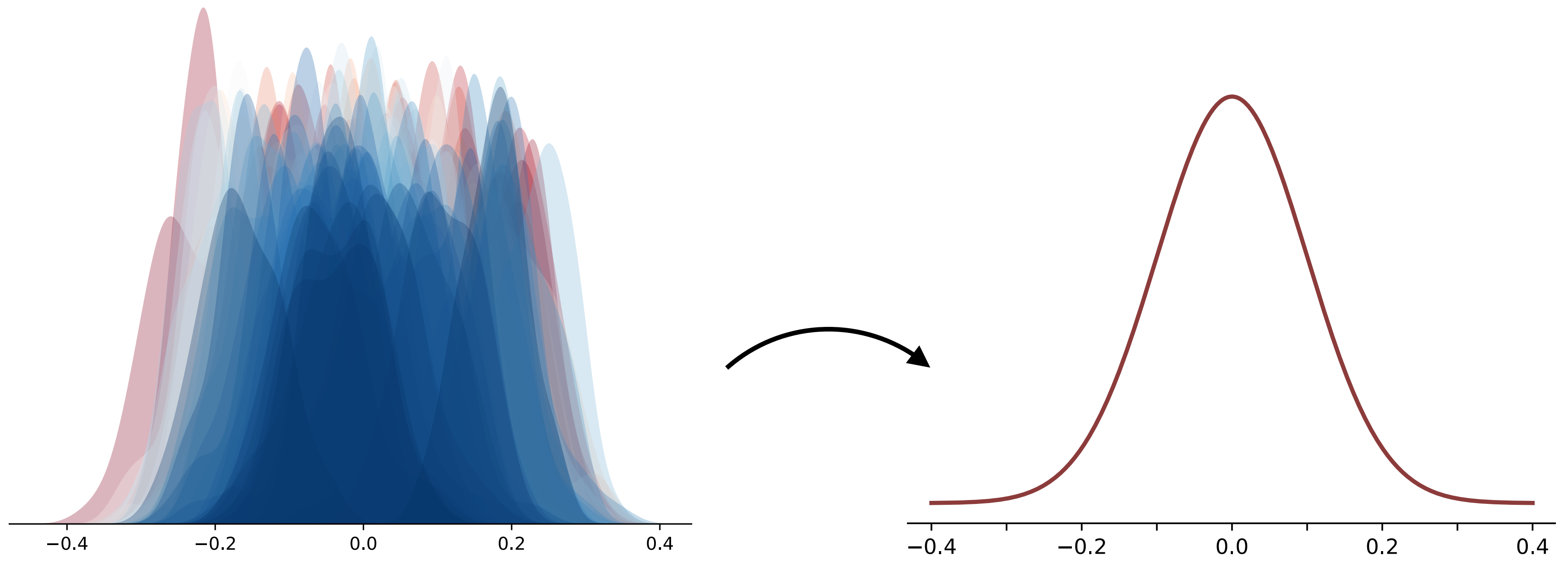


single observations — population distribution

- we use Bayesian analysis

$$p(\boldsymbol{\Lambda}|\{\mathbf{d}\}) \propto \mathcal{L}_H(\{\mathbf{d}\}|\boldsymbol{\Lambda})\, p(\boldsymbol{\Lambda})$$

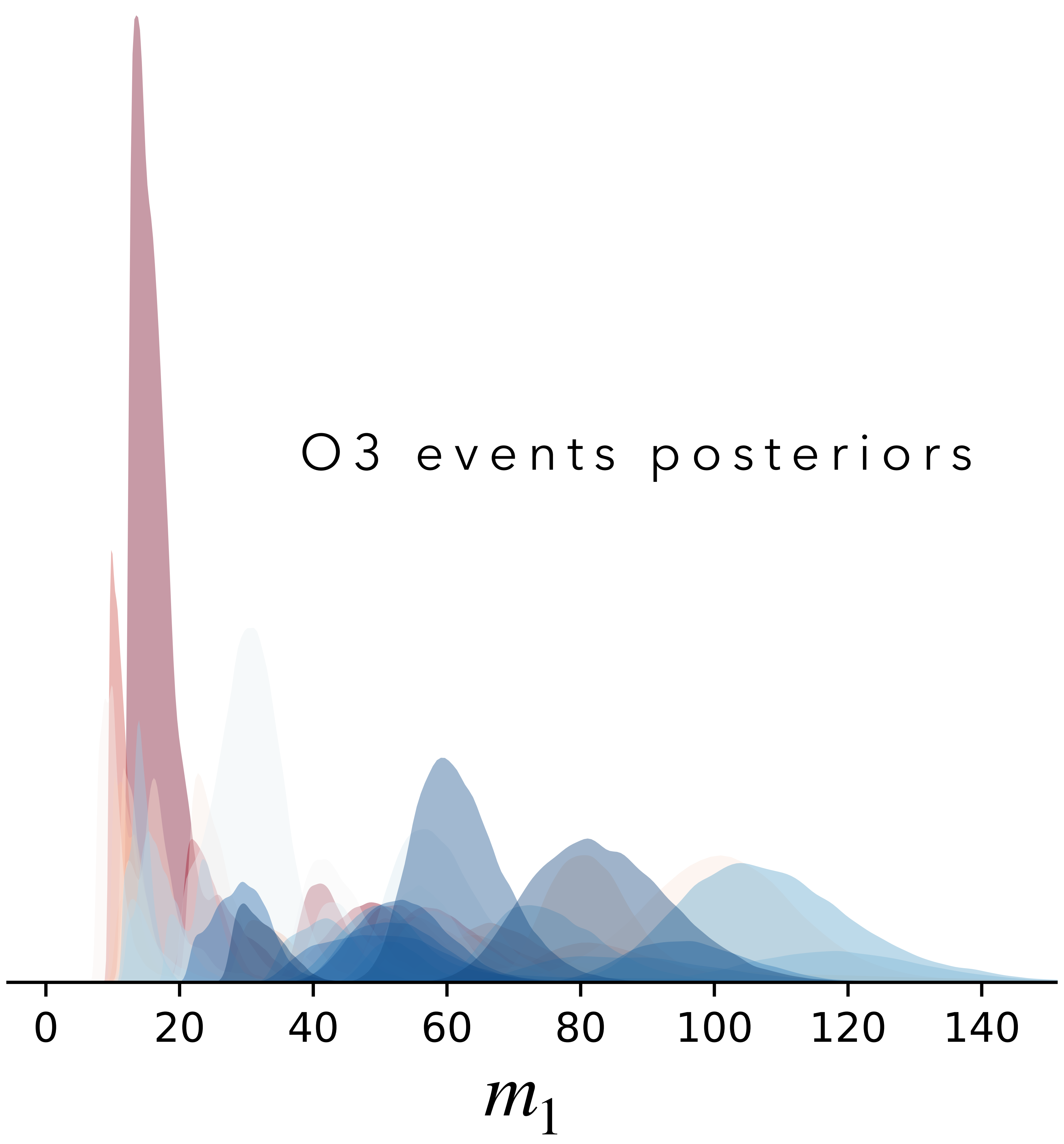
posterior — (hierarchical) likelihood — prior

$$p(\,model\,|\,data\,) \longleftrightarrow p(\,data\,|\,model\,)$$

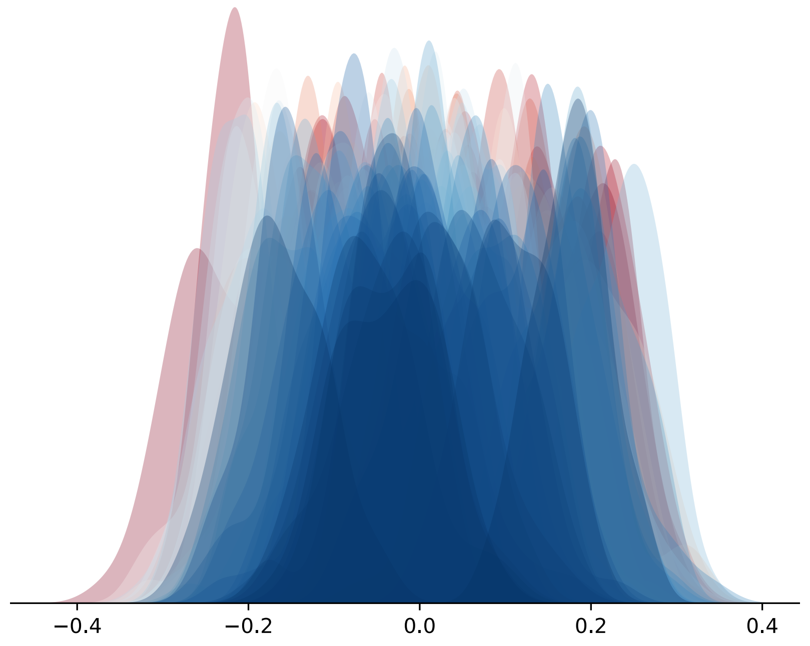
functional form for the BH distribution

GW events to be combined

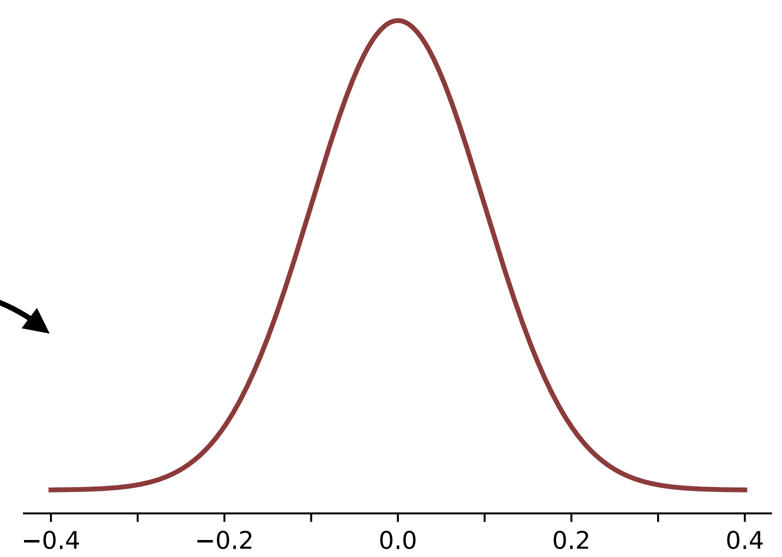
BLACK HOLES POPULATION



O3 events posteriors



single observations



population distribution

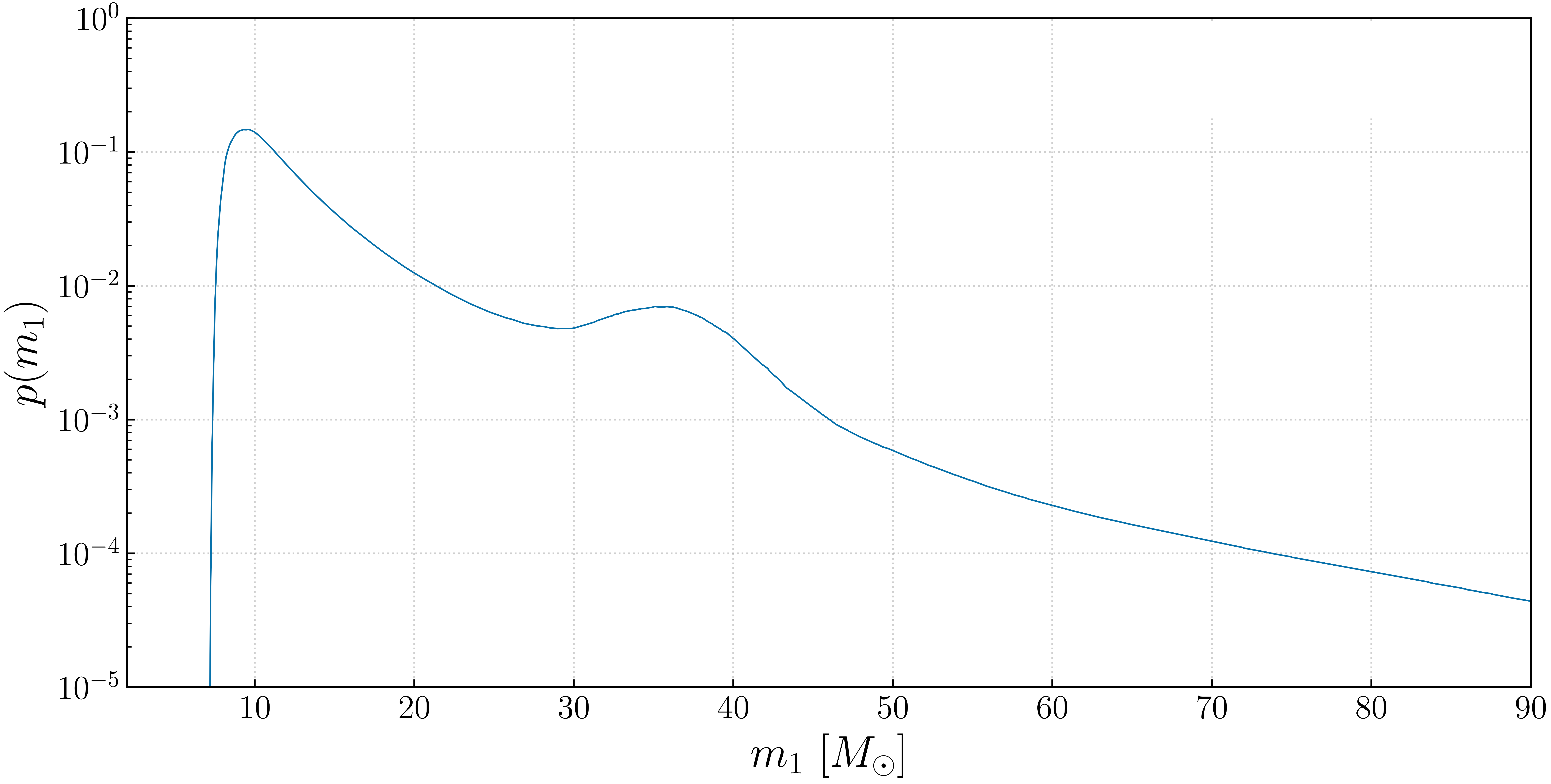
?

what is the predicted shape of the astrophysical BH distribution?

→ we don't know in advance!

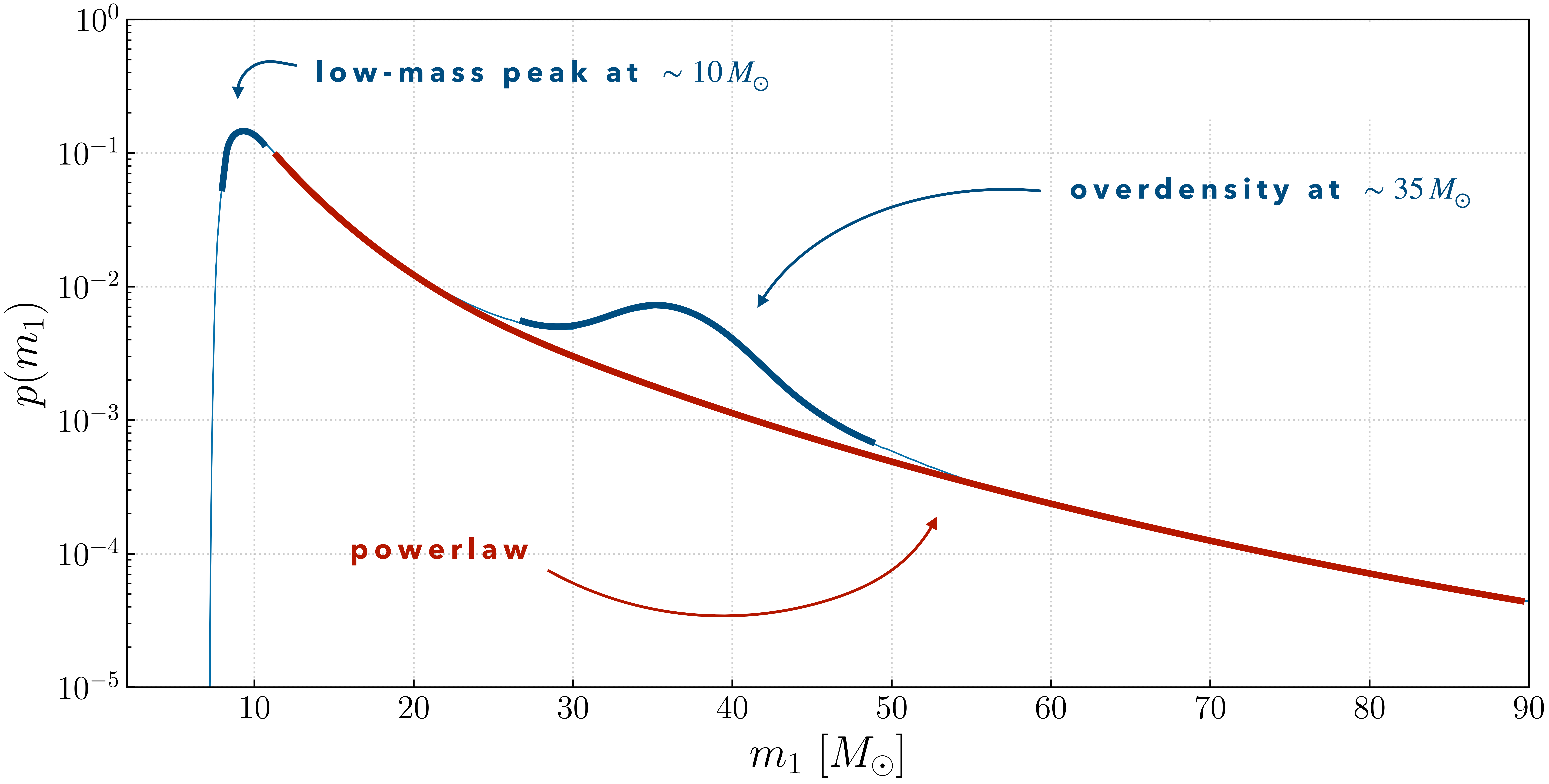
2. real data

(see Michela's talk)



LVK RESULTS GWTC-3

(see Michela's talk)

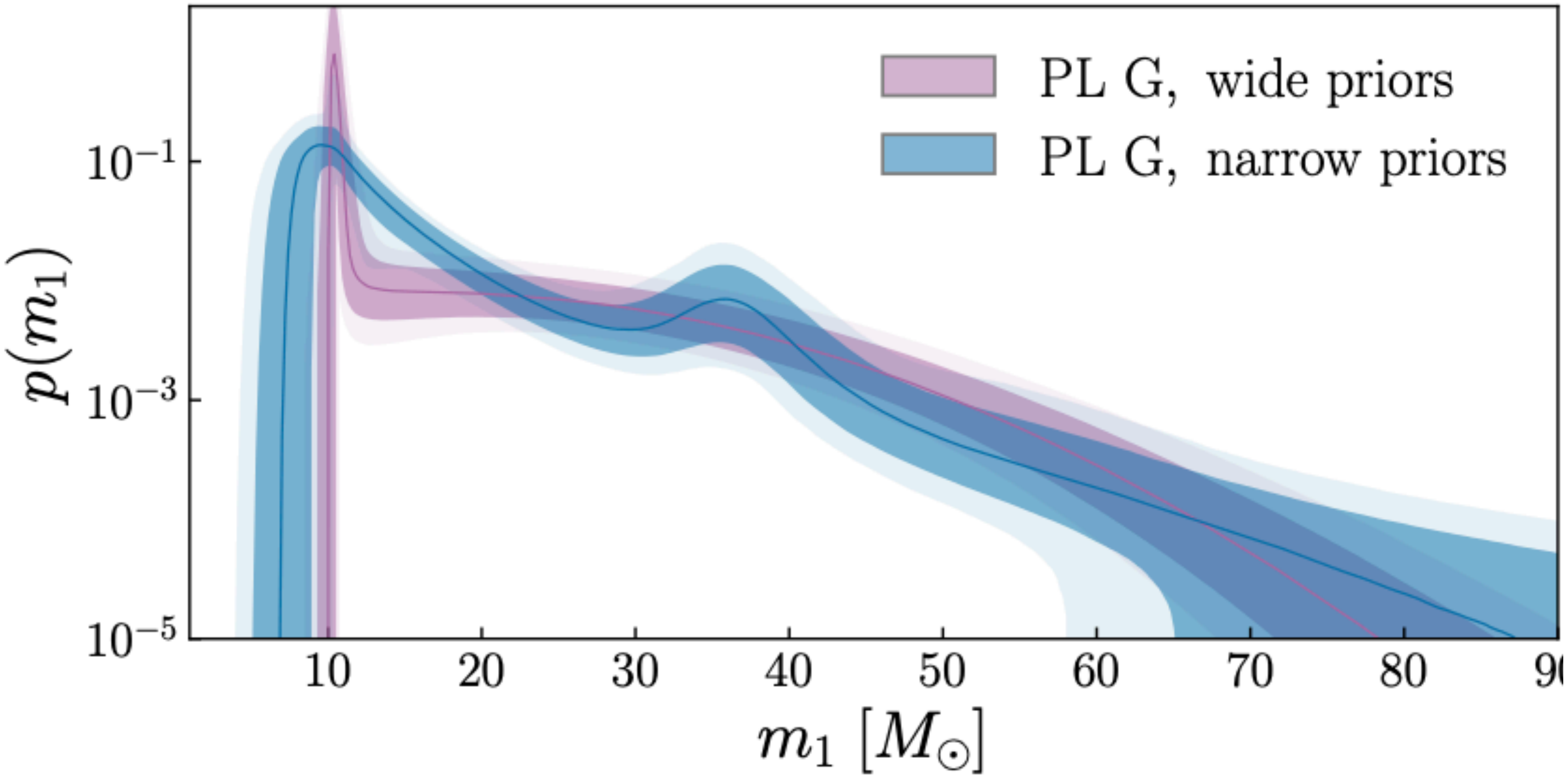


O3 DATA 2 FEATURES

mass model

Powerlaw

+ Gaussian



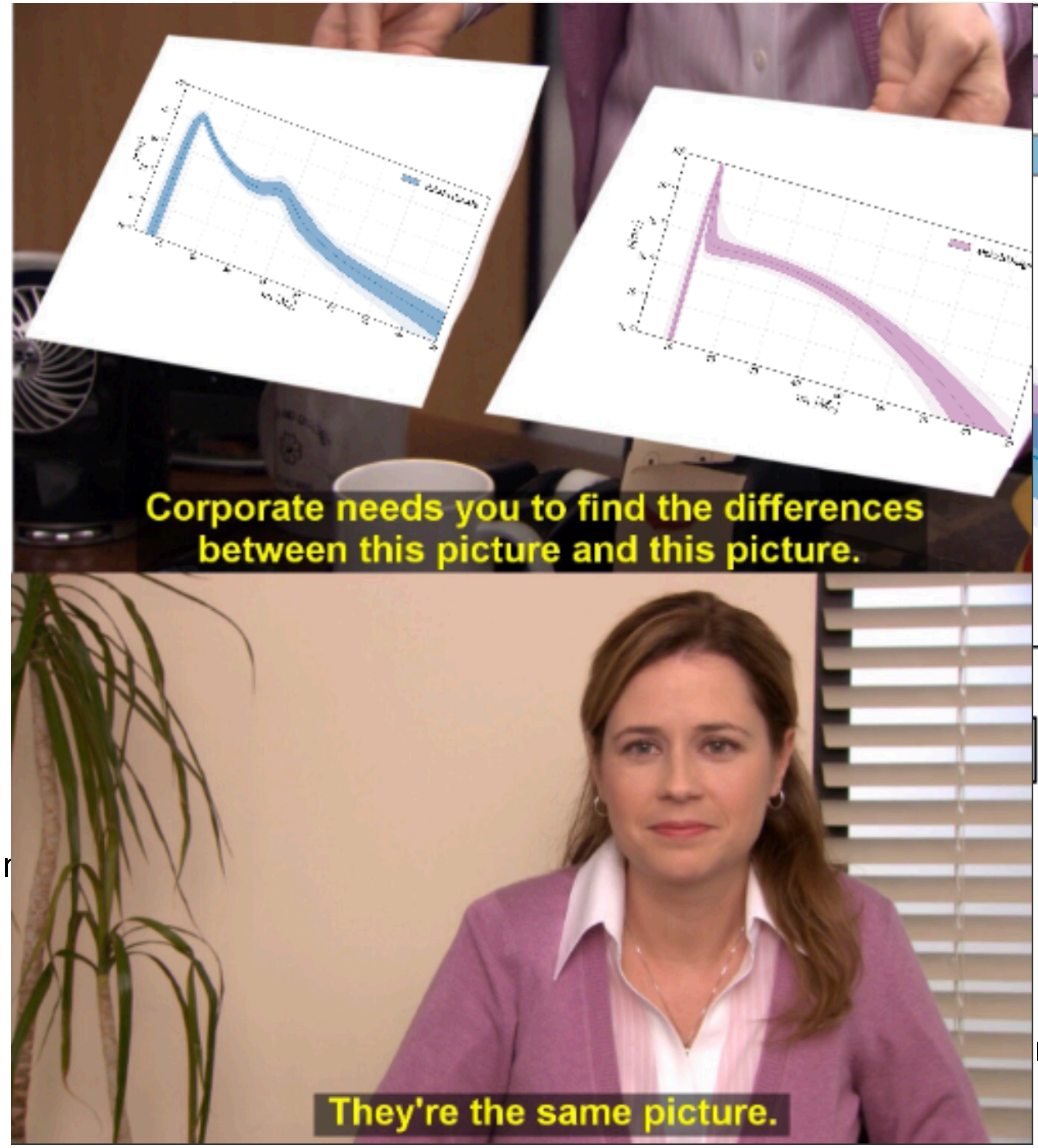
- results depend on the prior bounds
- with wide priors, find sharp low-mass peak and loose the $\sim 35 M_\odot$ overdensity
- the pink analysis is statistically preferred by Bayes factor ($lnB \simeq 5$)

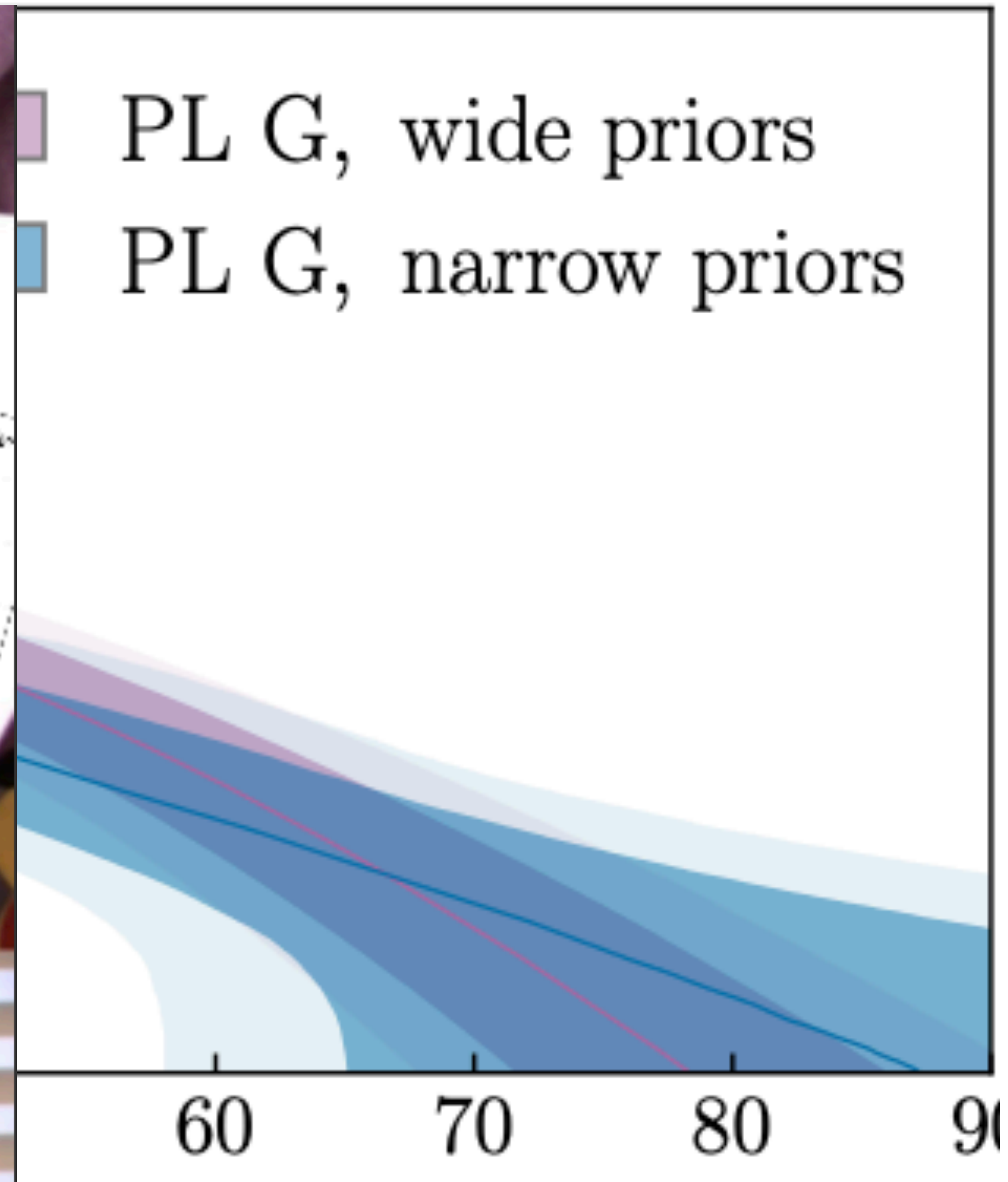
O3 DATA 2 FEATURES

mass model

Powerlaw

+ Gaussian





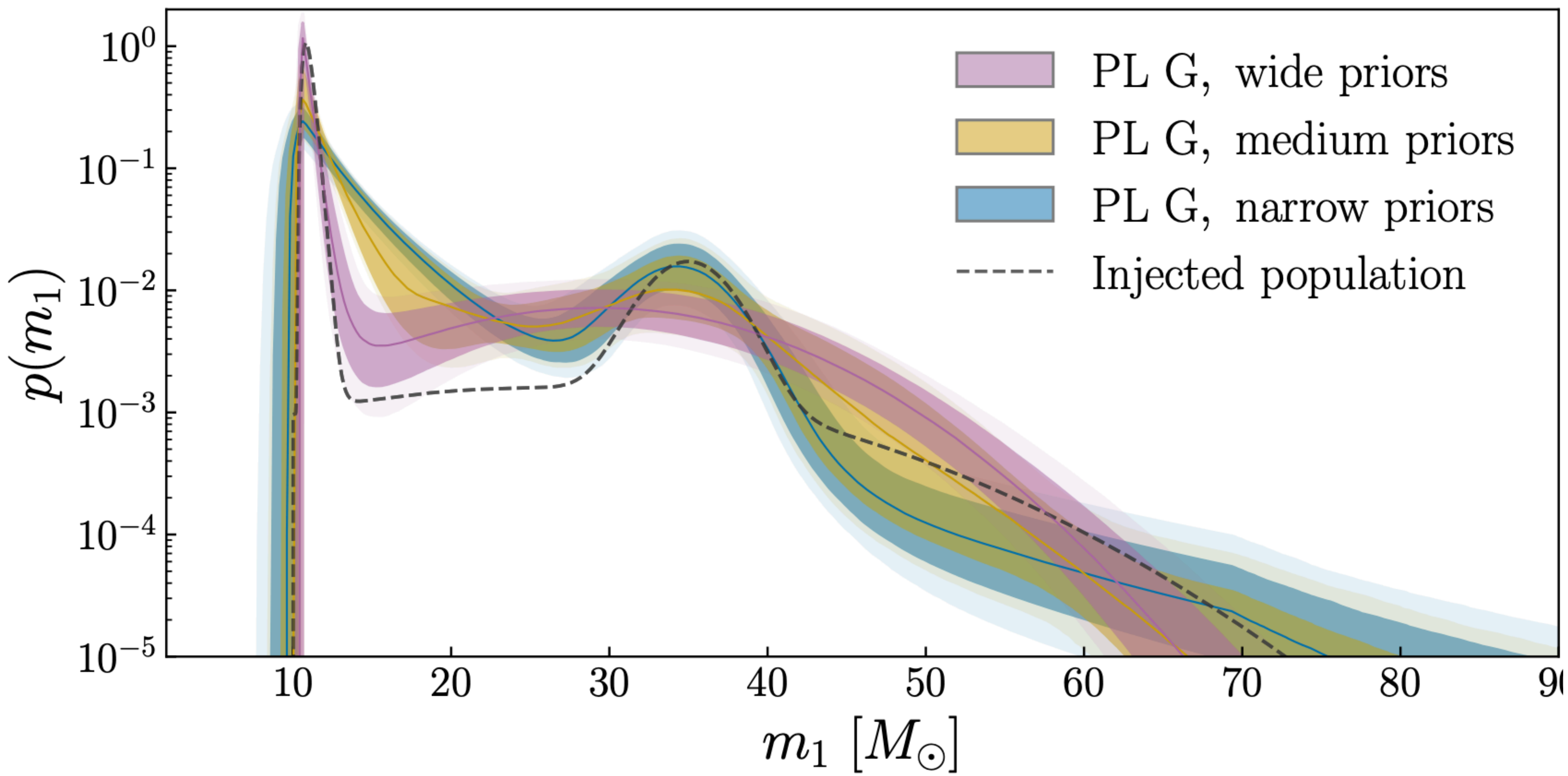
- results depend o
- with wide priors, $\sim 35\,M_\odot$ overdensity
- the pink analysis r ($lnB \simeq 5$)

SIMULATED DATA

mass model

Powerlaw

+ Gaussian



- simulate data from a population with both $10 M_\odot$ peak and $\sim 35 M_\odot$ overdensity
- reproduce the results on real data (bimodal hierarchical likelihood)

can we resolve both features simultaneously?

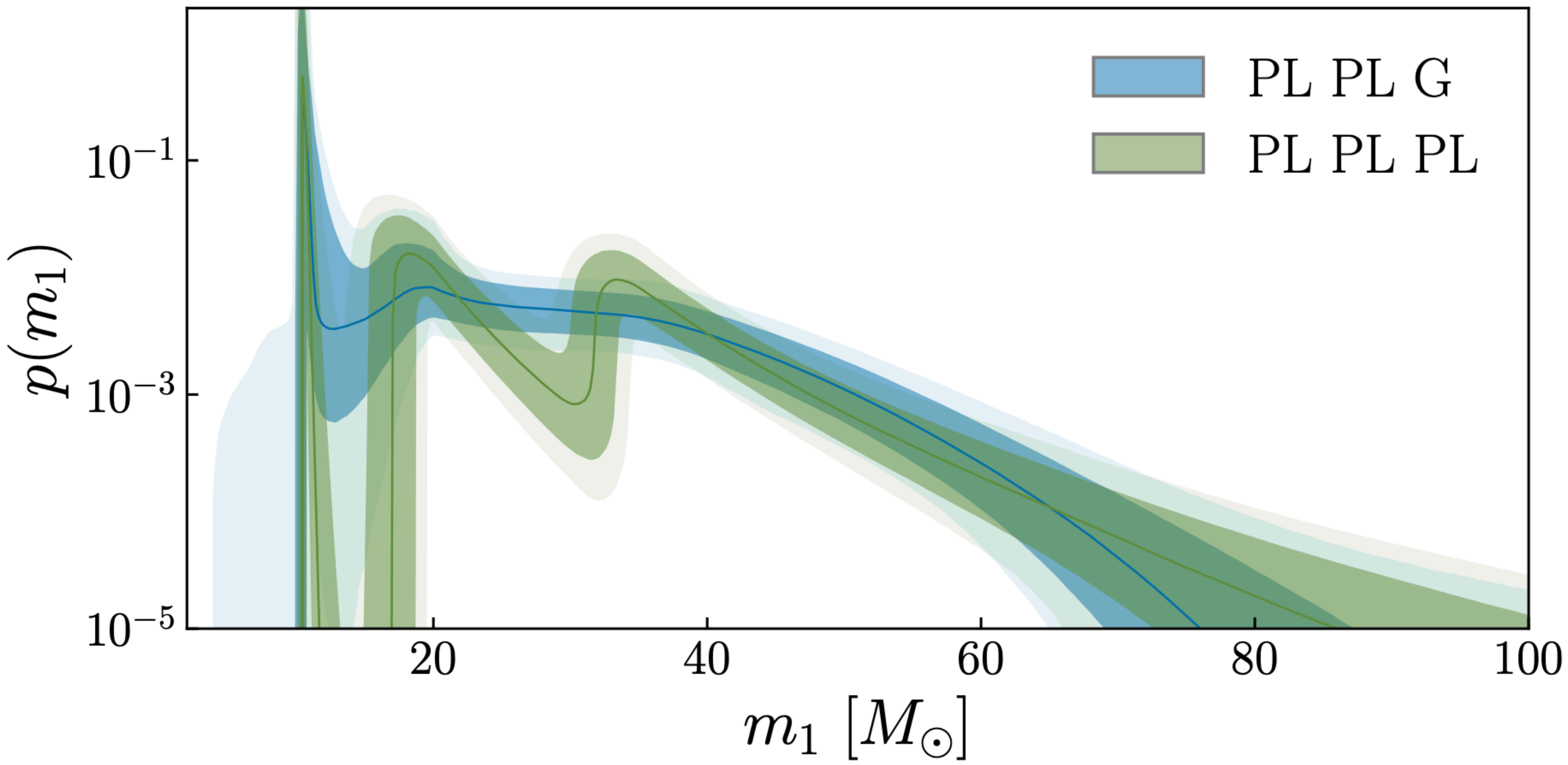
O3 DATA 3 FEATURES

mass model

Powerlaw

\+ Powerlaw

\+ Powerlaw



- additional features resolve both the sharp $10 M_\odot$ and $\sim 35 M_\odot$ peaks
- evidence for additional structure at $\sim 15 - 20 M_\odot$
- the PL-PL-PL is slightly preferred over the wide priors PL-G ($lnB \simeq 0.3$)

O3 DATA 3 FEATURES

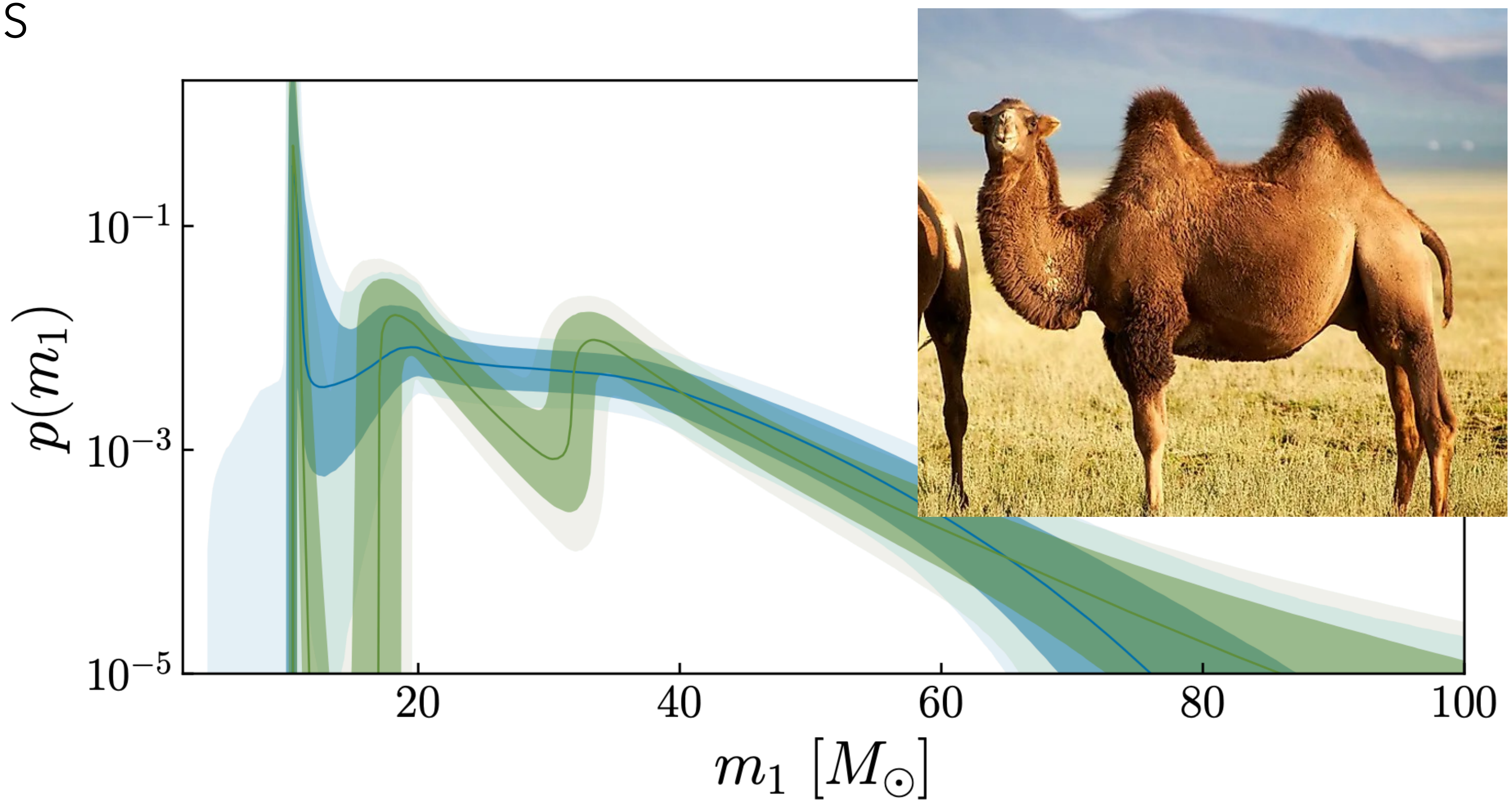
mass model

Powerlaw

+ Powerlaw

+ Powerlaw

(a.k.a. the **camel**)



- additional features resolve both the sharp $10\,M_\odot$ and $\sim 35\,M_\odot$ peaks
- evidence for additional structure at $\sim 15-20\,M_\odot$
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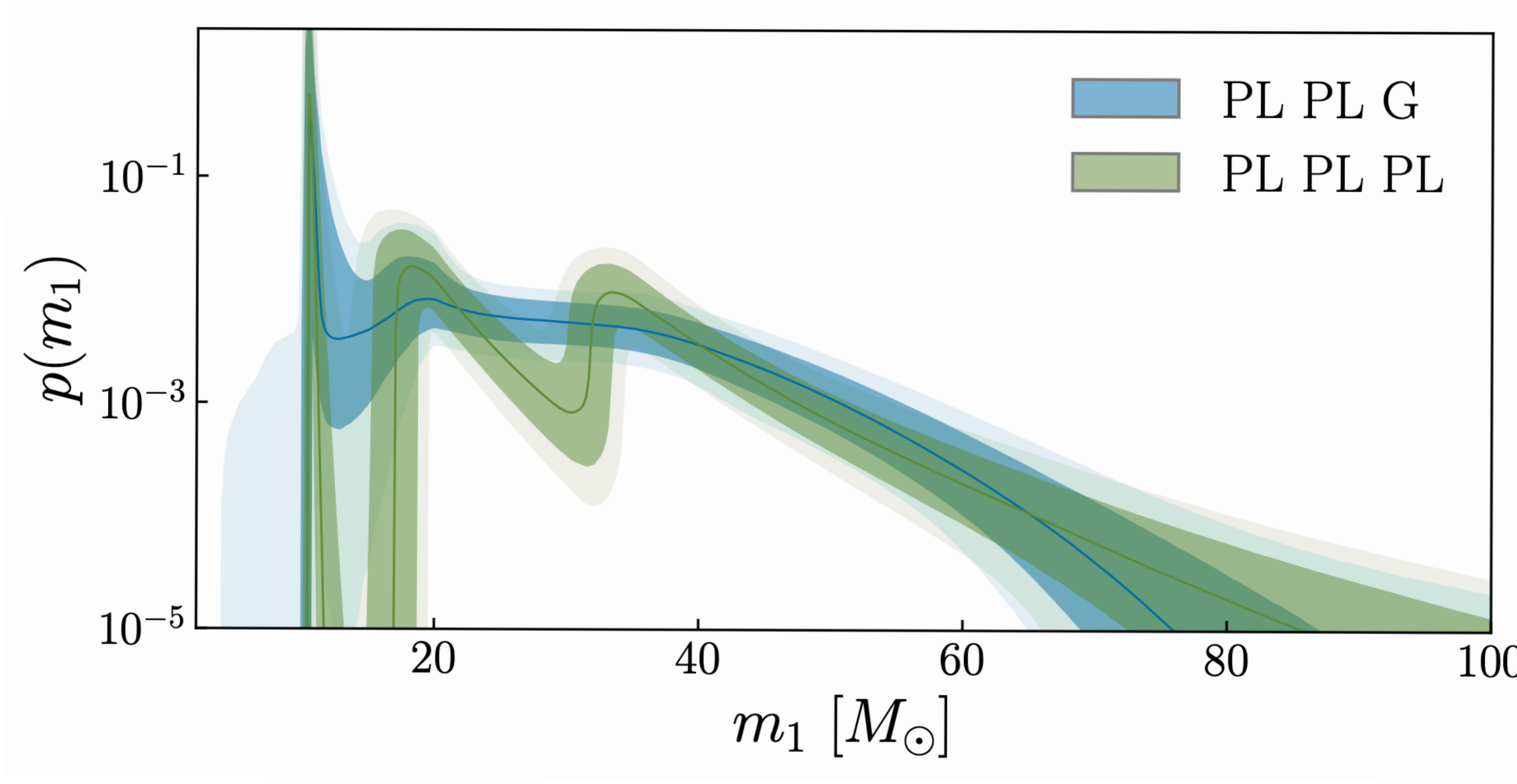
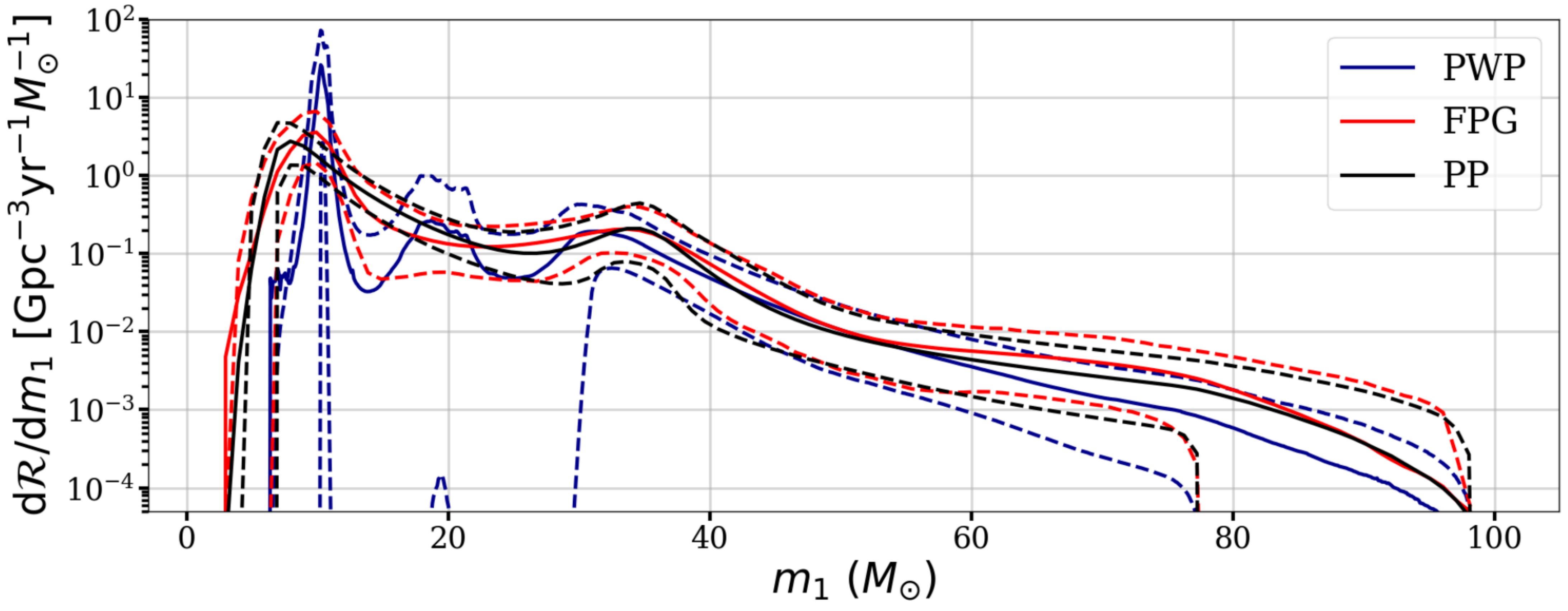
NON-PARAMETRIC COMPARISON

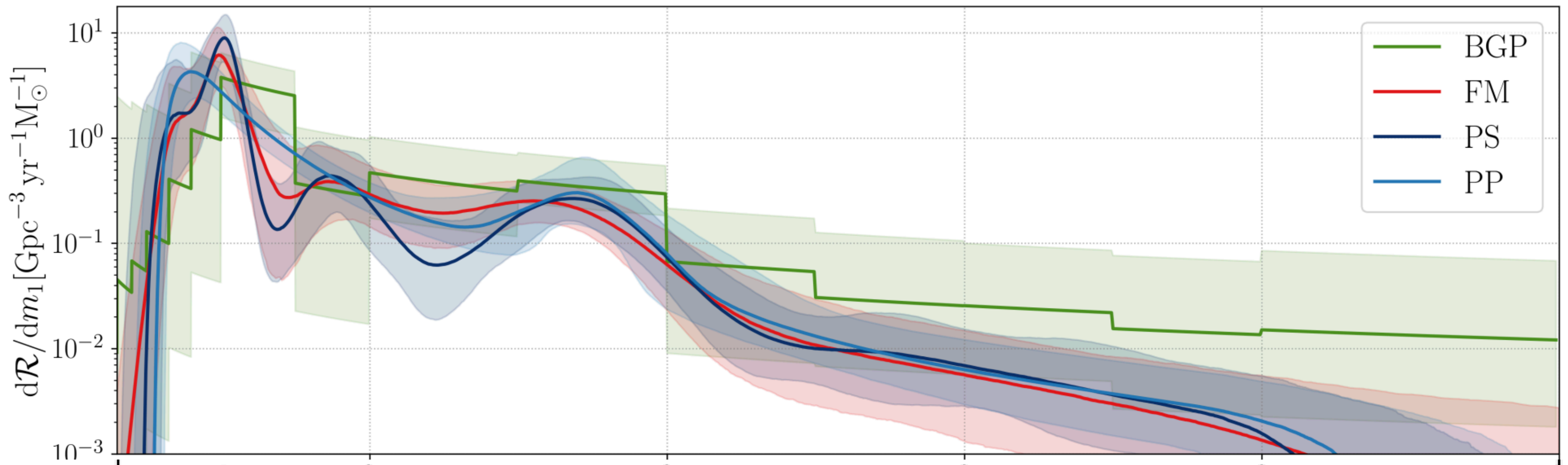
see also [2301.00834]



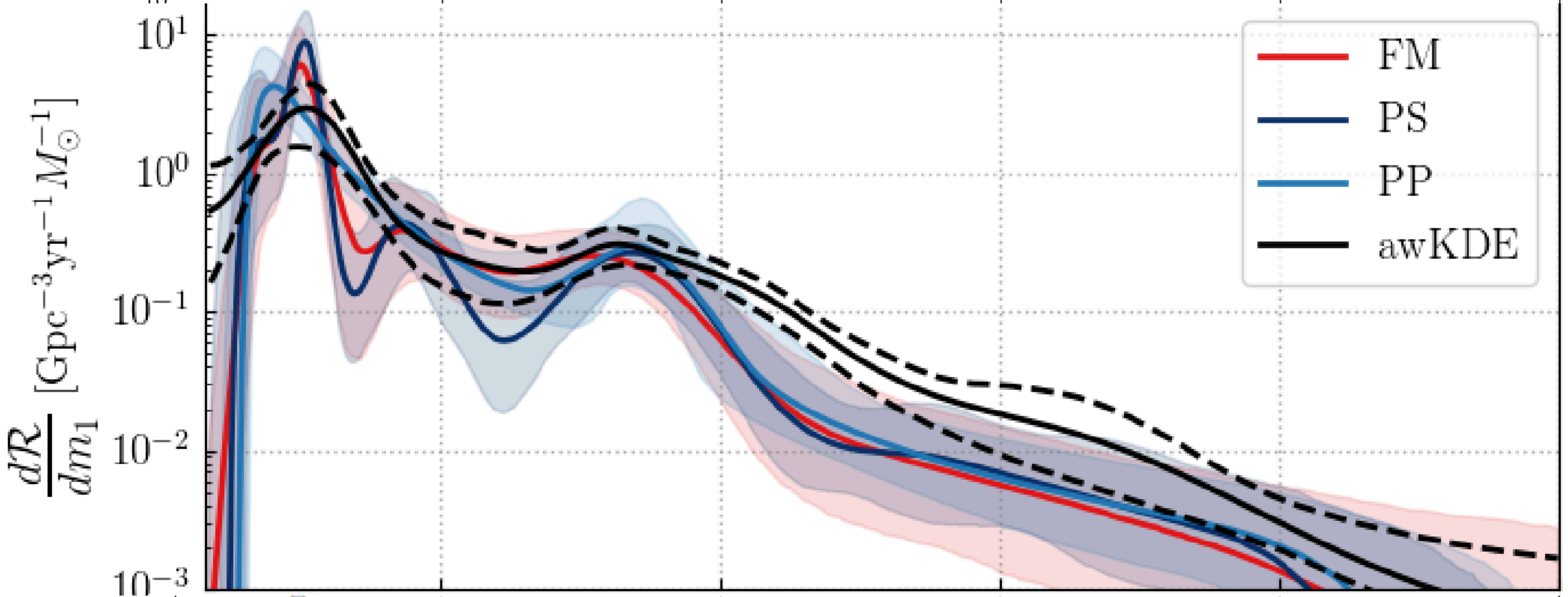
[2502.20445]



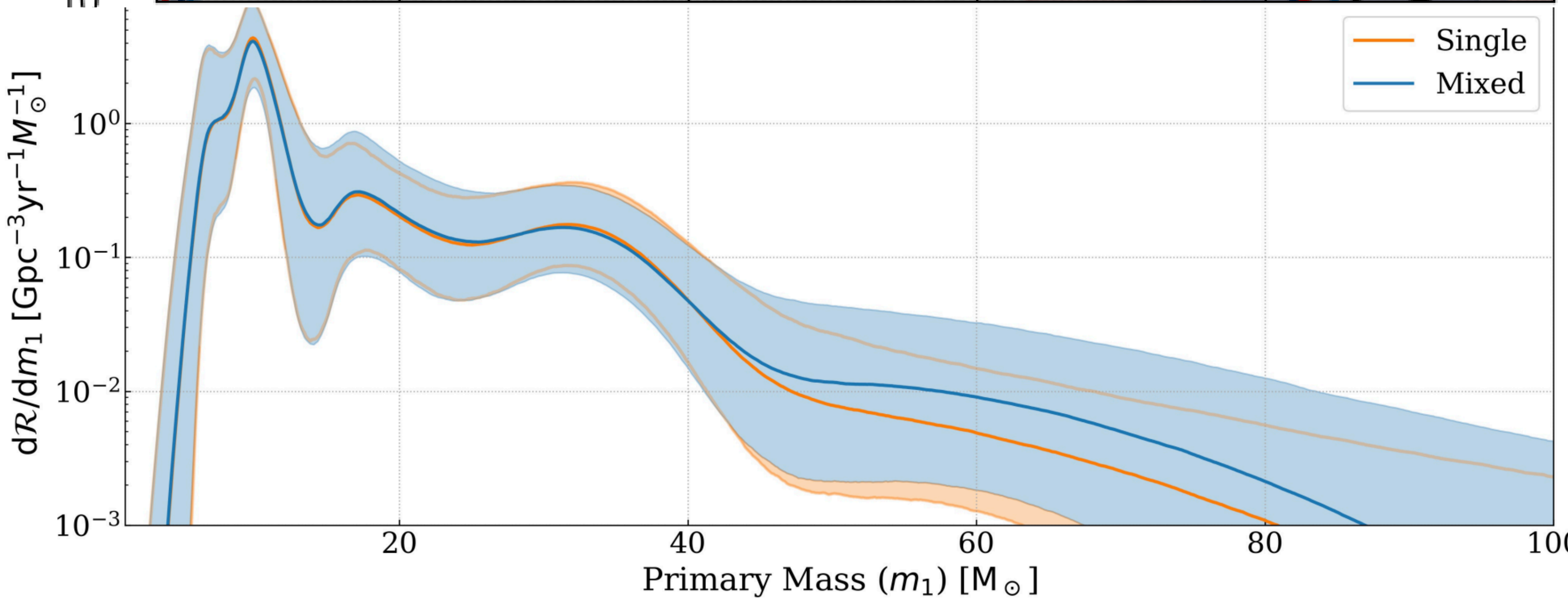
[2305.08909]



[2111.03634]



[2112.12659]



[2111.13991]

INTERPRETATION

- are the sharp $10 M_\odot$ peak and the $15 M_\odot$ drop physical?
    - likely model-induced biases, but that's what we have today
    - future data with more events will tell us more

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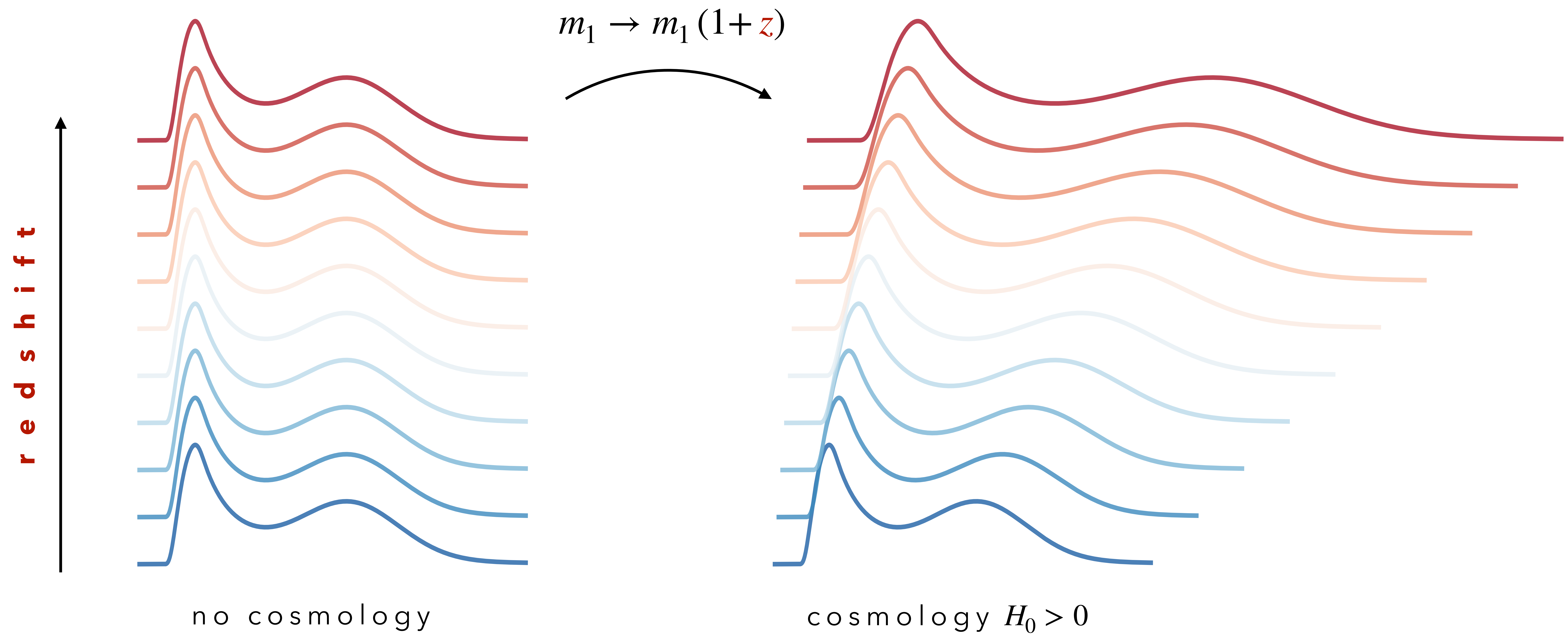
  - likely model-induced biases, but that's what we have today
  - future data with more events will tell us more

what can we use these results for?

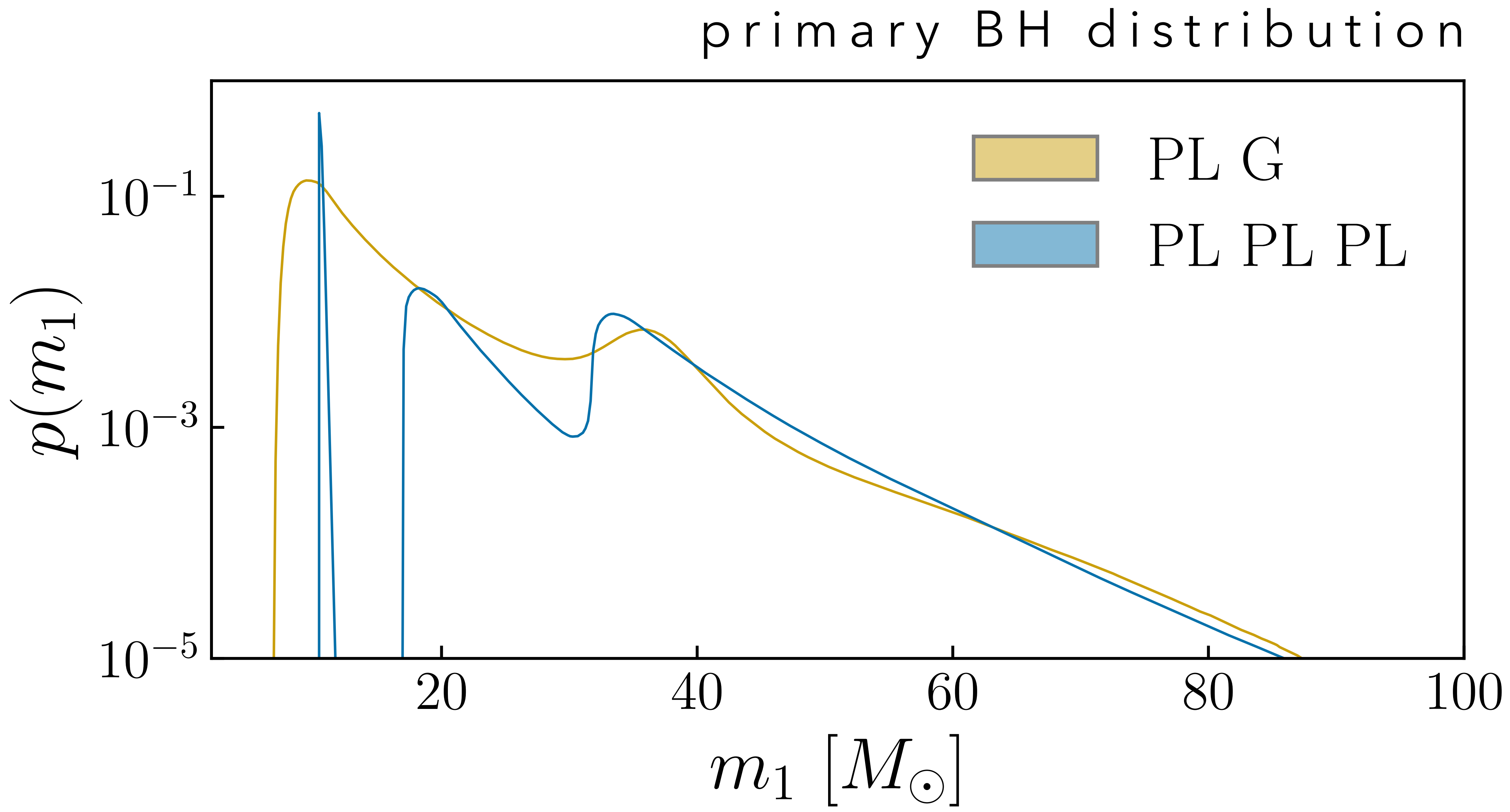
3. cosmology

SPECTRAL SIRENS IN 1 SLIDE

features in the population contain information on redshift

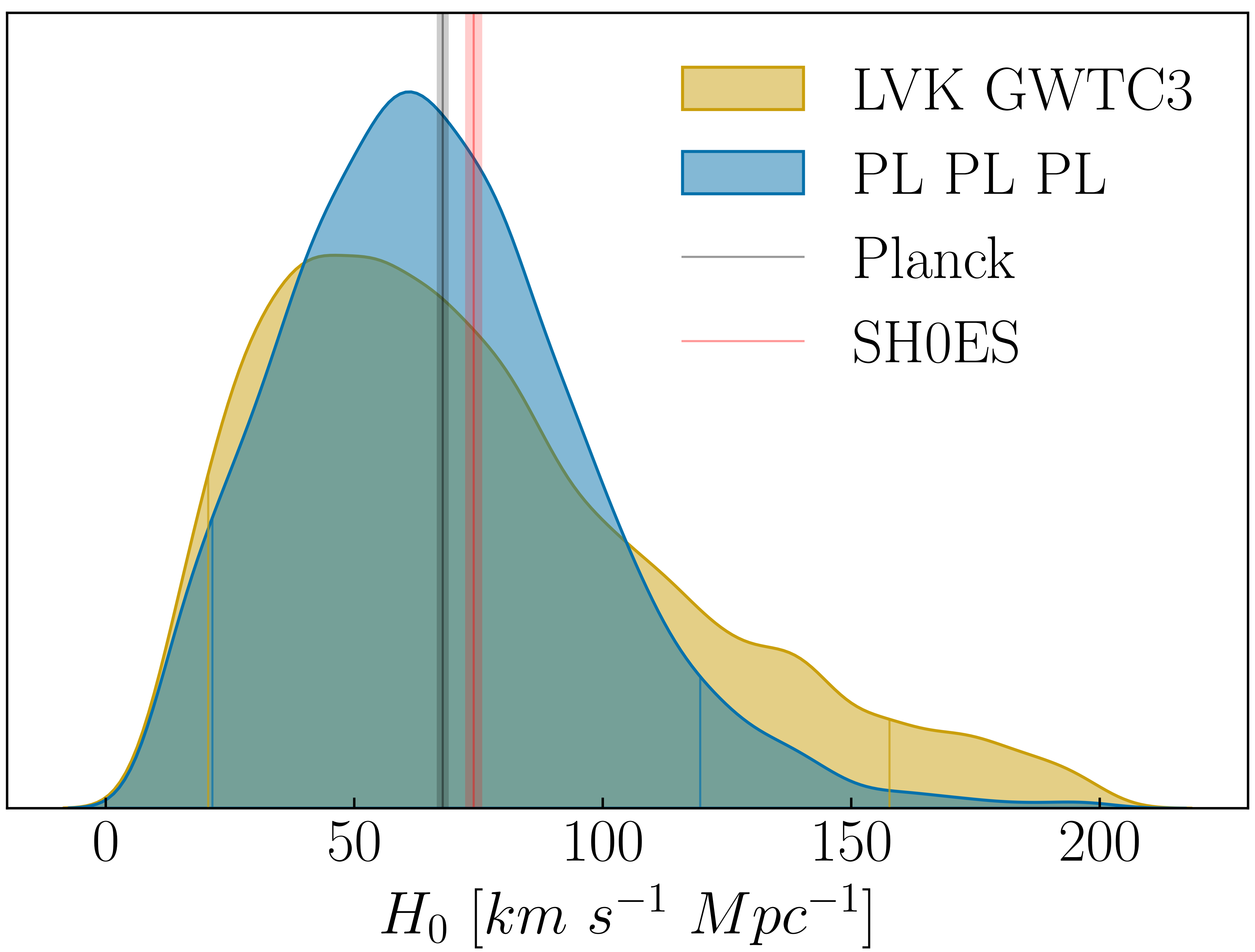


COSMOLOGY WITH 3 FEATURES



$H_0 = 66^{+91}_{-46}$ ⟶ $H_0 = 64^{+55}_{-42}$

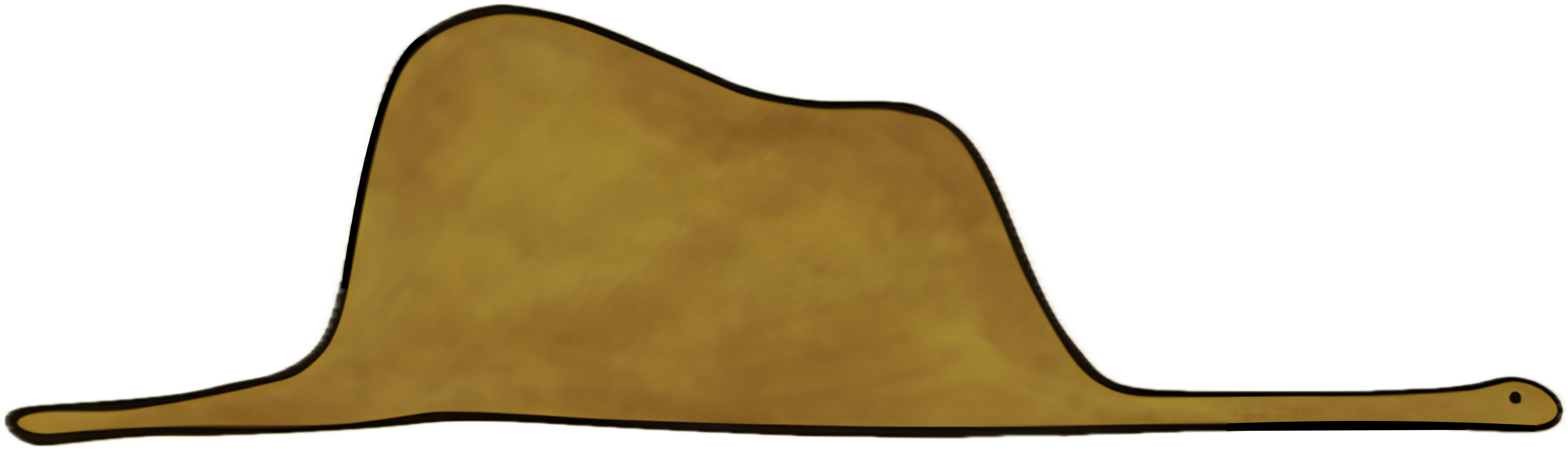
PRELIMINARY



$\sim 30\,\%$ improvement just by changing the mass model

CONCLUSIONS

- the population of black holes contains crucial information on *astrophysics* and cosmology
- the current fiducial Powerlaw-Gaussian model provides prior-dependent results
- multiple features resolve additional structure in agreement with more agnostic analyses
- such additional structure allows to improve current constraints on cosmology with only gravitational waves



ceci n'est pas une mass function.

backup slides

MODEL SETUP

$$\mathcal{L}_H(\{\mathbf{x}\}|\mathbf{\Lambda}) \propto e^{-N_{\exp}(\mathbf{\Lambda})} \prod_i^{N_{\rm gw}} T_{\rm obs} \int \mathrm{d}\boldsymbol{\theta}\mathrm{d}z\, \mathcal{L}_{\rm gw}(\mathbf{x}_i\,|\,\boldsymbol{\theta},\mathbf{\Lambda}) \boxed{\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\boldsymbol{\theta}\mathrm{d}z\mathrm{d}t_d}(\mathbf{\Lambda})}$$

$$\mathcal{R}(z\,|\,\mathbf{\Lambda})\boxed{p_{\rm pop}(\boldsymbol{\theta}\,|\,z,\mathbf{\Lambda})}\frac{1}{1+z}\frac{\mathrm{d}V_c}{\mathrm{d}z}$$

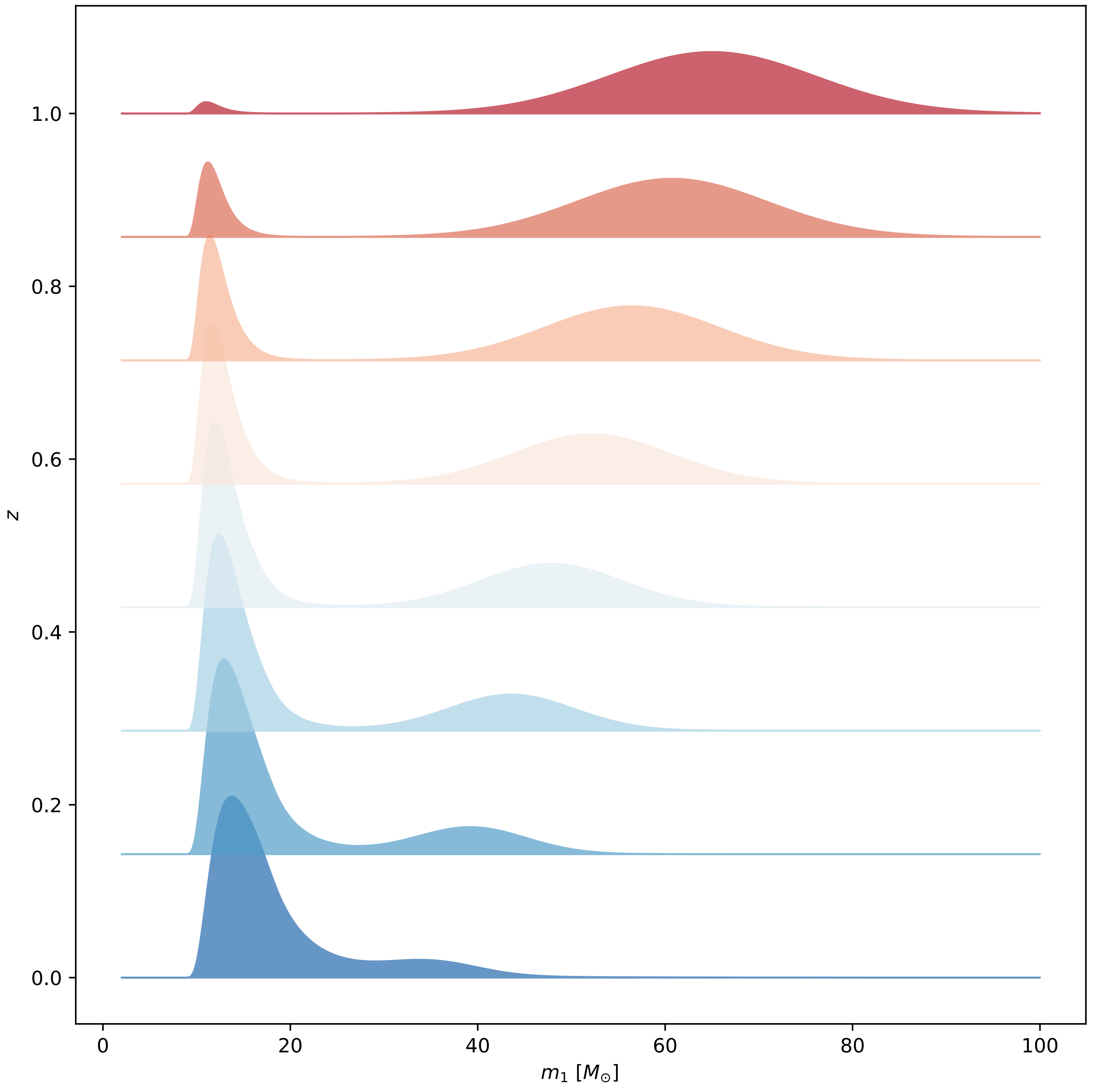
$$p_{\rm pop}(m_1,q\,|\,z,\mathbf{\Lambda}) = p_{\rm pop}(m_1\,|\,z,\mathbf{\Lambda})\,p_{\rm pop}(q\,|\,\mathbf{\Lambda})$$

- evolving parametrization
  - evolution of each feature
  - evolving mixture

$$p_{\rm pop}(m_1\,|\,z,\mathbf{\Lambda}) \sim \lambda(z)\,PL[\mathbf{\Lambda}_1(\mathbf{f}_1(z))] + [1-\lambda(z)]\,G[\mathbf{\Lambda}_2(\mathbf{f}_2(z))]$$

- we focus on **linear** redshift evolution

$$\mathbf{f}_i(z) = \mathbf{f}_{i,0} + \mathbf{f}_{i,1}\,z \qquad \lambda(z) = (\lambda_1-\lambda_0)\,z + \lambda_0$$

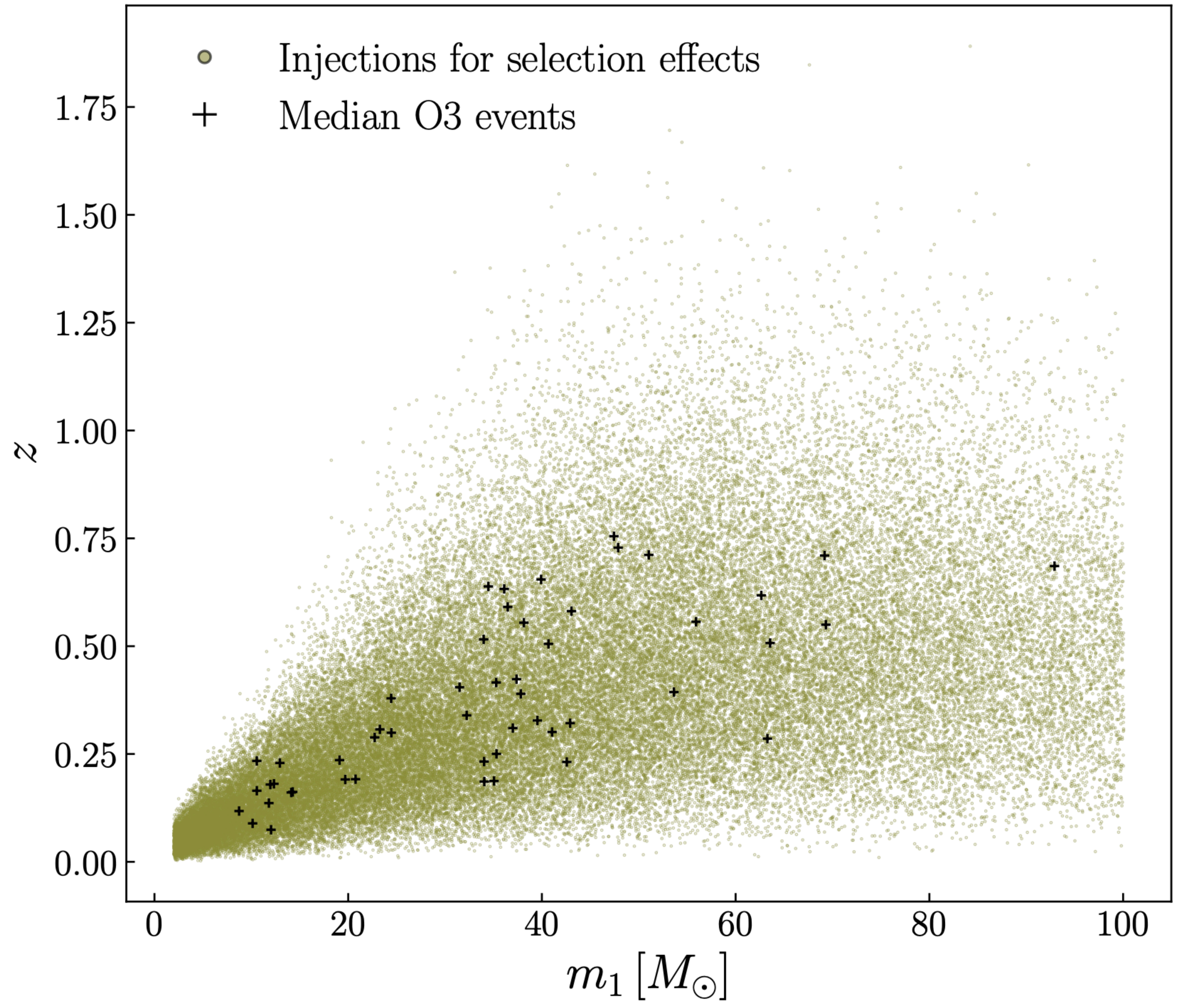


SETTINGS

- O3 data
  - 50 events (w/o GW190521 and GW190412)

- R&P O3 injections for selection effects
  - cut $IFAR > 4$

- ICAROGW pipeline
  - dynesty (2000 live points)
  - $N_{eff,PE} = 10,\ N_{eff,inj} = 200$
  - neglecting the spins
  - *fixed cosmology*

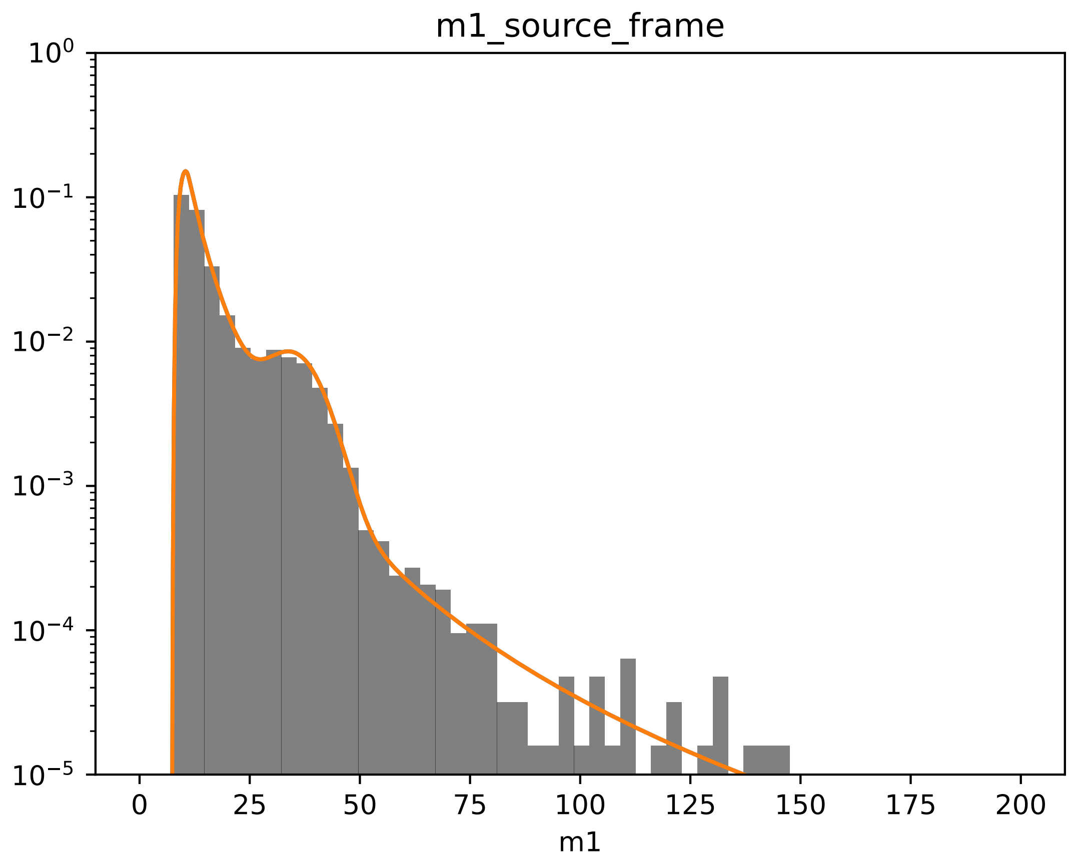


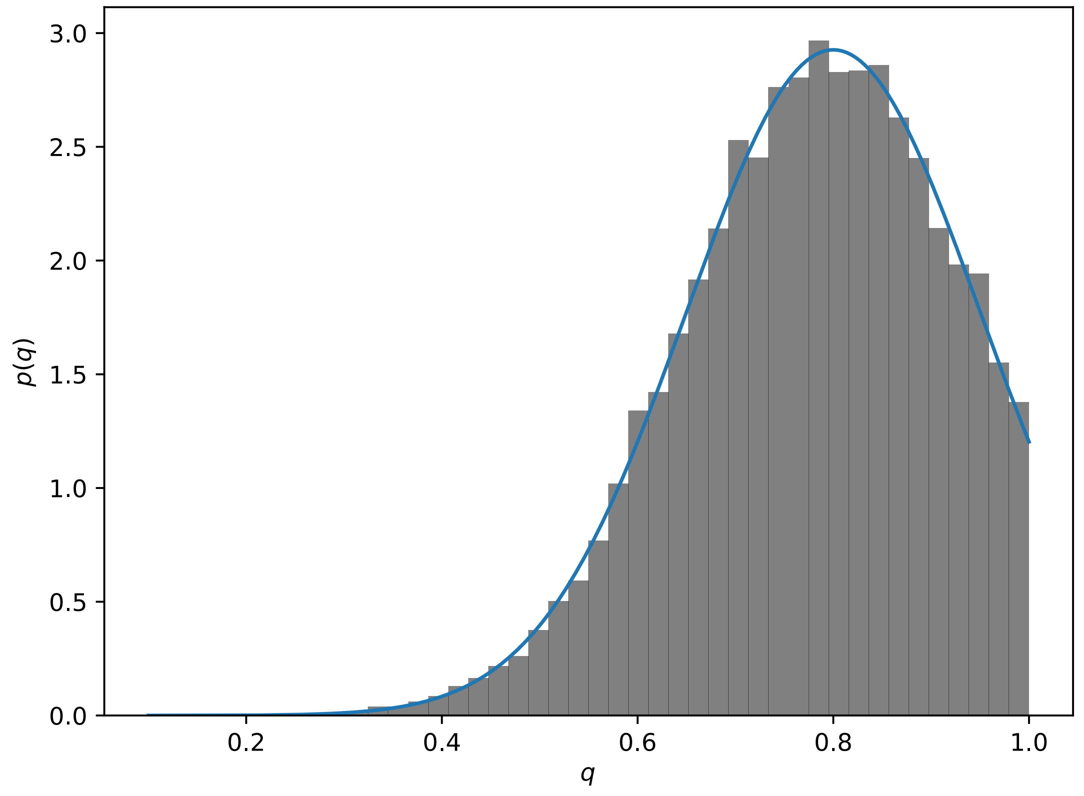
MODEL VALIDATION

α	m_{min}	m_{max}	μ_{z_0}	σ_{z_0}	mix_{z_0}	δ_m	μ_q	σ_q	γ
3.8	7	150	35	6	0.9	5	0.8	0.15	0

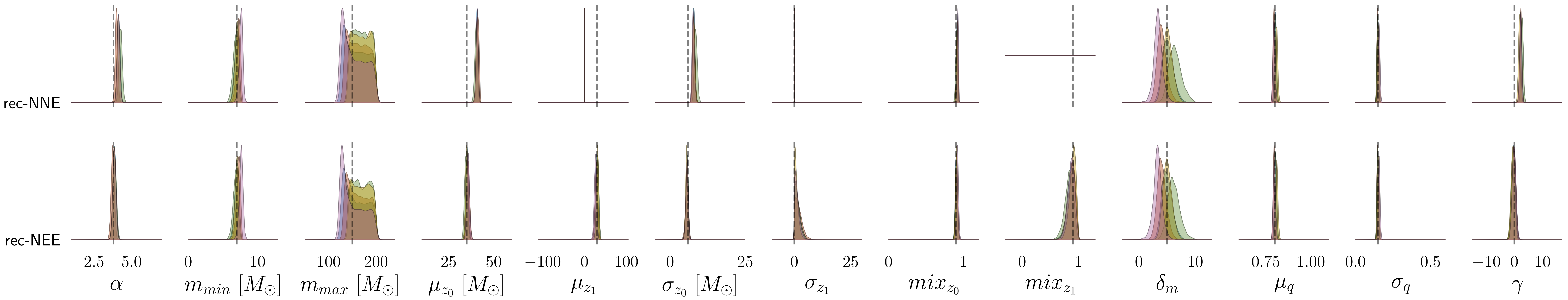
- injections from PowerLaw-Gaussian
  a. stationary $(\mu_{z_1} = 0)$
  b. evolving gaussian $(\mu_{z_1} = 30)$

- recovery
  a. non evolving $(\mu_{z_1} = 0)$
  b. evolving $(\mu_{z_1} = [-80, 80])$





$N_{events} \sim 1300$



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Primary stationary

`Powerlaw-Gaussian` wide priors

α	$m_{\min}$	$m_{\max}$	δ_m	μ	σ	mix
$(-4, 200)$	$(1, 100)$	$(30, 200)$	$(0, 10)$	$(1, 60)$	$(1, 30)$	$(0, 1)$

`Powerlaw-Gaussian` narrow priors

α	$m_{\min}$	$m_{\max}$	δ_m	μ	σ	mix
$(-4,\ 12)$	$(1, 100)$	$(30, 200)$	$(0, 10)$	$(1, 60)$	$(1,\ 6)$	$(0, 1)$

`Powerlaw-Gaussian-Gaussian`

α	$m_{\min}$	$m_{\max}$	μ_a	σ_a	μ_b	σ_b	mix_α	mix_β
$(-4, 200)$	$(1,\ 15)$	$(30, 200)$	$(10, 20)$	$(1, 30)$	$(20, 60)$	$(1, 30)$	$(0, 1)$	$(0, 1)$

`Powerlaw-Powerlaw-Gaussian`

α_a	$m_{\min_a}$	$m_{\max_a}$	α_b	$m_{\min_b}$	$m_{\max_b}$	μ	σ	mix_α	mix_β
$(-4, 200)$	$(1,\ 15)$	$(30, 200)$	$(-4, 20)$	$(10, 20)$	$(30, 200)$	$(20, 60)$	$(1, 30)$	$(0, 1)$	$(0, 1)$

`Powerlaw-Powerlaw-Powerlaw`

α_a	$m_{\min_a}$	$m_{\max_a}$	α_b	$m_{\min_b}$	$m_{\max_b}$	α_c	$m_{\min_c}$	$m_{\max_c}$	mix_α	mix_β
$(-4, 200)$	$(1,\ 15)$	$(30, 200)$	$(-4, 50)$	$(10, 20)$	$(30, 200)$	$(-4, 50)$	$(15, 60)$	$(30, 200)$	$(0, 1)$	$(0, 1)$

Primary evolving

`Powerlaw-Gaussian`

α_{z_0}	$m_{\min_{z_0}}$	$m_{\max_{z_0}}$	μ_{z_0}	σ_{z_0}	mix_{z_0}
$(-4, 200)$	$(1, 100)$	$(30, 200)$	$(1, 60)$	$(1, 30)$	$(0, 1)$
α_{z_1}	$m_{\min_{z_1}}$	$m_{\max_{z_1}}$	μ_{z_1}	σ_{z_1}	mix_{z_1}
$(-200, 200)$	$(-100, 100)$	0	$(-200, 200)$	$(0, 200)$	$(0, 1)$

`Powerlaw-Powerlaw-Powerlaw`

$\alpha_{a_{z_0}}$	$m_{\min_{a_{z_0}}}$	$m_{\max_{a_{z_0}}}$	$\alpha_{b_{z_0}}$	$m_{\min_{b_{z_0}}}$	$m_{\max_{b_{z_0}}}$	$\alpha_{c_{z_0}}$	$m_{\min_{c_{z_0}}}$	$m_{\max_{c_{z_0}}}$	$\text{mix}_{\alpha_{z_0}}$	$\text{mix}_{\beta_{z_0}}$
$(-4, 120)$	$(1,\ 20)$	$(30, 200)$	$(-4, 20)$	$(15, 25)$	$(30, 200)$	$(-4, 20)$	$(25, 60)$	$(30, 200)$	$(0, 1)$	$(0, 1)$
$\alpha_{a_{z_1}}$	$m_{\min_{a_{z_1}}}$	$m_{\max_{a_{z_1}}}$	$\alpha_{b_{z_1}}$	$m_{\min_{b_{z_1}}}$	$m_{\max_{b_{z_1}}}$	$\alpha_{c_{z_1}}$	$m_{\min_{c_{z_1}}}$	$m_{\max_{c_{z_1}}}$	$\text{mix}_{\alpha_{z_1}}$	$\text{mix}_{\beta_{z_1}}$
$(-100, 100)$	$(-100, 100)$	0	$(-100, 100)$	$(-100, 100)$	0	$(-100, 100)$	$(-100, 100)$	0	$(0, 1)$	$(0, 1)$

Mass ratio & rate evolution

`Gaussian` (truncated)

μ_q	σ_q
$(0.1, 1)$	$(0.01, 0.9)$

`Madau-Dickinson`

γ	κ	z_p	$\mathcal{R}_0$
$(-50, 30)$	$(-20, 10)$	$(0, 4)$	$(0, 100)$

O3 DATA STATIONARY SUMMARY

stationary model	$\ln \mathcal{B}$	$\mathcal{L}_{\max}$
`Powerlaw-Gaussian` wide priors	0	-1277.3
`Powerlaw-Gaussian` narrow priors	-5.2 ± 0.2	-1280.3
`Powerlaw-Gaussian-Gaussian`	-1.1 ± 0.2	-1276.2
`Powerlaw-Powerlaw-Gaussian`	-1.1 ± 0.2	-1270.9
`Powerlaw-Powerlaw-Powerlaw`	0.3 ± 0.2	-1270.9

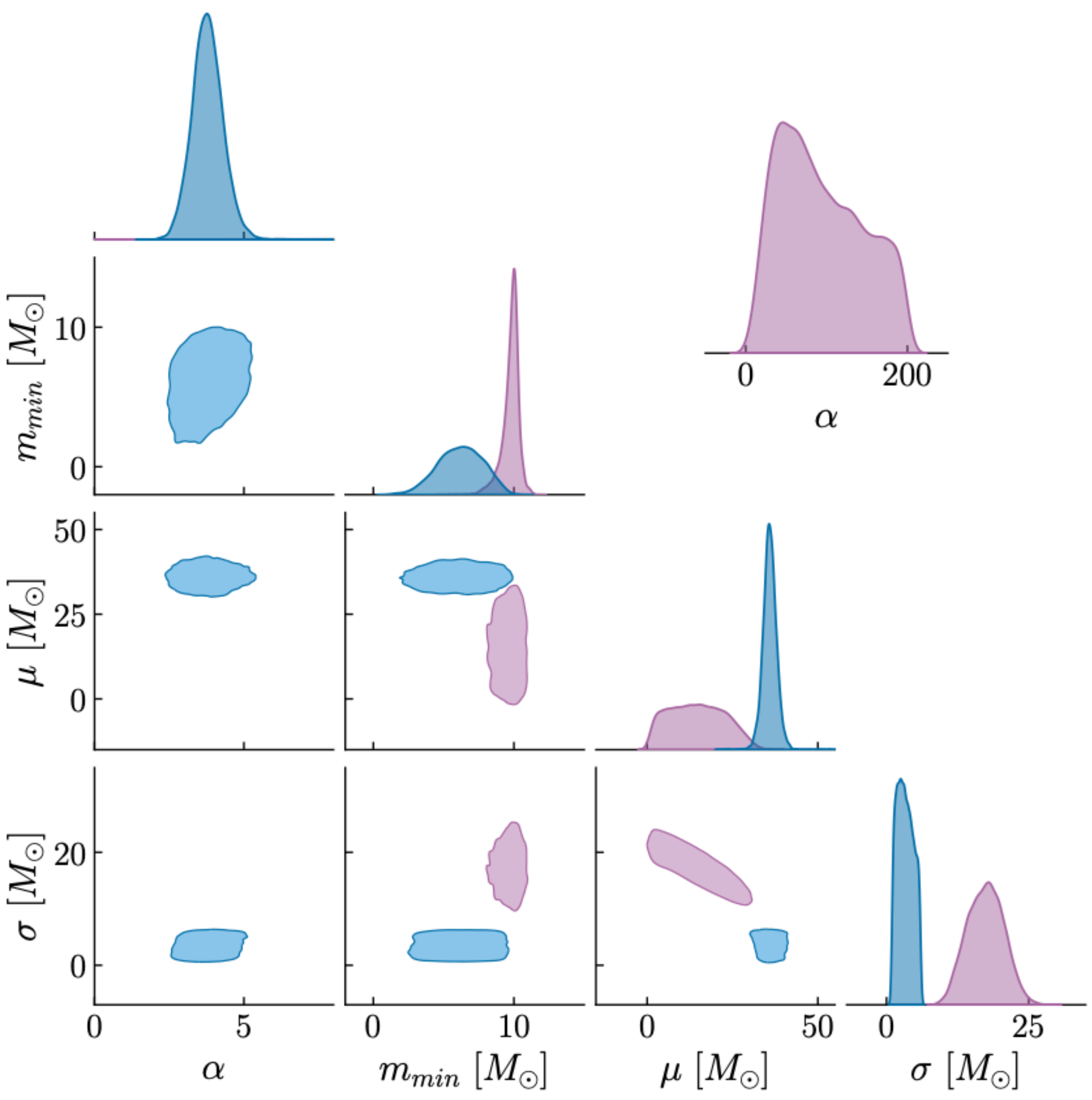
- PL-G results depend on prior bounds
  - wide priors are preferred by $\ln B \sim 5$
  - reproduced with simulation
- 3-feature models can resolve additional substructure
  - PL-PL-PL comparable with (wide priors) PL-G ($\ln B \sim 0.3$)
  - agreement with semi- and non-parametric literature

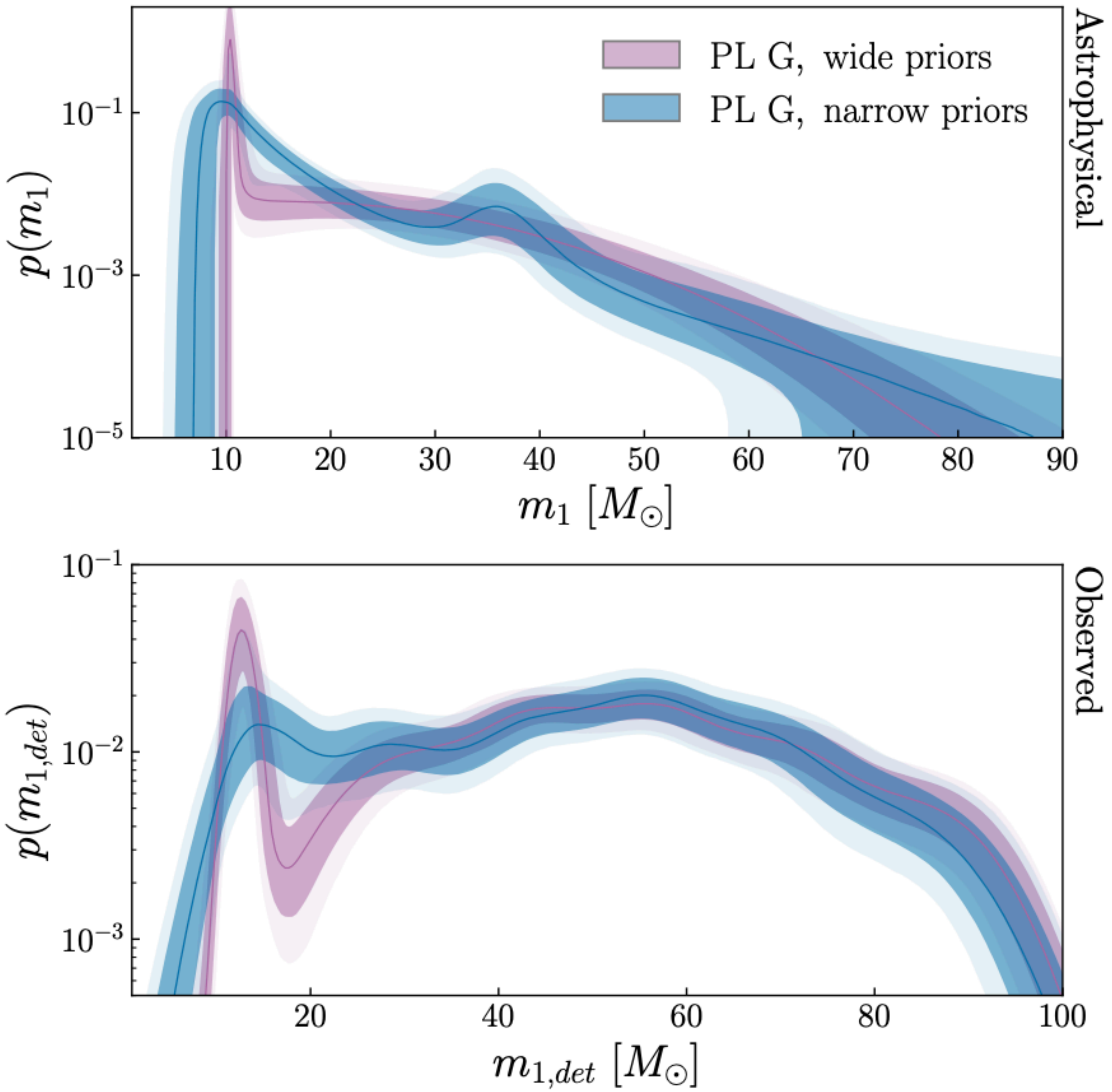
O3 DATA EVOLVING SUMMARY

evolving	$\ln \mathcal{B}$	$\mathcal{L}_{\max}$
`PL-G` stationary	0	-1276.4
`PL-G(z)`	-3.1 ± 0.2	-1276.3
`PL(z)-G`	-1.9 ± 0.2	-1275.2
`PL(z)-G(z)`	-5.1 ± 0.2	-1275.3
`PL-PL-PL` stationary	0	-1270.9
`PL-PL-PL(z)`	-1.2 ± 0.2	-1272.5
`PL-PL(z)-PL`	-2.7 ± 0.2	-1273.2
`PL(z)-PL-PL`	-1.9 ± 0.2	-1272.3
`PL(z)-PL(z)-PL(z)`	-5.6 ± 0.2	-1271.5

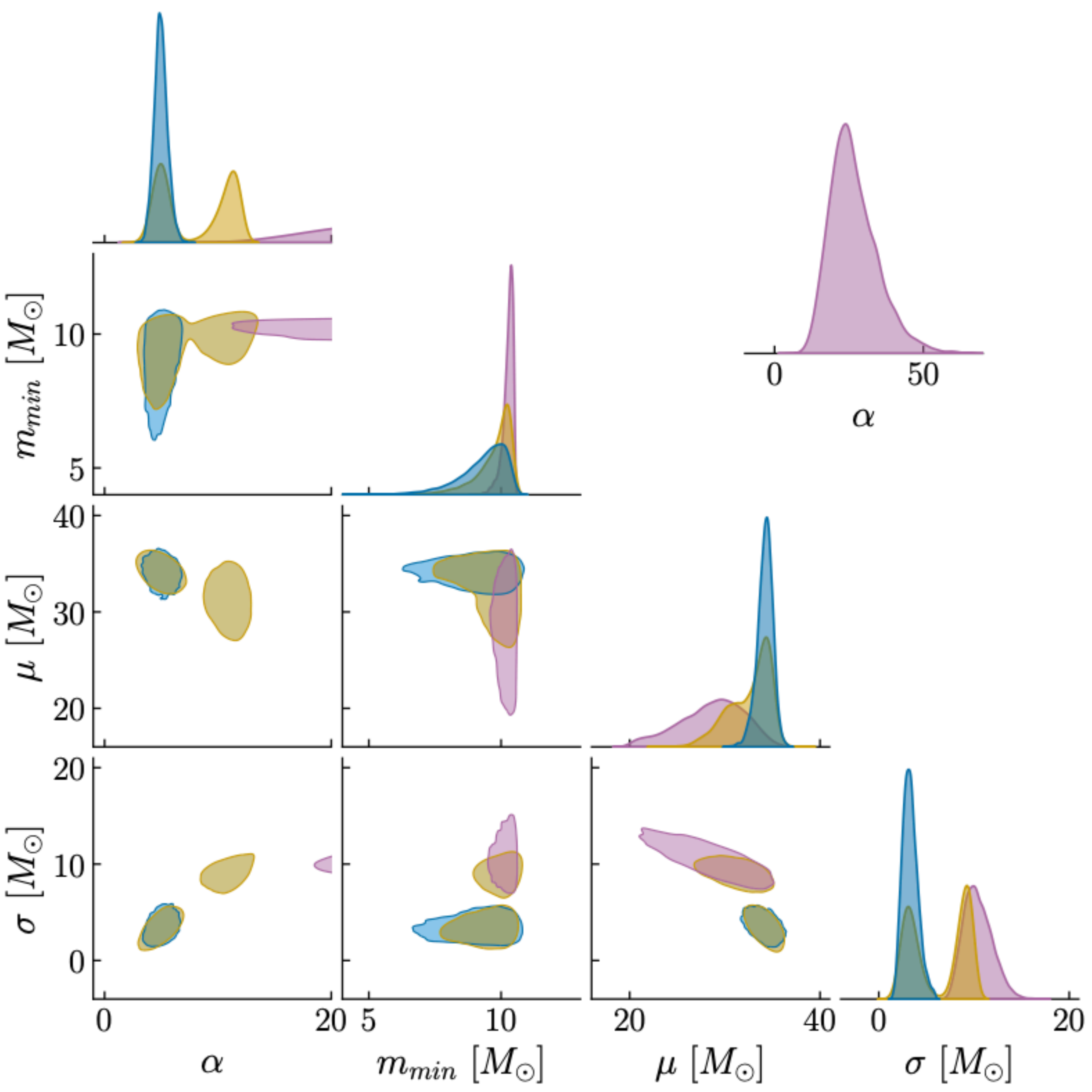
- no support for redshift evolution
  - negative BFs, evolving posteriors centered at zero
- interplay between evolution and selection effects
  - evolving features outside the detector horizon five same results

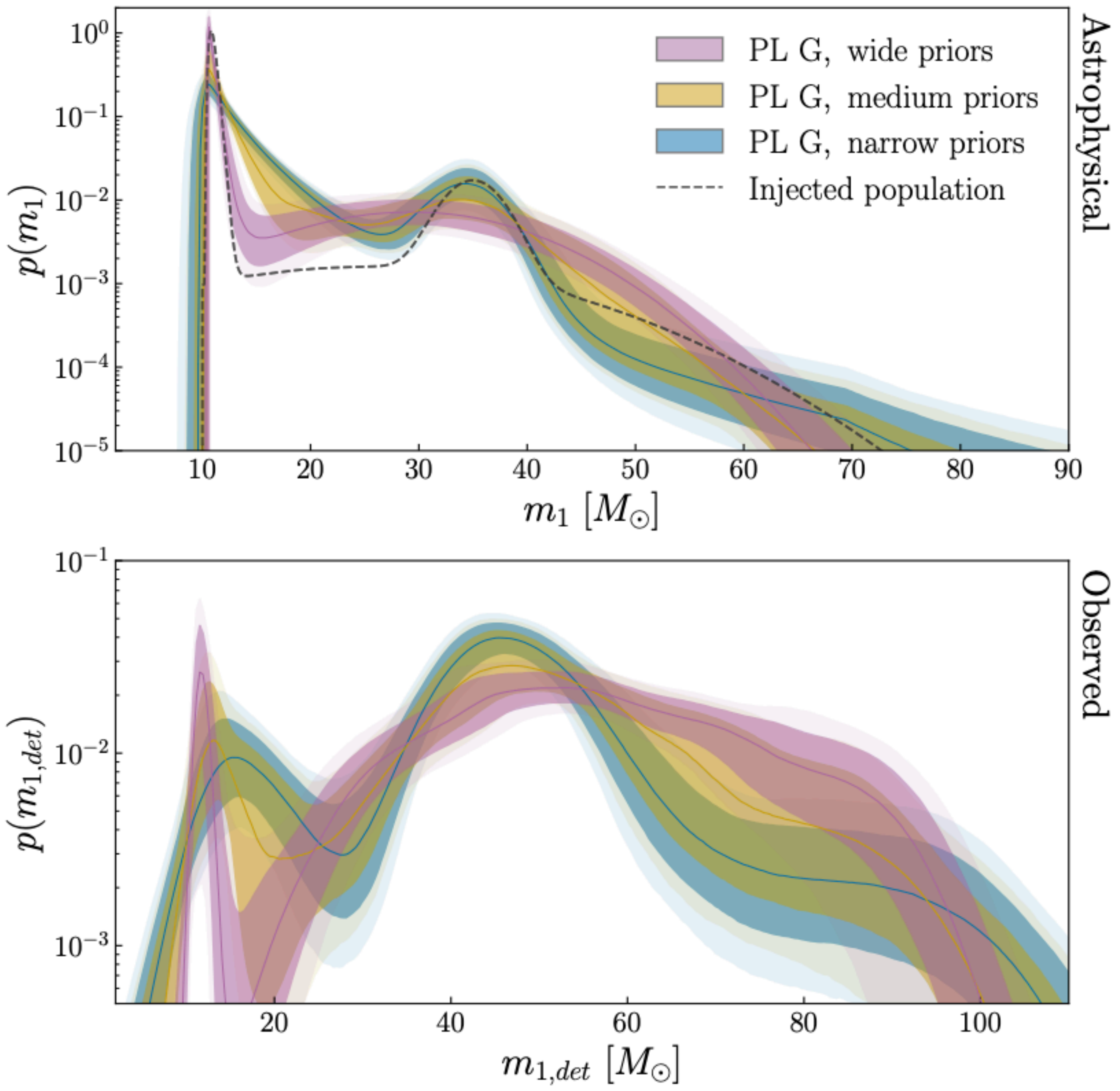
O3 DATA 2 FEATURES CORNER



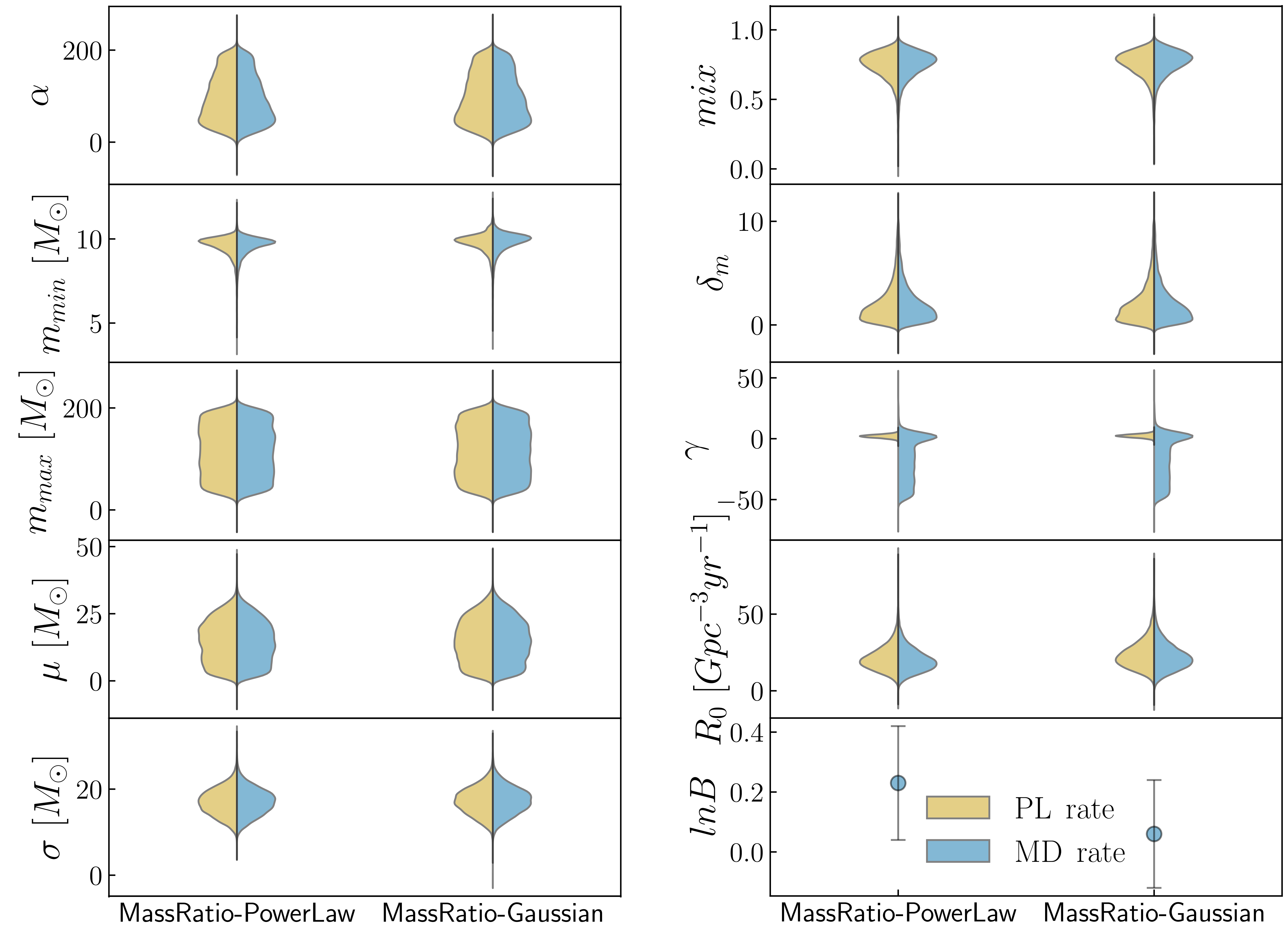


SIMULATED DATA CORNER





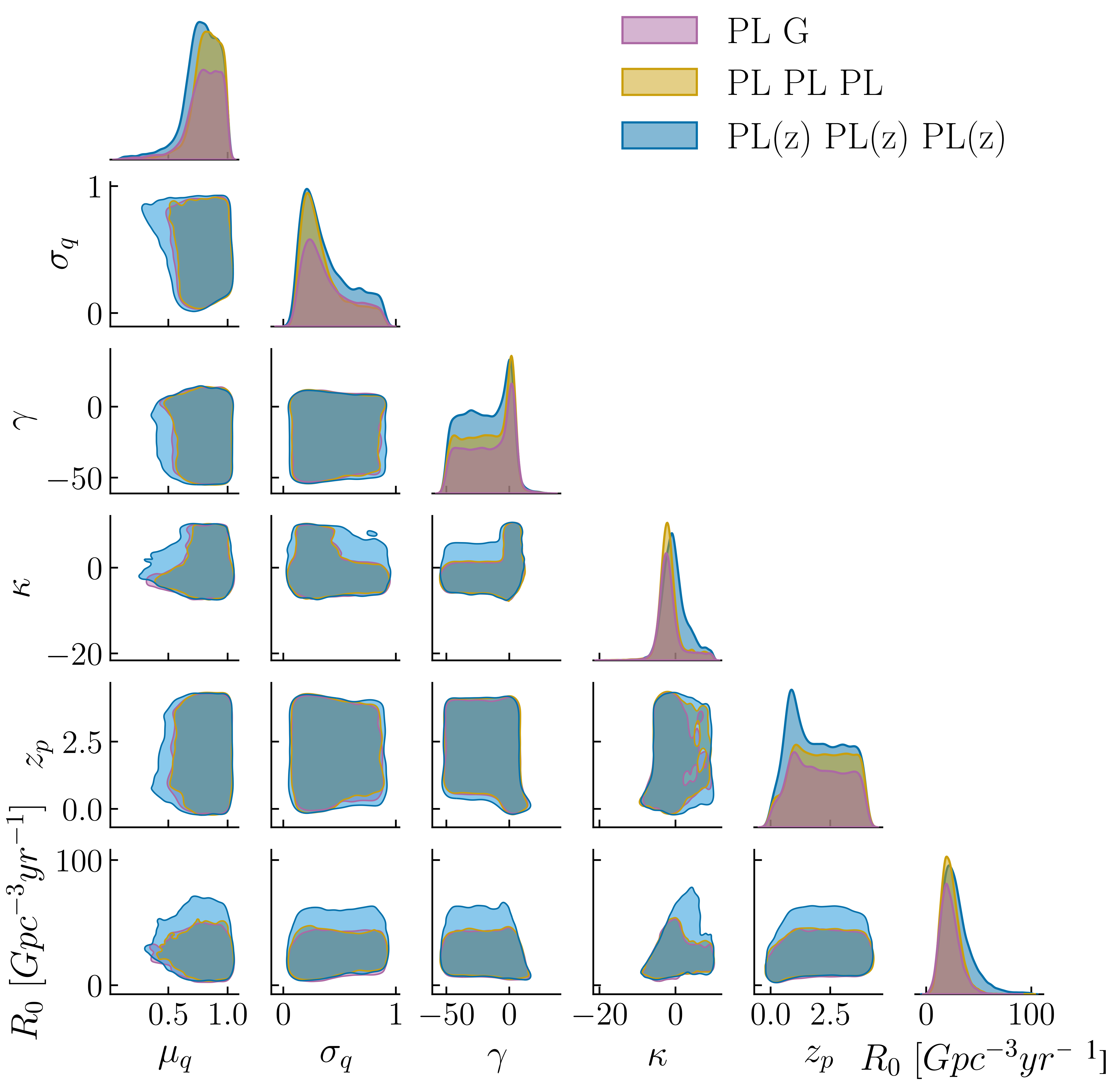
MASS RATIO & RATE EVOLUTION

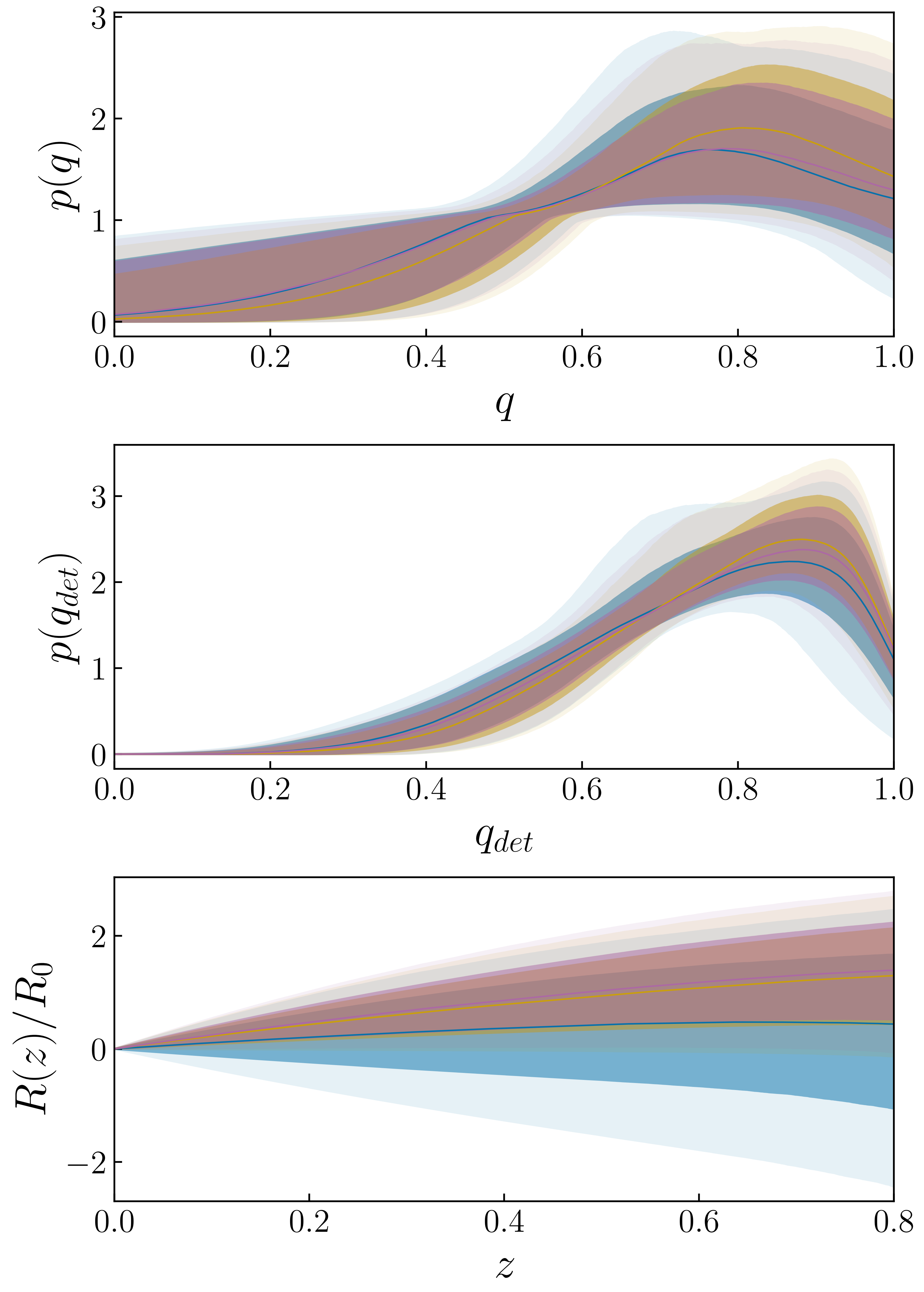


- rate evolution
  - Madau-Dickinson has and Powerlaw are comparable
- mass ratio
  - Powerlaw and (truncated) Gaussian are comparable

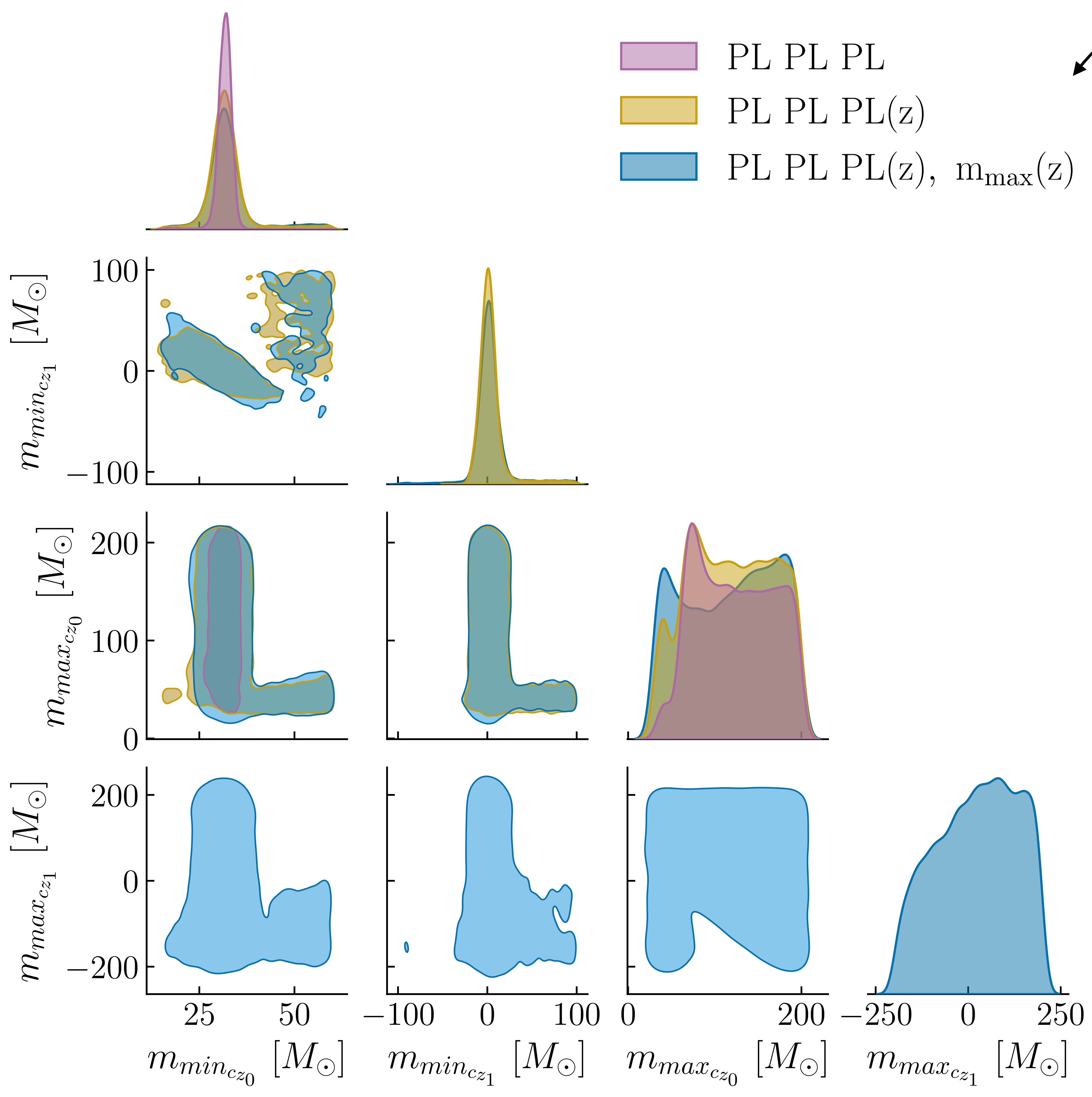
since there is no strong preference for any model, we use the Madau-Dickinson for the rate and the Gaussian for the mass ratio, because are more flexible

O3 DATA MASS RATIO & RATE EVOLUTION



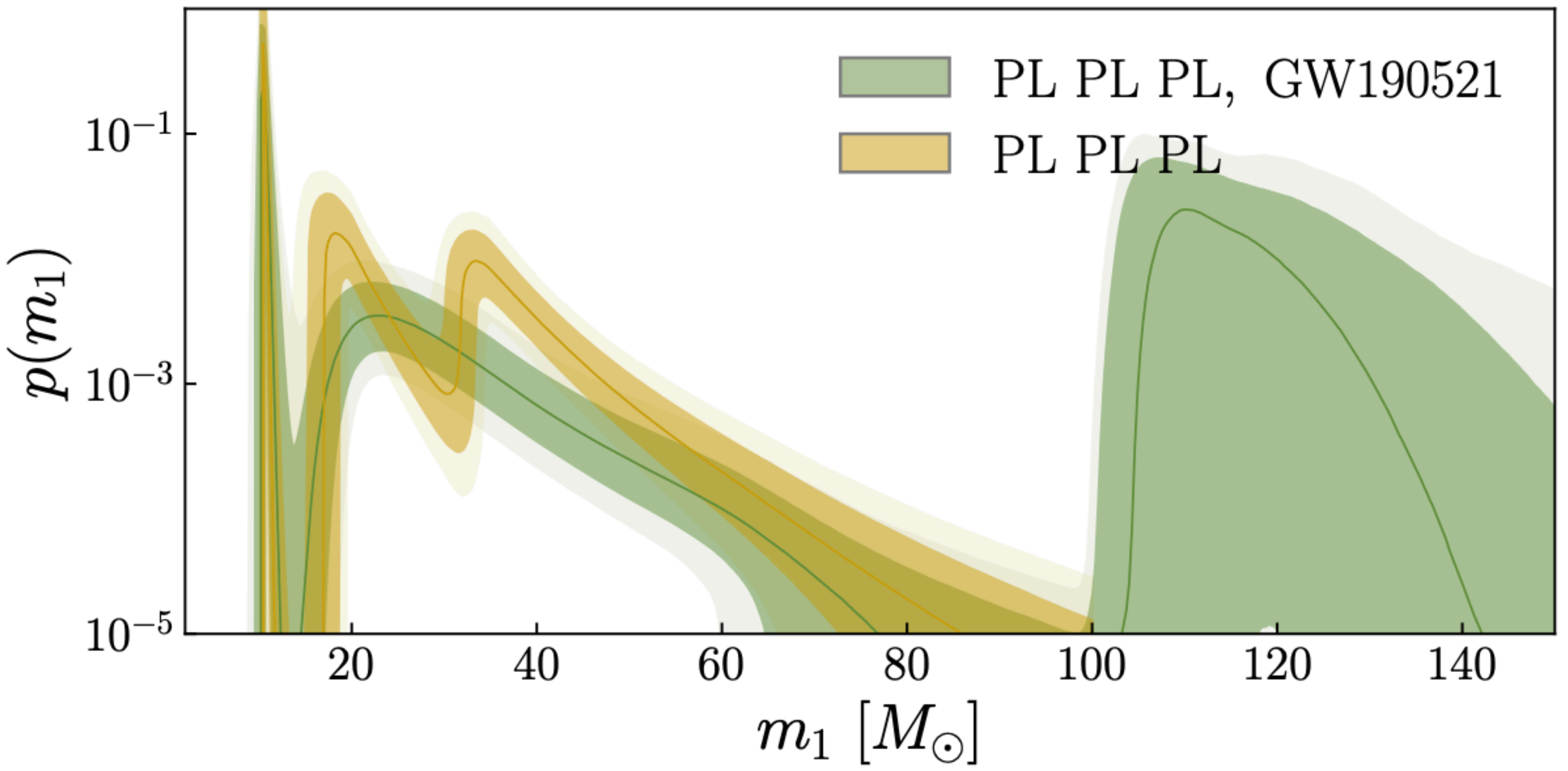


GW190521 and MAXIMUM MASS



- weak support for evolution in m_{max_c} posteriors ($\ln B = -0.1 \pm 0.2$)

- problems with the Powerlaws including GW190521



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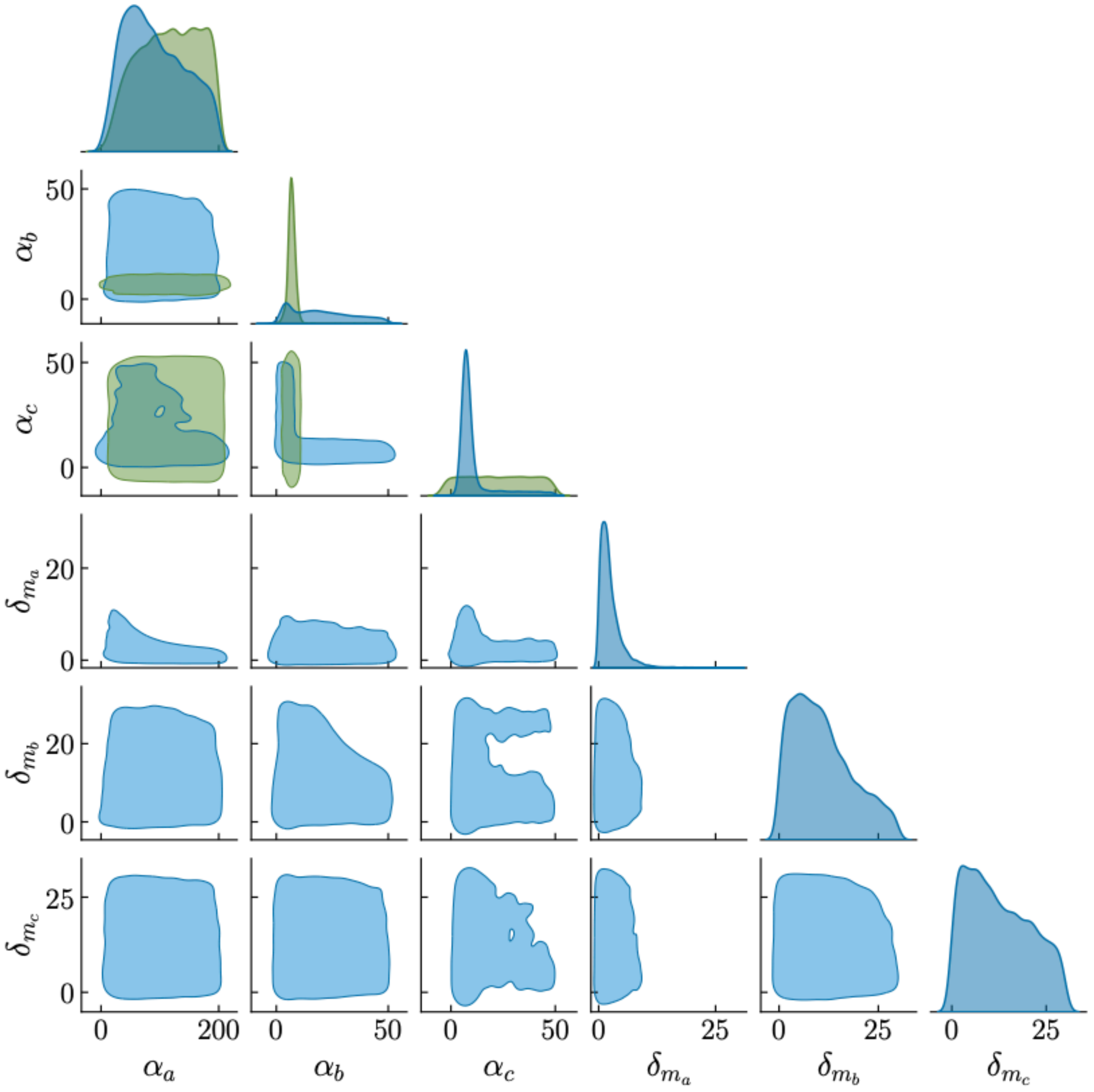
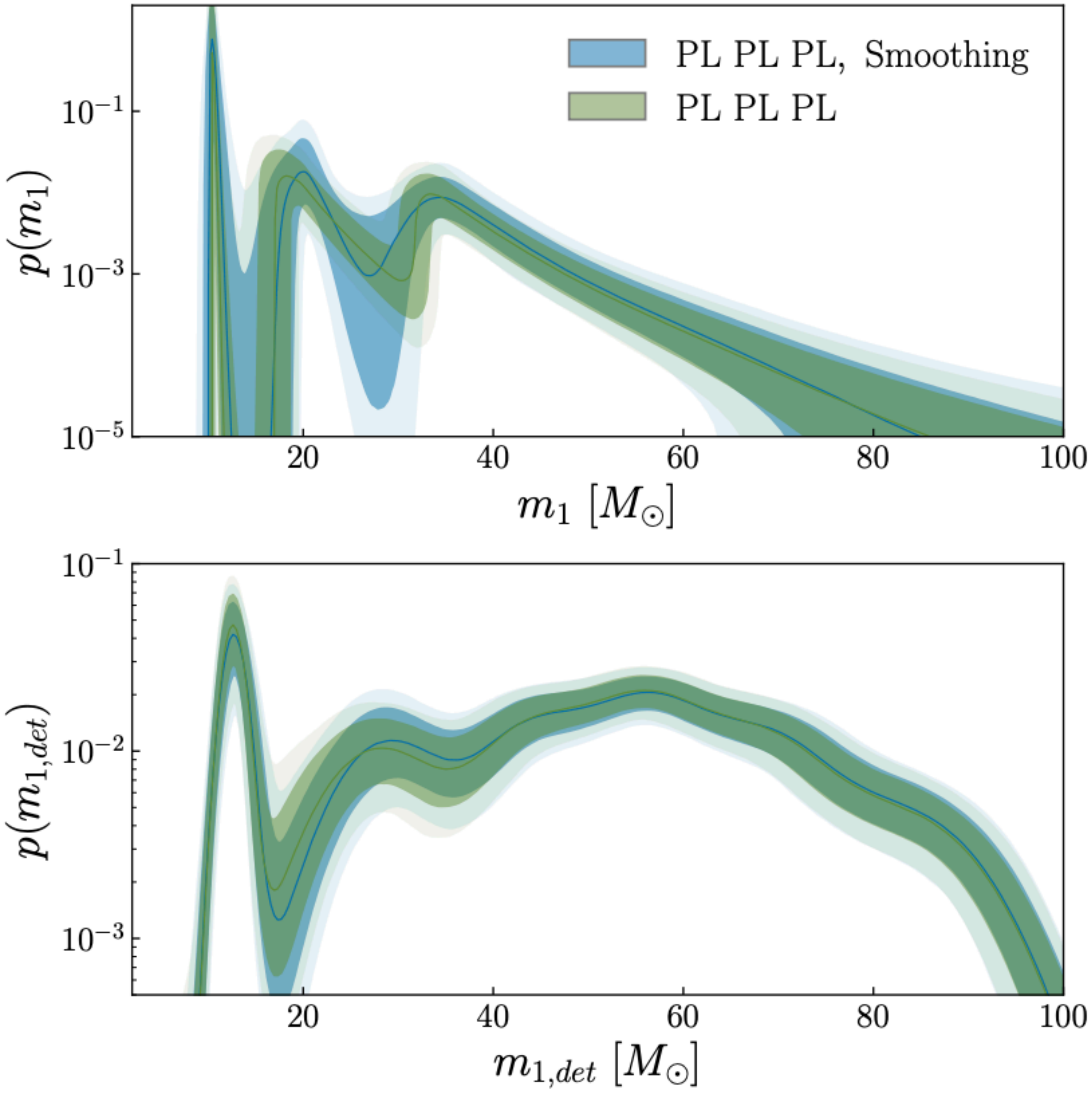
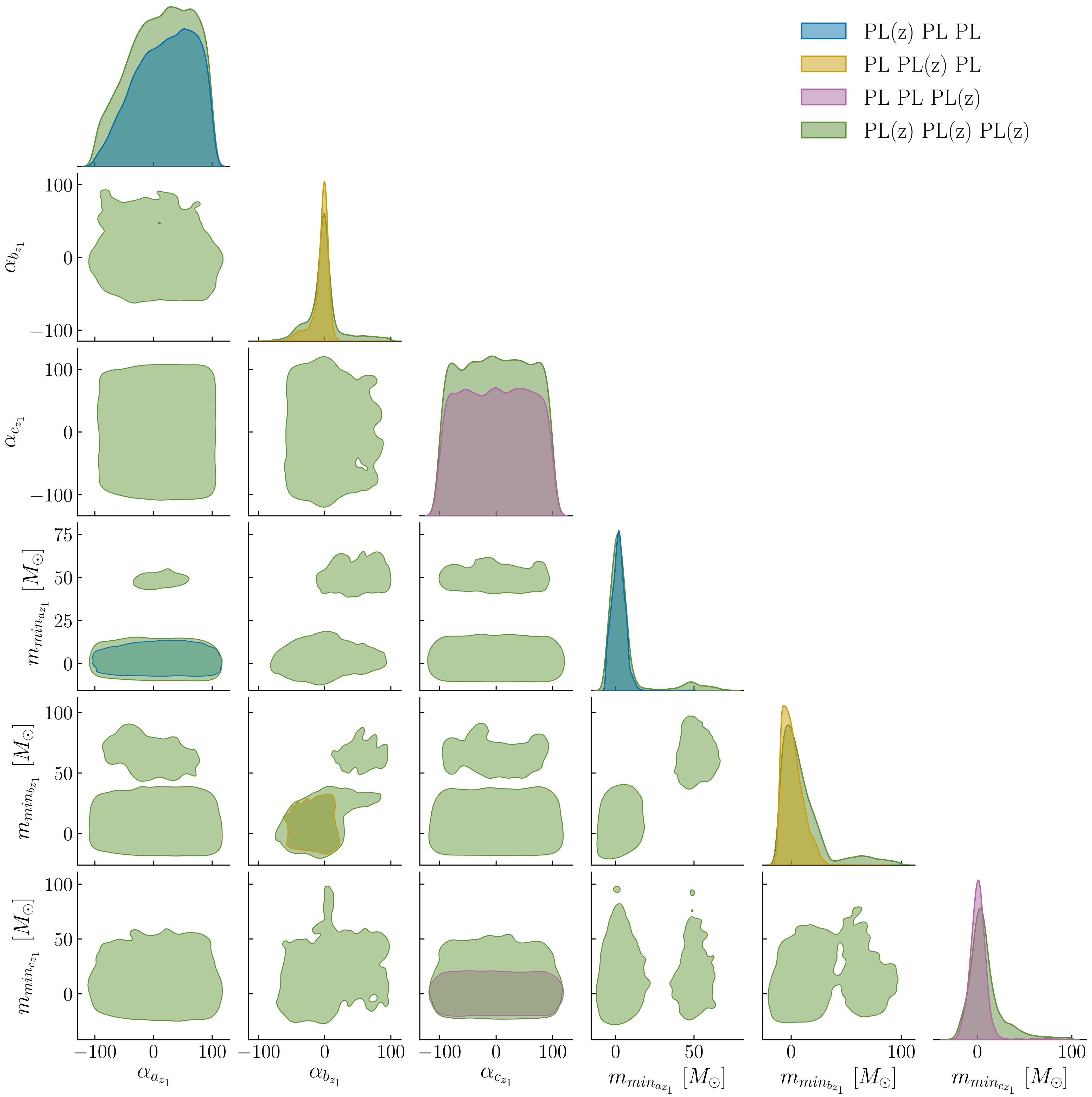
POWERLAW SMOOTHING $lnB = -1.9 \pm 0.2$

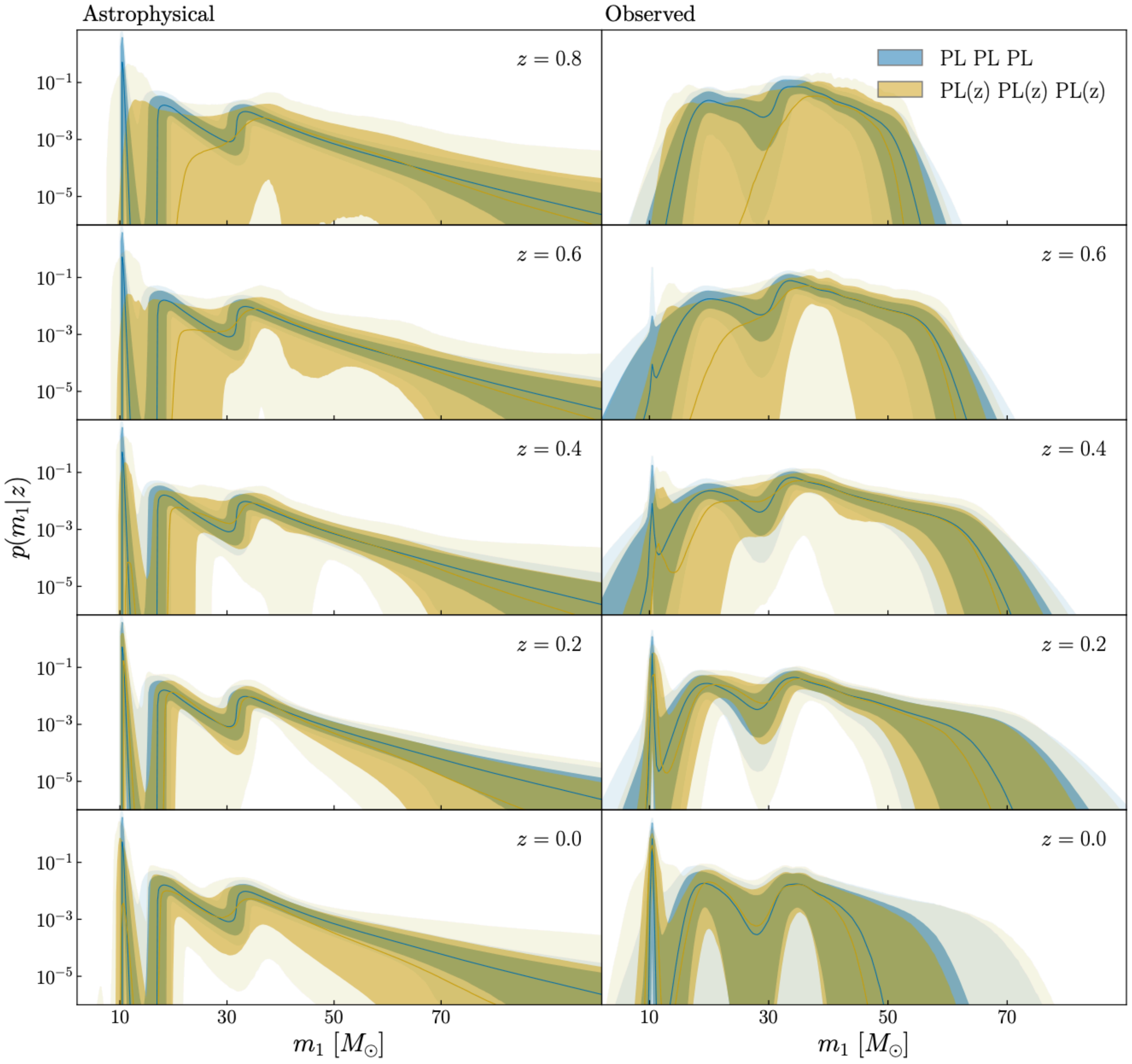




v. gennari

O3 DATA EVOLVING





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O3 DATA EVOLVING mix(z)