Theory of gravitational waves

Laura Bernard

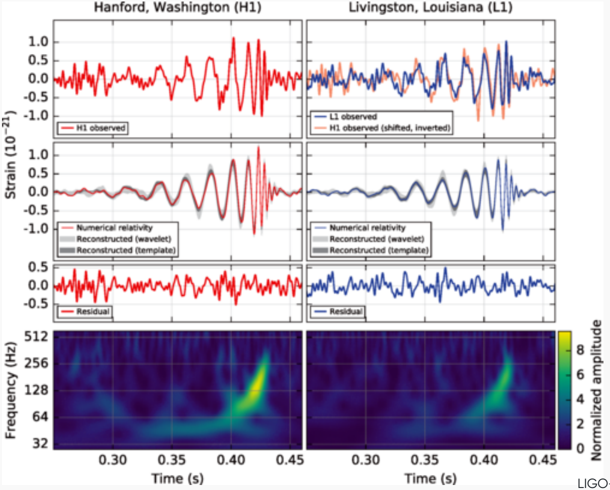
The first ACME workshop

The gravitational wave sky and complementary observations

7-11 April 2025, Toulouse

LUX Observatoire de Paris | PSL cnrs Sorbonne Université

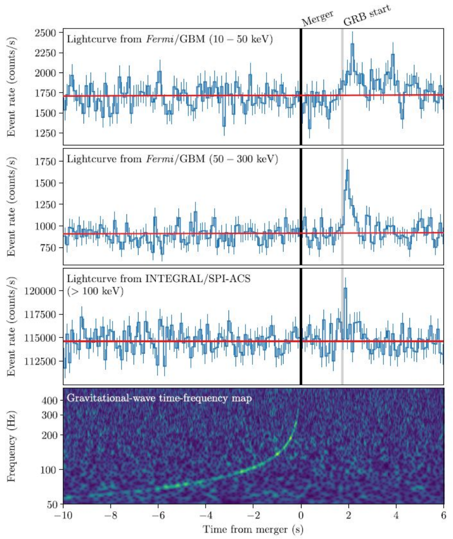
The first gravitational wave detections



LIGO-Virgo GW150914

Chirp: $\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\mathcal{M}^{5/3}f^{11/3}$, $\mathcal{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

The first gravitational wave detections



LIGO-Virgo GW170817

Chirp: $\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\mathcal{M}^{5/3}f^{11/3}\,, \qquad \mathcal{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

O4 and GW230529

LIGO/Virgo/KAGRA Public Alerts

- More details about public alerts are provided in the LIGO/Virgo/KAGRA Alerts User Guide.
- Retractions are marked in red. Retraction means that the candidate was manually vetted and is no longer considered a candidate of interest.
- Less-significant events are marked in grey, and are not manually vetted. Consult the LVK Alerts User Guide for more information on significance in O4.
- Less-significant events are not shown by default. Press "**Show All Public Events**" to show significant and less-significant events.

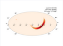
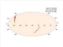
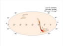
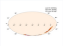
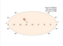
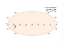
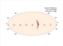
O4 Significant Detection Candidates: **142** (158 Total - 16 Retracted)

O4 Low Significance Detection Candidates: **2484** (Total)

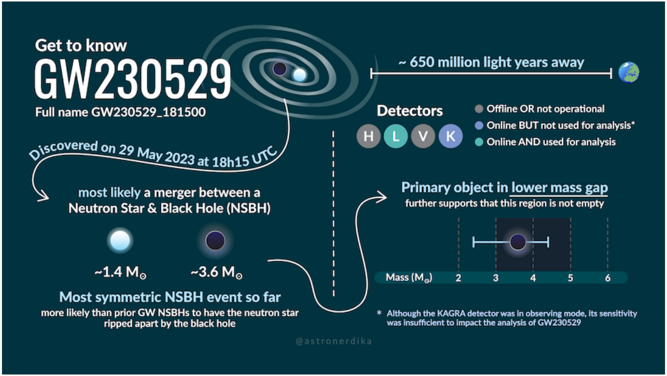
Show All Public Events

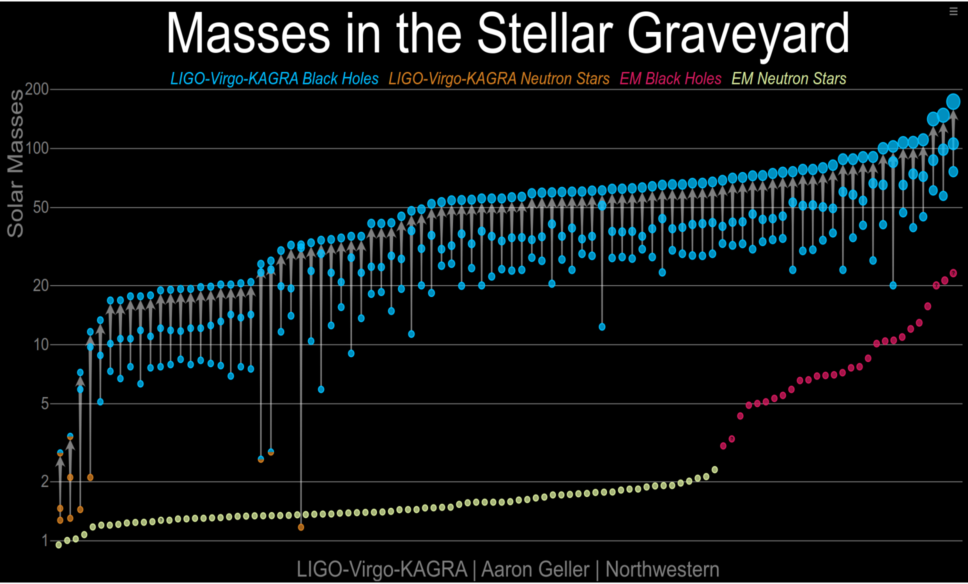
Page 1 of 11. next last »

SORT: EVENT ID (A-Z)

Event ID	Possible Source (Probability)	Significant	UTC	GCN	Location	FAR	Comments
S240930du	BBH (67%), Terrestrial (33%)	Yes	Sept. 30, 2024 23:46:14 UTC	GCN Circular Query Notices | VOE		1 per 2.4747 years	
S240930aa	BBH (>99%)	Yes	Sept. 30, 2024 03:59:59 UTC	GCN Circular Query Notices | VOE		1 per 1.0344e+11 years	
S240925n	BBH (>99%)	Yes	Sept. 25, 2024 00:58:09 UTC	GCN Circular Query Notices | VOE		1 per 7.9146e+11 years	
S240924a	BBH (>99%)	Yes	Sept. 24, 2024 00:03:16 UTC	GCN Circular Query Notices | VOE		1 per 12.869 years	
S240923ct	BBH (>99%)	Yes	Sept. 23, 2024 20:40:06 UTC	GCN Circular Query Notices | VOE		1 per 4.1462e+07 years	
S240922df	BBH (>99%)	Yes	Sept. 22, 2024 14:21:06 UTC	GCN Circular Query Notices | VOE		1 per 2.2729e+16 years	
S240921cw	BBH (>99%)	Yes	Sept. 21, 2024 20:18:35 UTC	GCN Circular Query Notices | VOE		1 per 39.517 years	
S240920dw	BBH (>99%)	Yes	Sept. 20, 2024	GCN Circular Query		1 per 3.2668e+43 years	

O4 and GW230529





Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Why do we need...

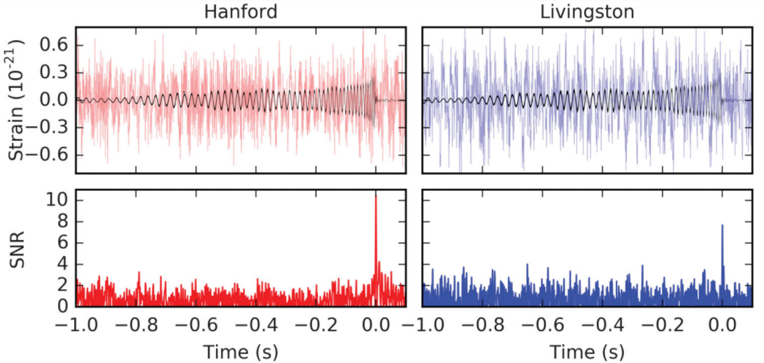
1. ...to have a bank of extremely precise waveform templates?

2. ...to use different modeling techniques?

3. ...to go beyond GR?

Why do we need...

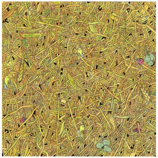
- to have a bank of extremely precise waveform templates?



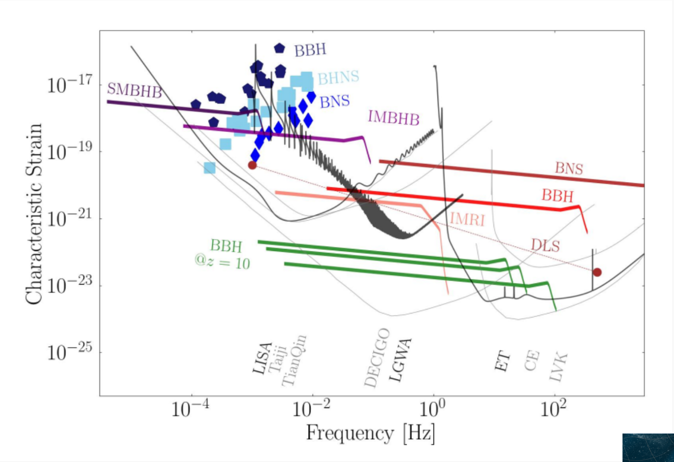
LIGO-Virgo GW151226

* chercher une aiguille dans une botte de foin

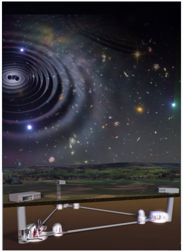
* looking for a needle in a haystack

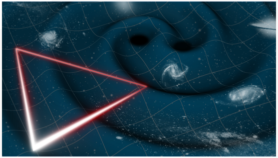


The future gravitational wave universe



Einstein Telescope blue book (2025)





Why do we need...

1. ...to have a bank of extremely precise waveform templates?
   - detection and parameter estimation

2. ... to use different modeling techniques?

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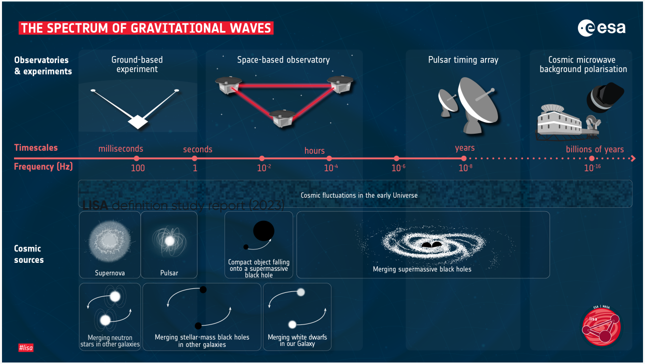
1. ...to have a bank of extremely precise waveform templates?
   - detection and parameter estimation

2. ... to use different modeling techniques?
   - very different sources



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1. ...to have a bank of extremely precise waveform templates?
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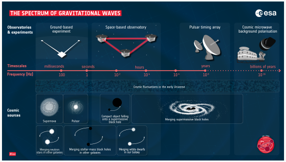
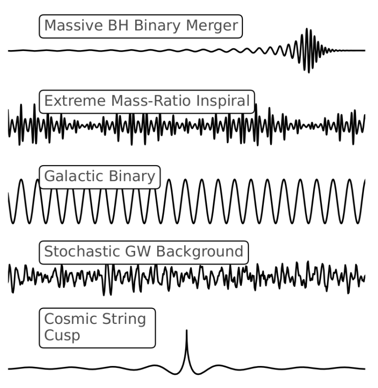
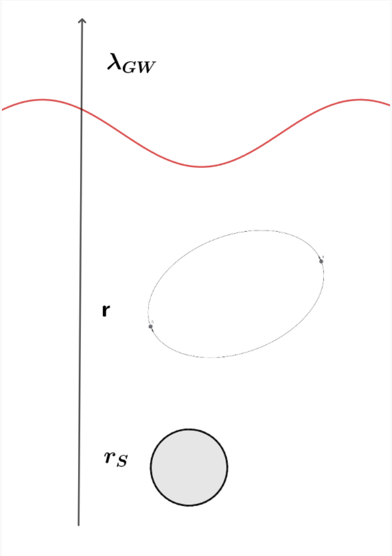


LISA definition study report (2023)



   - GR highly non linear $\Longrightarrow$ need numerical and analytical calculations

The b.a-ba of gravitational waves



The Einstein field equations

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi T^{\mu\nu}$$

The hierarchy of scales

$$r_S \ll r \sim \frac{r_S}{v} \ll \lambda_{GW} \sim \frac{r}{v}$$

The b.a-ba of gravitational waves

$$h^{\mu\nu} \equiv \sqrt{-g}\, g^{\mu\nu} - \eta^{\mu\nu}$$

$$\underbrace{\Box\, h^{\mu\nu}}_{\text{flat d'Alembertian } \eta^{\rho\sigma}\partial_\rho\partial_\sigma\, h} = \overbrace{\frac{16\pi G}{c^4}|g|T^{\mu\nu}}^{\text{matter fields}} + \underbrace{\Lambda^{\mu\nu}[h, \partial h, \partial^2 h]}_{\text{non-linearities: } \Lambda \sim h\partial^2 h + \partial h \partial h + h\partial h \partial h + \cdots}$$

matter eoms: $\nabla_\nu T^{\mu\nu} = 0$

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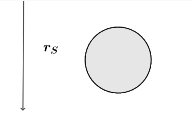
Internal zone $r \leq r_s$

- point-particle approximation

$$S_{\rm m} = -\sum_a m_a \int \mathrm{d}\tau_a$$

- finite-size effects: tides, spins



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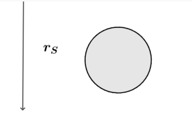

Near zone zone $\lambda_{GW} \gg r \gg r_s$

- 2-body dynamics

$$\boxed{\mathbf{a}_1 = -\frac{G\, m_2}{r_{12}^3}\mathbf{n}_{12} + \mathcal{O}\left(\frac{1}{c^2}\right)}$$

The b.a-ba of gravitational waves

Gravitational wave field

$$\nu \equiv \frac{m_1 m_2}{m_1+m_2}, \quad x = \frac{G(m_1+m_2)}{rc^2}$$

$$H_{ij}^{TT} = \frac{2G}{c^4 R} P_{ijkl}(\mathbf{N}) \left\{ \ddot{U}_{kl}\left(T - \frac{R}{C}\right) + \mathcal{O}\left(\frac{1}{c}\right) \right\}$$

$$U_{ij} = \sum_a m_a \, x_a^{<ij>}$$

Energy balance equation

$$\langle \frac{\mathrm{d}E}{\mathrm{d}t} \rangle = -\langle \mathcal{F} \rangle \qquad \text{with} \qquad \mathcal{F} \equiv \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)^{\mathrm{GW}} = \frac{G}{c^5}\left[\dddot{U}_{ij} \dddot{U}_{ij} + \mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

$$= \frac{32 c^5 \nu^2 x^5}{5G}$$

The b.a-ba of gravitational waves

Gravitational wave field

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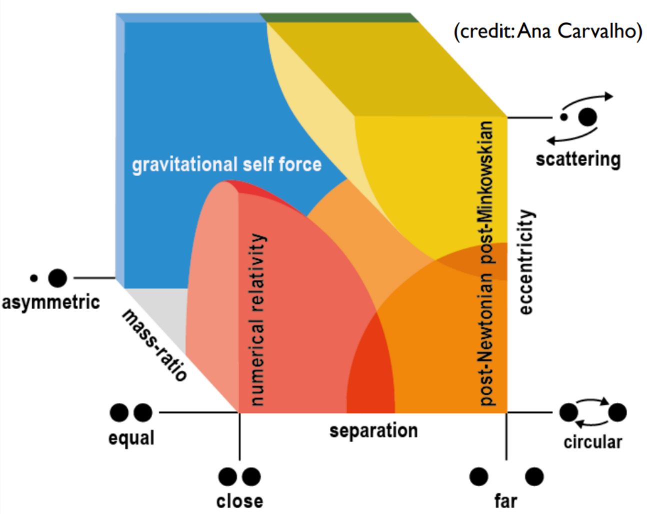
$$= \frac{32 c^5 \, \nu^2 \, x^5}{5G}$$

Dynamics period decay $\frac{\mathrm{d}P}{\mathrm{d}t}$, eccentricity $\frac{\mathrm{d}e}{\mathrm{d}t}$

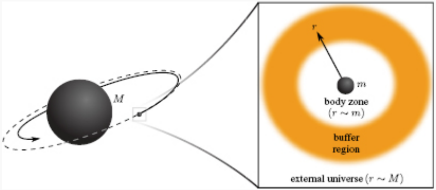
GW modes amplitude $a(t) \propto (t_c - t)^{1/4}$, phase $\phi(t) \propto (t_c - t)^{5/8}$

The different methods

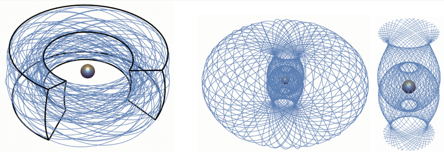


- Inspiral-Merger-Ringdown (IMR): *effective-one-body, phenomenological & surrogate models*

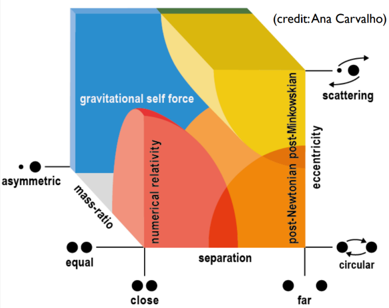
The different methods: gravitational self-force



- extreme mass ratio inspiral
- expansion in $q = \frac{m_1}{m_2} \ll 1$
- resonances, par ex. 2:3



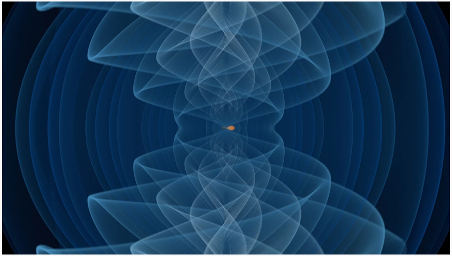
Barack & Pound '18



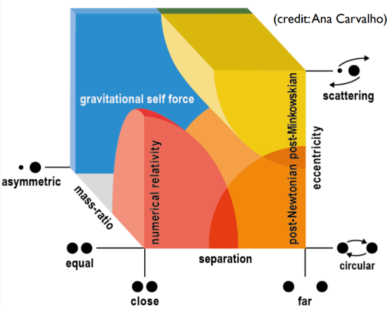
we need to go to **second order self-force** for LISA's accuracy

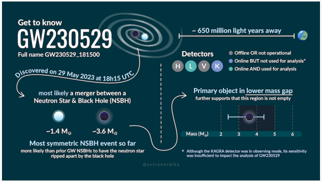
The different methods: numerical relativity

- solving the full Einstein equations
- computationally expensive
- add spins, eccentricity, *etc.*

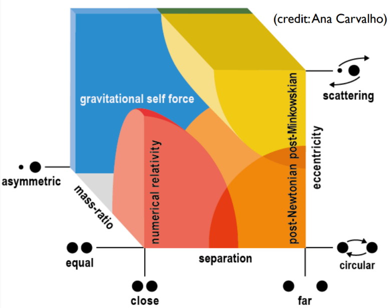


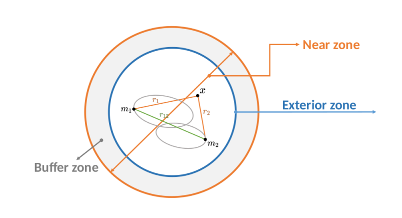
I. Markin, T. Dietrich, H. Pfeiffer, A. Buonanno (Potsdam University and Max Planck Institute for Gravitational Physics)



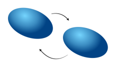


The different methods: post-Newtonian





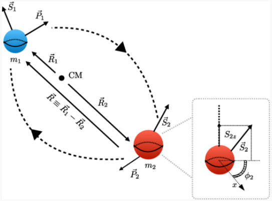
- expansion in $\epsilon = \frac{v_{12}^2}{c^2} \sim \frac{G\,m_{1,2}}{r_{12}c^2} \ll 1$
- point-particle approximation



- add spins, tides, *etc.*

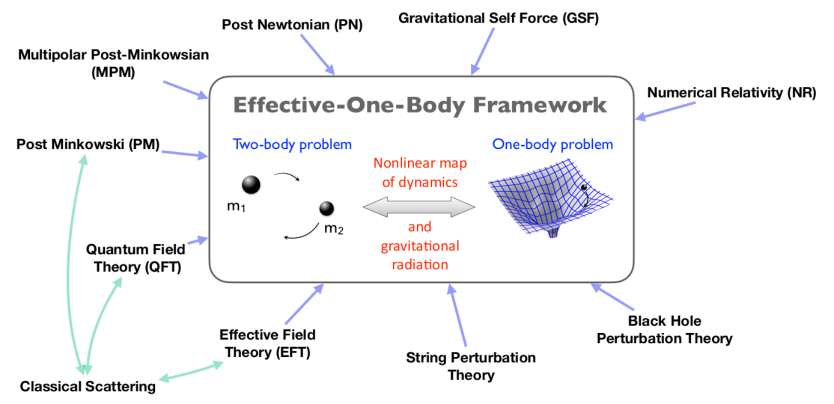
Current state-of-the-art:

- **4.5PN for p.p.**
- NNLO for tides
- $\sim$ 3PN for spins



Tanay et al. '23

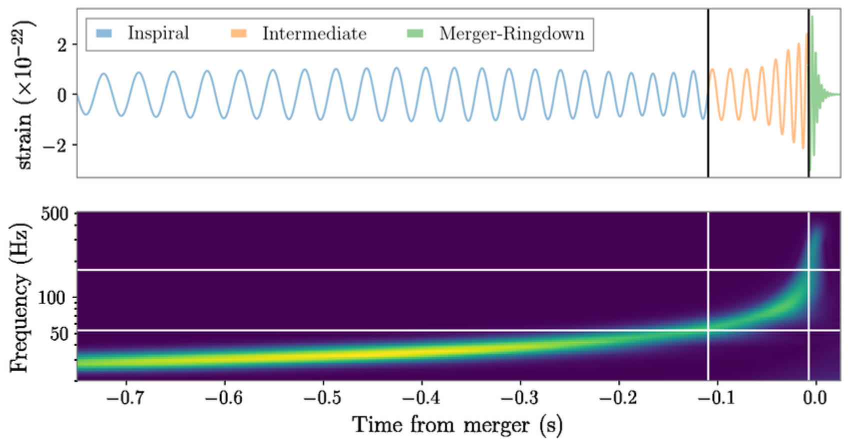
Full IMR waveform: the EOB class



LISA waveform white paper '23

Full IMR waveform: the Phenom class

$$h(f) = \mathcal{A}(f)\, e^{\psi_n(f)} \qquad \psi_n = \{\varphi_{0,..7},\, \sigma_{0..4},\, \beta_{1..3},\, \alpha_{0..5}\}$$

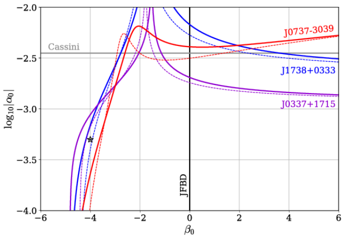


Kwok et al. '21

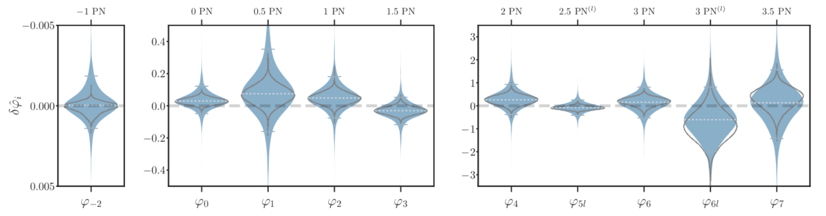
Why do we need...

1. ...to have a bank of extremely precise waveform templates?
   - detection and parameter estimation
2. ...to use different modeling techniques
   - different sources, Einstein eqs. hard to solve
3. ...to go beyond GR?
   - better understanding of GR
   - to answer fundamental physics and/or cosmology inconsistencies
   - to test the gravitational interaction

GR: a beautiful and successful theory

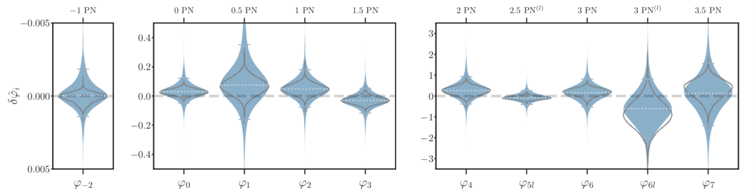


Kramer et al. '21



LIGO-Virgo '21

Limitation of current tests



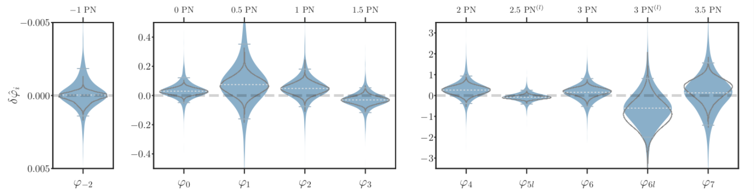
LIGO-Virgo '21

- agnostic searches: $h(f) = \mathcal{A}(f)\, \mathrm{e}^{\psi_n(f) + \delta\psi_n(f)}$, $\delta\psi_n = \{\delta\varphi_{-2,0,..7}\}$
- limited theory-specific tests, ex:
  - ST theories: 3PN dynamics, 2.5PN radiation, NNLO tides
  - EsGB, Chern-Simmons: LO correcion, including spin

But:

- ★ we can miss some deviations
- ★ no systematic way to connect deviations to new physics

Limitation of current tests



LIGO-Virgo '21

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But:

★ we can miss some deviations

★ no systematic way to connect deviations to new physics

What if we had a dictionnary?

The dictionary

$$S = \frac{\Lambda^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\left\{R + \sum_{p\geq 3}\alpha_{(p-2)}\,M_\Lambda^{2p-2}\,\left(R_{\mu\nu\rho\sigma}\right)^p\right\}$$

- M_Λ new physics scale
- m characteristic mass of the system ~ lightest component

The dictionary

$$S = \frac{\Lambda^2}{2} \int \mathrm{d}^4x \, \sqrt{-g} \left\{ R + \sum_{p\geq 3} \alpha_{(p-2)} \, M_\Lambda^{2p-2} \, \left(R_{\mu\nu\rho\sigma}\right)^p \right\}$$

- M_Λ new physics scale

$$\boxed{h \sim h_{\mathrm{GR}} \cdot \delta \cdot \left(\frac{v^2}{c^2}\right)^{3p-4,\,5^*} \cdot \left(\frac{M_\Lambda}{m}\right)^{2p-2} \cdot \alpha_{(p)}}$$

- m characteristic mass of the system ~ lightest component
  - stronger for low mass systems: higher curvature
  - stronger in the inspiral
- leading correction at $3p - 4$ or 5PN (tides) if no extra degree of freedom

The dictionary

$$S = \frac{\Lambda^2}{2} \int d^4x \sqrt{-g} \left\{ R + \sum_{p \geq 3} \alpha_{(p-2)} M_\Lambda^{2p-2} \left(R_{\mu\nu\rho\sigma}\right)^p \right\}$$

- M_Λ new physics scale

$$\boxed{h \sim h_{\rm GR} \cdot \delta \cdot \left(\frac{v^2}{c^2}\right)^{3p-4,\, 5^*} \cdot \left(\frac{M_\Lambda}{m}\right)^{2p-2} \cdot \alpha_{(p)}}$$

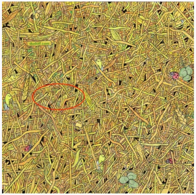
- m characteristic mass of the system $\sim$ lightest component
  - stronger for low mass systems: higher curvature
  - stronger in the inspiral
- leading correction at $3p-4$ or 5PN (tides) if no extra degree of freedom

★ systematic way to **distinguish new physics from systematics**

★ can be **directly integrated** into existing data analysis methods

Conclusion: we need...

1. ...to have a bank of **extremely precise** waveform templates
   - detection and parameter estimation
2. ...to use **different modeling techniques**
   - different sources, Einstein eqs. hard to solve
3. ...to go **beyond GR**
   - better understanding of gravity, fundamental physics and cosmology questions



Thank you!